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Modeling Low-Frequency Hamiltonian Noise

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Noisy Quantum Evolution

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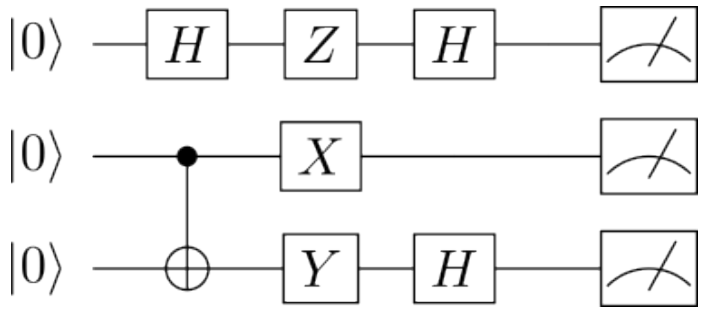
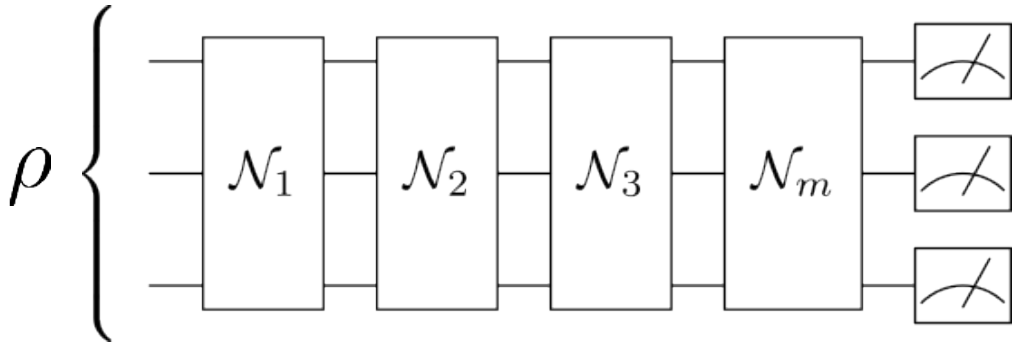
What We Want



What we get:



- Real-world quantum processors never implement the exact operations we want.
- Instead of perfect state preparation in the ground state we get some mixed state ρ
- Instead of a sequence of specified unitaries we get some noisy non-unitary evolution $\mathcal{N}$
- Measurement operations themselves can introduce noise and decoherence
- Characterization through tomographic protocols is one of the most powerful tools we have for understanding the dynamics of as-built quantum computers, but nearly all of the most widely used techniques make the assumption that the dynamics of a system are Markovian.

Non-Markovianity

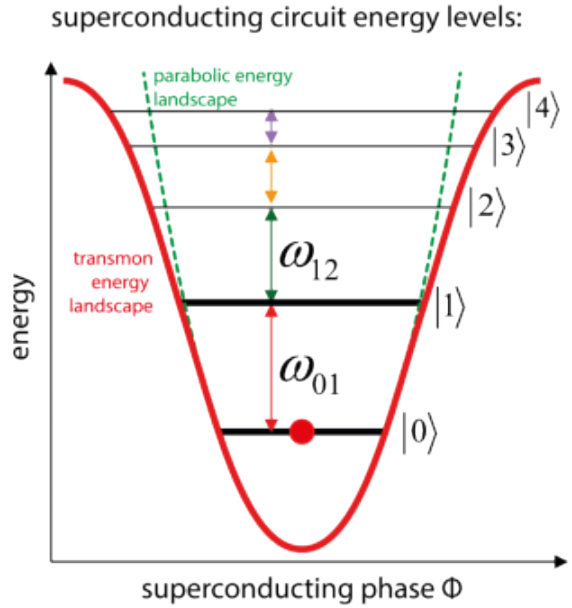
- A process that acts on the state ρ is Markovian if the state of the system after the process, ρ', is entirely determined by ρ.

