

Towards Robust Image Denoising with Scale Equivariance

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Abstract

Despite notable advances in image denoising, existing models often struggle to generalize beyond in-distribution noise patterns, particularly when confronted with out-of-distribution (OOD) conditions characterized by spatially variant noise. This generalization gap remains a fundamental yet underexplored challenge. In this work, we investigate *scale equivariance* as a core inductive bias for improving OOD robustness. We argue that incorporating scale-equivariant structures enables models to better adapt from training on spatially uniform noise to inference on spatially non-uniform degradations. Building on this insight, we propose a robust blind denoising framework equipped with two key components: a Heterogeneous Normalization Module (HNM) and an Interactive Gating Module (IGM). HNM stabilizes feature distributions and dynamically corrects features under varying noise intensities, while IGM facilitates effective information modulation via gated interactions between signal and feature paths. Extensive evaluations demonstrate that our model consistently outperforms state-of-the-art methods on both synthetic and real-world benchmarks, especially under spatially heterogeneous noise. Code will be made publicly available.

Introduction

The field of image denoising has progressed significantly, evolving from traditional prior-based methods to deep learning frameworks that deliver state-of-the-art performance. However, these advances are largely confined to a supervised learning paradigm, which exposes a fundamental vulnerability, *i.e.*, models trained on paired data with in-distribution (ID) noise often fail to generalize when confronted with out-of-distribution (OOD) noise patterns that are more complex and spatially varying. As illustrated in Fig. 1, performance degrades sharply under such mismatched conditions. This generalization gap is commonly attributed to overfitting, *i.e.*, networks tend to entangle signal representations with the noise characteristics seen during training. Although recent self-supervised and zero-shot methods offer improved flexibility, they typically underperform in terms of denoising quality and remain fragile under true OOD settings. As such, the field faces a critical dilemma: *how to attain the high fidelity of supervised denoising without inheriting its brittleness.*

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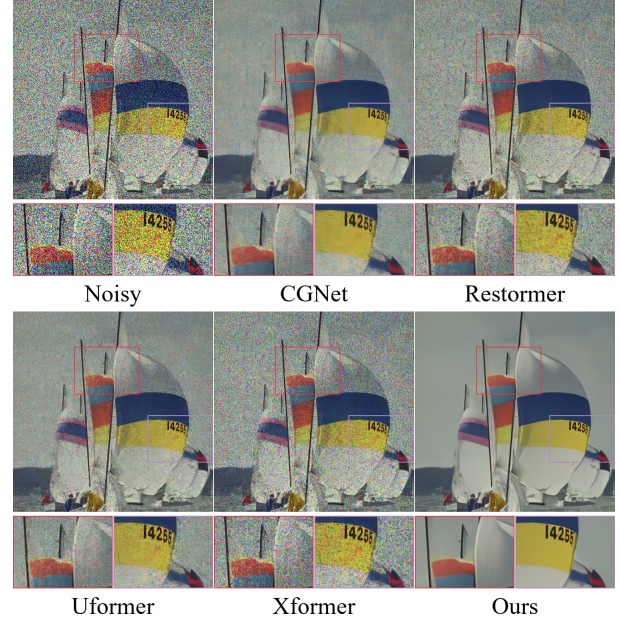


Figure 1: Visual comparison on a sample with OOD noise.

To tackle this issue, existing approaches have explored several directions. One straightforward strategy is to diversify the training noise distribution, using synthesized complex noise such as mixtures (Guo et al. 2019; Zhang et al. 2023b) or adversarial perturbations (Chen et al. 2023). Another line of work focuses on reformulating the learning objective, *e.g.*, by encouraging distortion-invariant representations through MaskedDenoising (Chen et al. 2023) or CLIP-based alignment (Cheng, Liang, and Tan 2024). A third direction leverages noise adaptation, where pretrained models are fine-tuned for specific OOD conditions (Kim, Kim, and Baik 2024; Ha et al. 2024; Chihaoui and Favaro 2025). While effective to some extent, these methods primarily focus on data, objectives, or adaptation protocols, leaving the network architecture itself largely unexamined. In contrast, we argue that OOD robustness should be viewed as an inductive property of the network design rather than merely a byproduct of training strategies.

The core challenge in generalizing from uniform to non-uniform noise lies in the entanglement of features with spatially varying noise level maps. Ideally, a denoising model

should “factor out” the influence of the noise level. However, most existing architectures (see Fig. 1) are ill-equipped to do so, lacking an explicit mechanism to disentangle this interaction. Motivated by this, we revisit a fundamental design principle, namely *scale equivariance*, which is a property that ensures consistent behavior under input scaling and is closely related to first-order homogeneity. We find that enforcing scale equivariance within the network architecture can suppress the impact of spatially varying noise levels, leading to improved generalization and train-test consistency. To this end, we design operations and modules that promote scale-equivariant behavior.