$$G : \rho \rightarrow \rho'$$

- Once characterized, no additional context information beyond G's identity is required to know how it acts on a state.
- G must by linear, completely positive and trace preserving. i.e. it can be written as a process matrix.
- This is not the only definition of Markovianity that exists in the literature (alternative definitions are more narrow).
- The dynamics of real quantum computers in the laboratory are nearly never purely Markovian.
- This is detectable using tools from statistics for evaluating model violation, and model violation is regularly seen even when fitting the largest possible class of Markovian models supported by our methods, indicating the presence of truly non-Markovian dynamics

Common Forms of Non-Markovianity

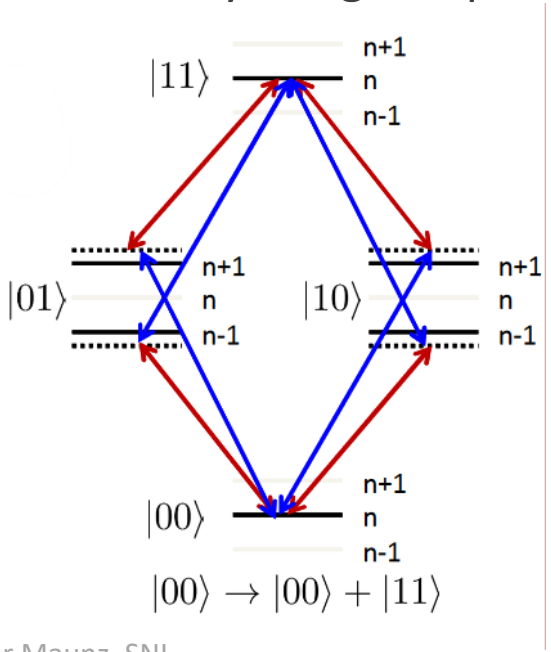
- Leakage: In many platforms the $|0\rangle$ and $|1\rangle$ states are simply the two lowest energy levels of many. Dynamics can drive a system out of this qubit subspace resulting in leakage to higher levels.



Credit: https://blog.qutech.nl/2017/08/13/how-to-make-artificial-atoms-out-of-electrical-circuits-part-ii-circuit-quantum-electrodynamics-and-the-transmon/

- Serial Context Dependence: The behavior of a gate often depends on which gates/operations preceded it.

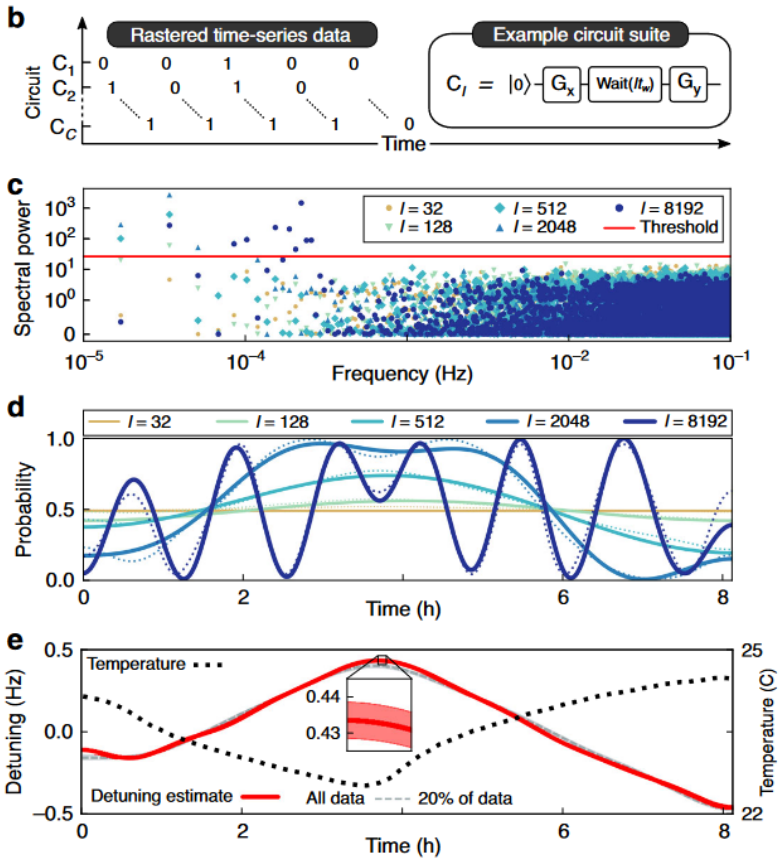
- In trapped ion systems Molmer-Sorenson gates are used as two-qubit native entangling operations. Because of heating the performance of an MS gate depends on how many MS gates preceded it.



Credit: Peter Maunz, SNL
https://www.osti.gov/servlets/purl/1428158

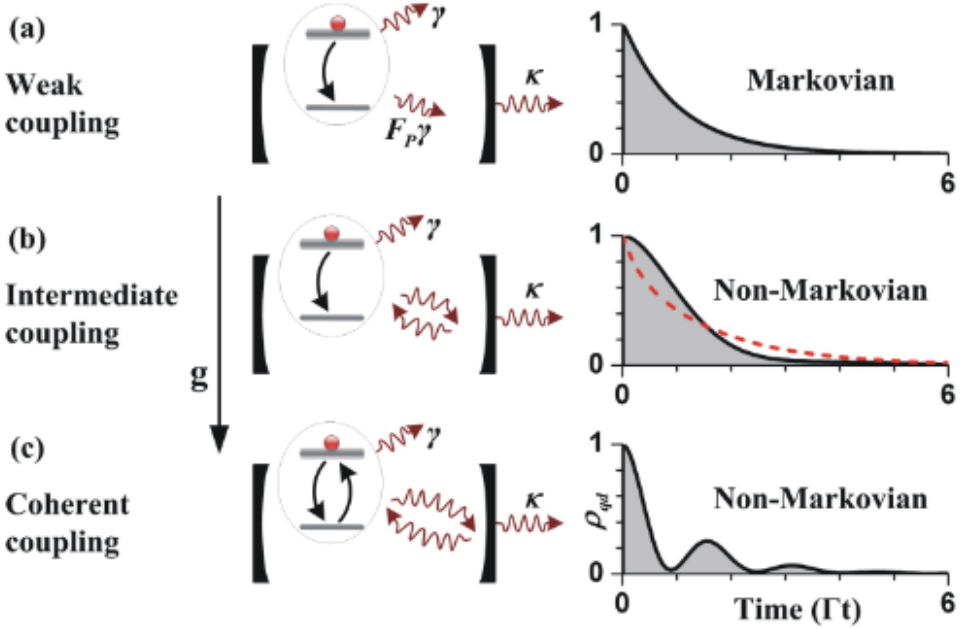
Common Forms of Non-Markovianity

- Drift: Over the course of an experiment control parameters and environments can change over time, resulting in a device falling out of calibration.



Proctor et al. Nature Communications, 11, 5396 (2020)

- Quasistatic-DC Noise: Temporal correlations in randomly fluctuating noise parameters can induce non-Markovian decoherence/dephasing with characteristic non-exponential fidelity decays.