Modern denoising networks often adopt normalization layers to stabilize training. However, conventional normalization schemes (e.g., BatchNorm, LayerNorm) are scale-independent and can inadvertently amplify the entanglement between feature maps and noise levels. We address this by proposing a simple Constant Scaling (CS) operation, which normalizes features by the number of channels, reducing variance without introducing scale distortion. Moreover, pixel-wise OOD noise introduces inconsistent distortions across feature dimensions. To mitigate this issue, we introduce the Normalized Self-Modulator (NSM), which transforms pixel features into a normalized space and modulates them using their original values to maintain scale equivariance. Together, CS and NSM constitute our Heterogeneous Normalization Module (HNM). Another critical component is the activation function. Standard nonlinearities, especially those involving exponential terms, can disrupt scale properties. Apart from common activations, the star operation/gating mechanism (Ma et al. 2024) achieves high-dimensional nonlinear mapping in a low-dimensional space but is typically second-order homogeneous. To retain first-order scale equivariance, we modify and extend the gating design via introducing the Interactive Gating Module (IGM). This module uses a dual-signal scaling strategy, allowing information flow to adapt jointly to both feature and gating signals, thereby enhancing selective filtering under non-uniform noise conditions.

Our major contributions are summarized as follows:

- We identify *scale equivariance* as a key role for mitigating the train-test inconsistency caused by non-uniform noise levels. Guided by this insight, we design a robust blind denoising network tailored for OOD scenarios.
- We propose the Heterogeneous Normalization Module, which integrates feature scaling and normalized self-modulation to strike a stable feature representation and flexible feature correction. Moreover, the Interactive Gating Module is proposed to provide powerful nonlinear representations and improve the feature selection.
- Extensive experiments reveal the superior denoising robustness of our network against OOD noises, especially in high-level noise conditions. Our model consistently outperforms existing approaches with advanced perceptual quality across diverse degradation patterns.

Related Work

This part will briefly review representative works related to deep learning-based and robust image denoising.

Deep learning-based image denoising. Deep learning has become the dominant paradigm in image denoising, with early successes driven by Convolutional Neural Networks (CNNs) (Zamir et al. 2020; Chang et al. 2020; Zhang et al. 2021; Shen et al. 2023; Ghasemabadi et al. 2024), and more recent advances powered by Vision Transformer (ViT) architectures (Wang et al. 2022; Zamir et al. 2022; Li et al. 2023; Zhang et al. 2024; Chen et al. 2024), which leverage sophisticated self-attention mechanisms and normalization strategies. While these designs excel on ID data, their supervised nature makes them prone to overfitting, thus limiting their OOD robustness. To address this, alternative paradigms such as self-supervised learning (Zhang et al. 2023a; Liao et al. 2024; Li, Zhang, and Zuo 2025; Liu et al. 2025) and zero-shot denoising (Quan et al. 2020; Mansour and Heckel 2023; Ma et al. 2025) have emerged. These approaches enable model training directly on unpaired or noisy data without requiring clean targets, for offering greater flexibility. However, they often fall short in denoising quality and still face OOD limitations, as their training remains implicitly tied to the noise distribution in the available data. Thus, a fundamental challenge remains, saying how to retain the high performance of supervised frameworks while overcoming their inherent lack of robustness under OOD noise. This question continues to motivate much of the current research in robust image denoising.

Robust image denoising. Robust image denoising refers to a model’s ability to maintain high performance when exposed to unseen or OOD noise patterns. Existing efforts to enhance such robustness can be broadly categorized as follows. One common approach is to enrich the training data by simulating complex noise, such as via mixture noise modeling (Guo et al. 2019; Zhang et al. 2023b) or adversarial perturbations (Chen et al. 2023). Although these strategies help improve diversity during training, they also risk contaminating the evaluation setup by inadvertently exposing the model to OOD-like noise during training, thus complicating generalization assessment. A second direction involves learning degradation-invariant representations, encouraging models to focus on clean image content rather than the noise distribution. Representative methods include MaskedDenoising and CLIPDenoising (Cheng, Liang, and Tan 2024). However, these methods often come at the cost of reduced image quality, particularly under high noise levels. A third class of methods emphasizes noise adaptation. For instance, LAN (Kim, Kim, and Baik 2024) and noise translation techniques (Ha et al. 2024) attempt to map OOD noise into a known or ID domain. However, such methods may suffer from scalability limitations or require prior exposure to real-world noise distributions. MID (Chihaoui and Favaro 2025) presents another strategy by fine-tuning a denoiser trained solely on simple synthetic noise using self-supervised objectives. While effective, this approach relies on ensemble inference, making it computationally intensive.

In contrast to the mentioned schemes that primarily focus on data augmentation, objective design, or adaptation

techniques, this work takes a fundamentally different perspective, *i.e.*, how *network architecture itself* can serve as an inductive bias for robustness.