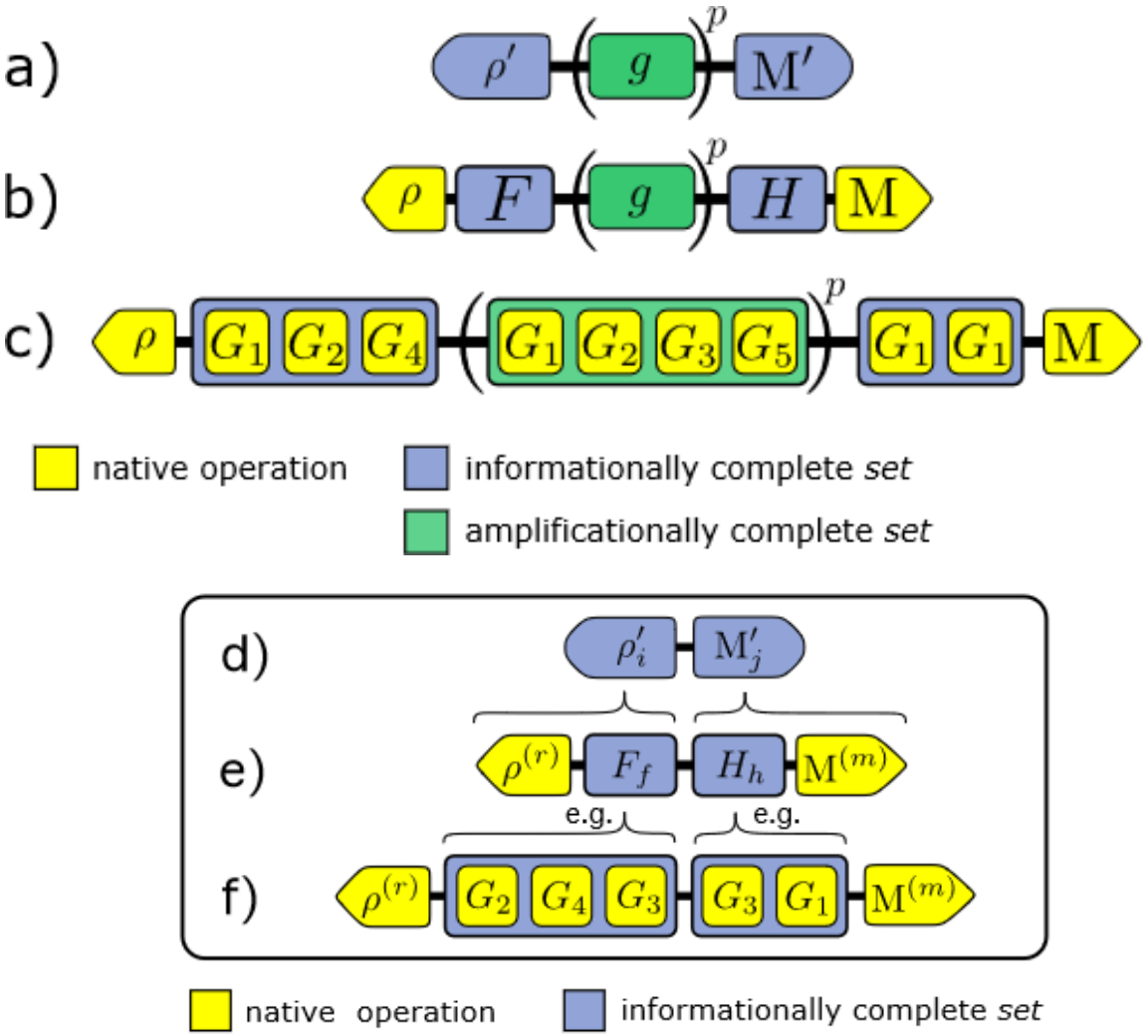


Madsen et al. Phys. Rev. Lett. 106, 233601 (2011)

Long Sequence Gate Set Tomography (GST)

- Long sequence GST (GST) generates high precision self consistent tomography of an entire gate set by performing tomography on circuits constructed using repeated short sequences of gates called “germs.”
- Germs are selected such that they amplify perturbations to the ideal parameter values of the target gate set.
- Coupled with an informationally complete of state preparations and measurements GST can estimate the parameters of a gate set with precision $\mathcal{O}(\frac{1}{L\sqrt{N}})$ where L is the maximum circuit depth used.

Structure of a GST Experiment



Non-Markovianity in GST Fits (Quasistatic DC Noise)

GST fits to a data set for a 1-qubit XY gate set with canonical non-Markovian dephasing. For each shot of each circuit we sample a normally distribution coherent X error for the X gate and coherent Y error for the Y gate with a standard deviation of .03 radians.

Figure 1. Model violation summary.

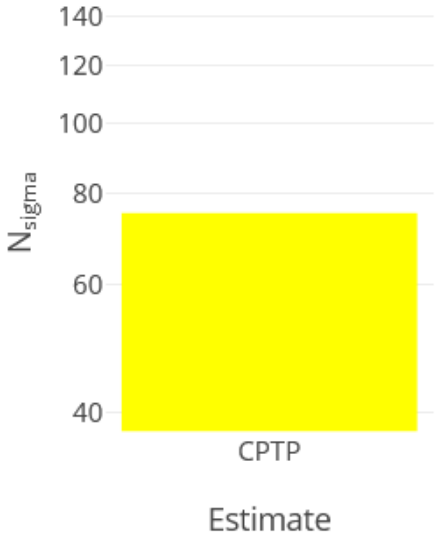


Figure 2. Model violation per iteration.

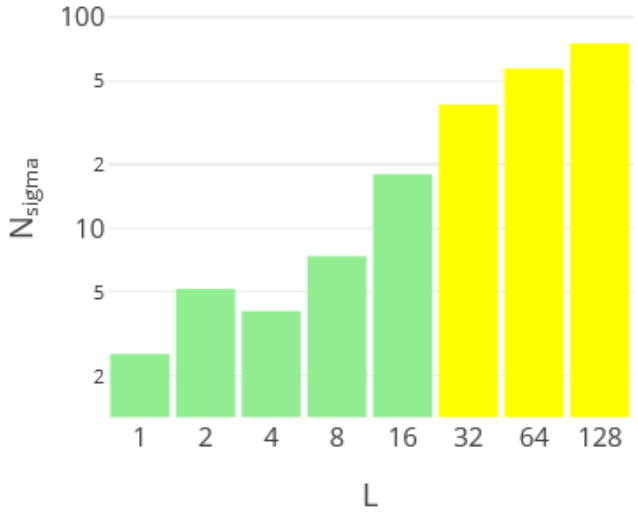
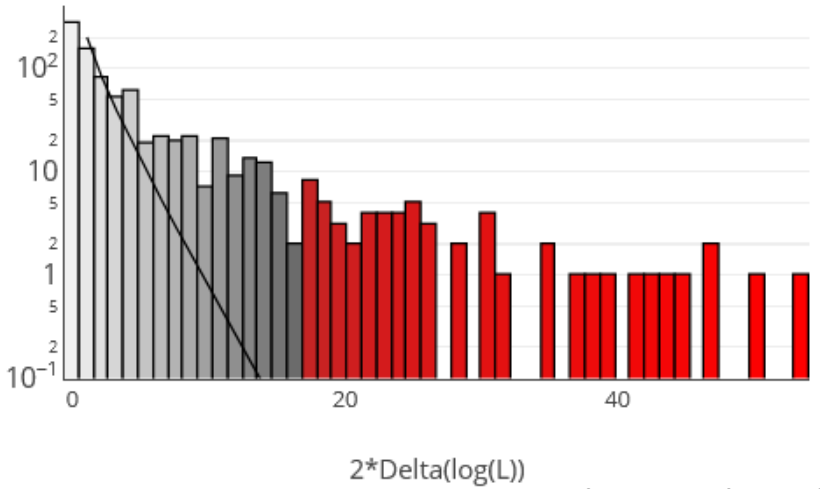


Figure 3. Histogram of per-circuit model violation.



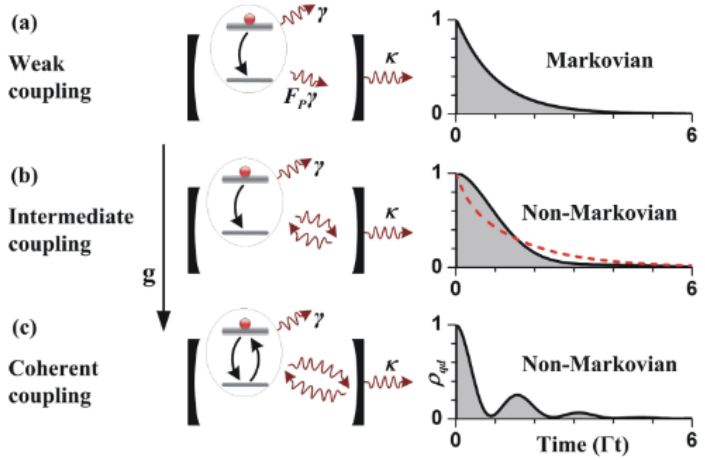
Theorem. *Let $\mathcal{L}$ be the likelihood that an estimated model produces a data set and let $\mathcal{L}_{max}$ be the likelihood of a saturated model which fits the data exactly. If the estimated model is valid the log-likelihood ratio statistic will be χ^2_k distributed,*

$$LLRS = 2(\log(\mathcal{L}_{max}) - \log(\mathcal{L})) \sim \chi^2_k$$

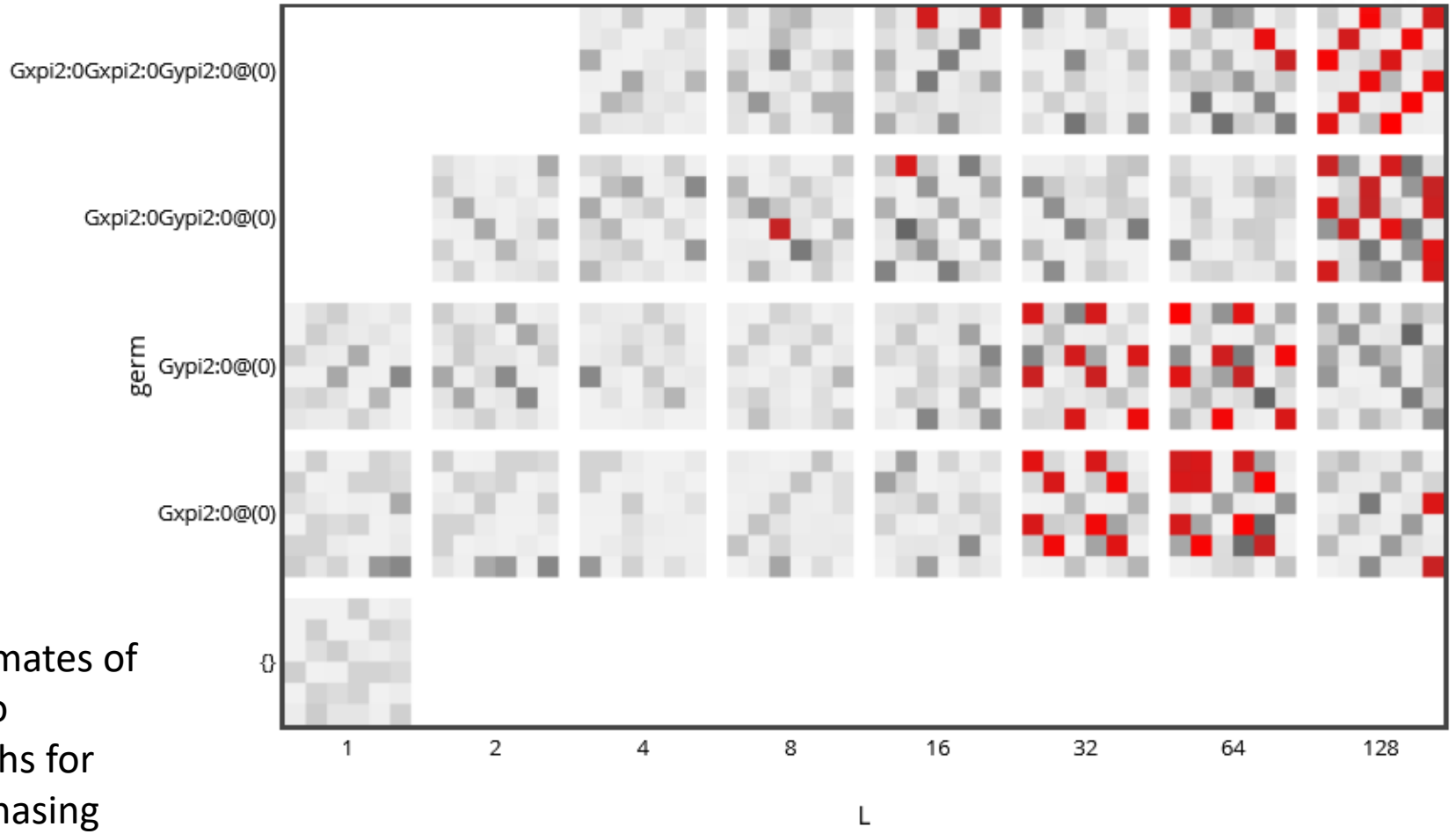
where k is the difference in the number of parameters between the models.

For a 2-outcome measurement this implies that the per-circuit contributions to the log-likelihood ratio should be approximately χ^2_1 distributed if a model is valid.

Non-Markovianity in GST Fits (Quasistatic DC Noise)

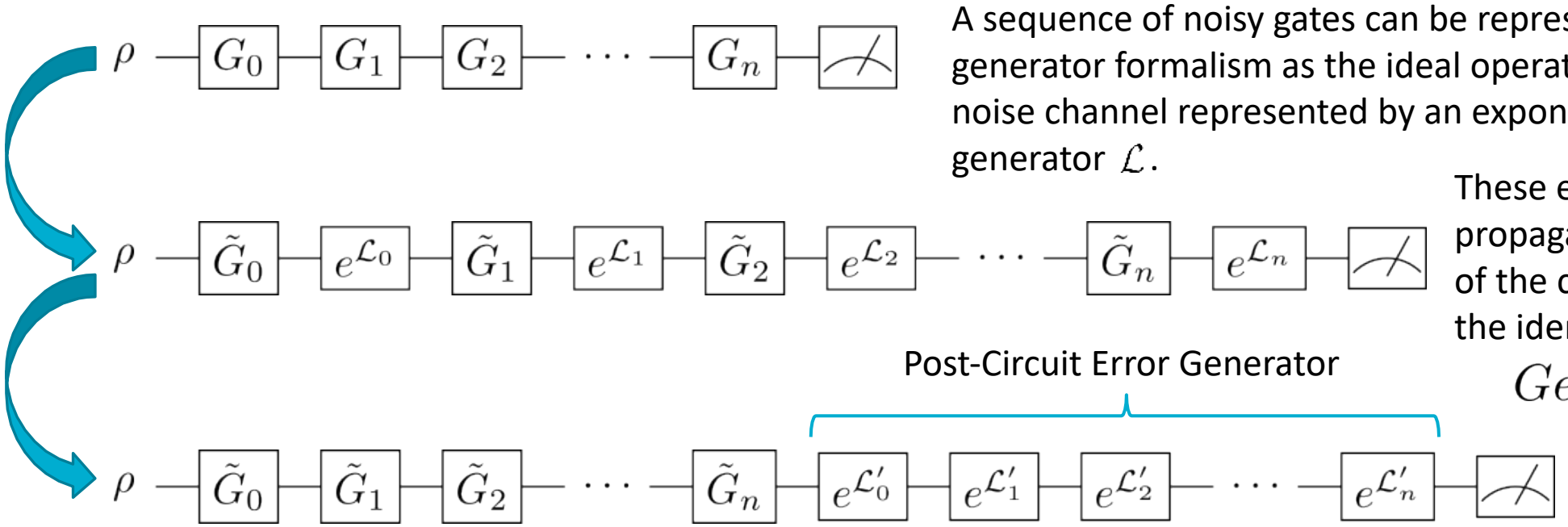


Madsen et al. Phys. Rev. Lett. 106, 233601 (2011)



Non-Markovian dephasing corrupts estimates of Markovian dephasing rates. This leads to structured model-violation at large depths for circuits most sensitive to stochastic dephasing errors.

Extending GST for Modeling Quasistatic DC Noise



A sequence of noisy gates can be represented in the error generator formalism as the ideal operation followed by a noise channel represented by an exponentiated error generator $\mathcal{L}$.

These error generators are propagated through to the end of the circuit by repeatedly using the identity:

$$Ge^{\mathcal{L}} = e^{G\mathcal{L}G^{-1}}G$$

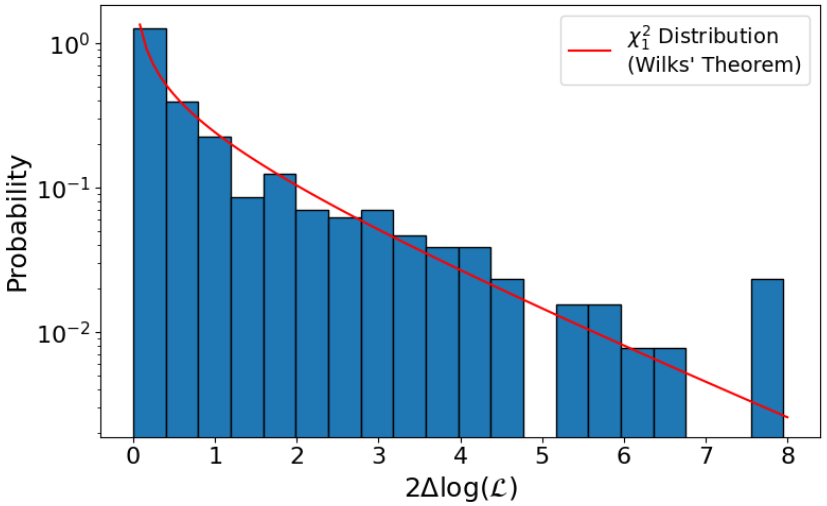
- Canonical non-Markovian dephasing arises from fluctuating Hamiltonian error generator rates (i.e. fluctuating coherent errors) with time correlations on the order of the time for a single shot of a circuit.
- This can be modeled in GST by letting the Hamiltonian rates of each gate correspond to gaussian random variables with some mean and standard deviation that are resampled after every shot of every circuit and then propagating these through the circuit to form an effective post-circuit error generator.
- For sufficiently large numbers of shots what we actually observe is the expectation value of the post-circuit error channel taken over the distribution of Hamiltonian rates.

Modeling Quasistatic DC Noise (Results)

GST fits to a data set for a 1-qubit XY gate set with canonical non-Markovian dephasing. Data is sampled according to a noise model consisting of normally distributed coherent X, Y and Z errors that fluctuate on the time scale of a single shot.

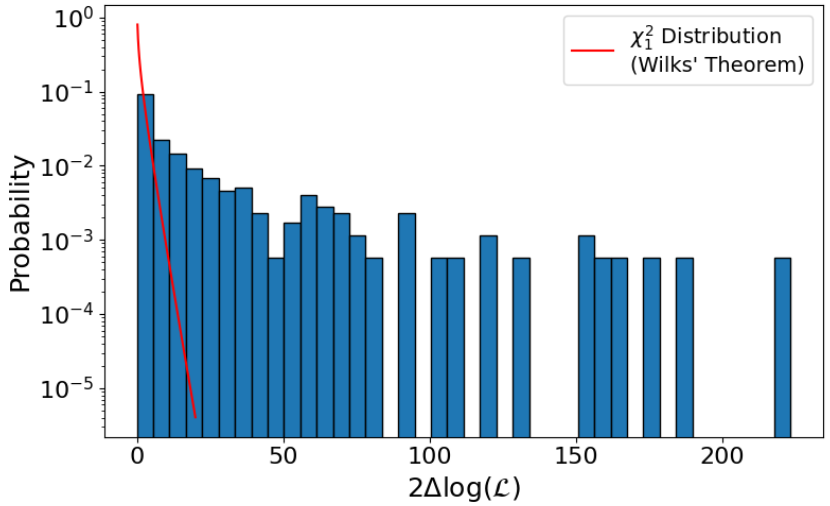
Gate	(μ_X, μ_Y, μ_Z)	$(\sigma_X, \sigma_Y, \sigma_Z)$
$R_x(\pi/2)$	(.05, -.05, .025)	(.025, .025, .025)
$R_y(\pi/2)$	(-.05, -.025, .025)	(.025, .025, .025)

Quasistatic Noise Model



Total log-likelihood: $l_{\mathcal{C}} = -492528$

Standard Markovian Model

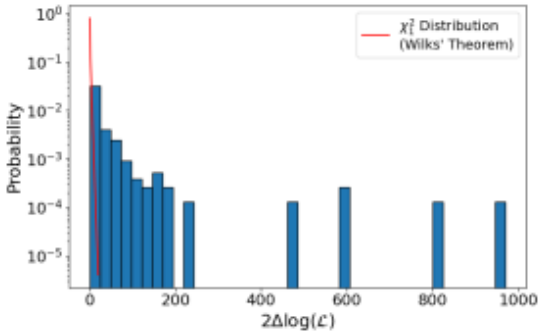


Total log-likelihood: $l_{\mathcal{C}} = -497020$

$\Delta l_{\mathcal{C}} = 4493 \; \gamma = 1498$

Evidence Ratio: $\gamma = \frac{2\Delta l_{\mathcal{C}}}{\Delta k}$, where Δk is the change in the number of parameters.

By adding only a small number of additional parameters to our model (just 6 standard deviations for the Hamiltonian rates) we can characterize a common form of non-Markovianity that standard Markovian models are unable to capture.



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