Problem Analysis

The canonical model for denoising (Liang et al. 2021; Li et al. 2023)¹ models an observed noisy image Y as the sum of its underlying clean signal X and an additive white Gaussian noise component $\lambda \cdot N$:

$$Y = X + \lambda \cdot N, N_{ij} \sim \mathcal{N}(0, 1), \quad (1)$$

where λ is a scalar representing a uniform noise level. While this model is widely adopted for training deep denoisers, it introduces a critical mismatch with real-world noise characteristics, which are rarely spatially uniform. In practice, due to sensor response variations and environmental influences (*e.g.*, lighting, exposure, ISO), the noise level varies across spatial locations. Consequently, the scalar λ should be replaced by a spatially varying map Λ , leading to a distributional shift between training and inference. This train-test discrepancy from uniform to non-uniform noise lies at the heart of poor OOD generalization. We identify a core reason for this degradation as feature entanglement. Conventional denoising networks tend to learn features that are tightly coupled to the magnitude of the noise. When the noise level becomes spatially varying, this coupling breaks down, leading to destabilized representations and degraded performance. To achieve robustness under such conditions, the key is to decouple feature representations from the noise level map. We argue that this can be systematically addressed by enforcing scale equivariance in the network design. Also known as first-order homogeneity, scale equivariance ensures that the network’s output scales linearly with the input, formally expressed as follows:

Definition 1 (Scale Equivariance). *A function $\mathcal{H}$ obeying the $\mathcal{H}(0) = 0$ is said to be first-order homogeneous if, for input I and a scalar $k > 0$, it satisfies the following condition:*

$$\mathcal{H}(k \cdot I) = k \cdot \mathcal{H}(I).$$

For a denoising network $\mathcal{F}$ endowed with the above property, its behavior during training under a uniform noise level λ can be described as follows:

$$\mathcal{F}(X + \lambda \cdot N) = \lambda \cdot \mathcal{F}\left(\frac{X}{\lambda} + N\right) \xrightarrow{\text{target}} X. \quad (2)$$

As shown in Eq. (2), the network no longer learns a direct mapping from a noisy image to its clean counterpart. Instead, the effective learning task becomes denoising the input $X/\lambda + N$ to produce X/λ , thereby removing only the additive noise N . This formulation decouples the denoising mechanism from the global noise magnitude, embedding a scale-equivariant inductive bias directly into the network architecture. This decoupling extends naturally to the more challenging OOD setting with spatially variant noise Λ :

$$\mathcal{F}(X + \Lambda \odot \tilde{N}) = \Lambda \odot \mathcal{F}\left(\frac{X}{\Lambda} + \tilde{N}\right) \xrightarrow{\text{target}} X, \quad (3)$$

¹Without loss of generality, we adopt it through this work.

where $\odot$ denotes Hadamard product and $\tilde{N}$ is a base noise term that may originate from a different distribution. Remarkably, the network’s internal processing remains structurally consistent: regardless of whether the noise level is uniform or spatially varying, the effective input is normalized to unit noise variance across the image. This normalization aligns with recent observations (Chen et al. 2023; Cheng, Liang, and Tan 2024) that denoisers inherently prioritize learning the noise pattern over image content. By enforcing a consistent computational pattern across domains, scale equivariance provides a principled and architecture-level path to achieving robustness.

The importance of strict first-order homogeneity is best understood by examining common violations, which we term architectural pathologies for this task: 1) *Zero-Order Homogeneity (Scale Independence)*. Consider a network with this property, *i.e.*, $\mathcal{F}(X + \lambda \cdot N) = \mathcal{F}(X/\lambda + N)$, its learning objective becomes mapping $X/\lambda + N$ to X . This imposes a flawed dual task: the network must simultaneously denoise the input and learn the global scaling λ , which forces an entanglement between features and the noise level seen during training. This entanglement is precisely what fails when λ becomes a spatially-varying map; 2) *Higher-Order Homogeneity*. A network with second-order homogeneity, *i.e.*, $\mathcal{F}(X + \lambda \cdot N) = \lambda^2 \cdot \mathcal{F}(X/\lambda + N)$ involves similar entanglement phenomena. The objective becomes mapping $X/\lambda + N$ to X/λ^2 , creating a distorted target. Moreover, the second-order property involves a quadratic growth in feature variance, increasing risk of learning collapse; 3) *Exponential Activations*. Many exponential-based activation functions like GELU break homogeneity. They introduce input-dependent scaling errors that compound throughout the network, rendering OOD performance fragile.

A First-Principles Validation. With scale equivariance equipped, even a simple network should generalize well under distributional shifts. To test this in blind denoising, we build a baseline U-Net only from demonstrably scale-equivariant components: standard convolutions and ReLU activation. Despite its simplicity and lack of advanced modules, this network serves as a clean testbed to observe the effect of scale equivariance without confounding factors.

Lemma 1 (Homogeneity of Basic Operators). *Both convolution and ReLU are first-order homogeneous.*

Proof. Given an input I and a scalar $k > 0$, the convolution output at position (i, j) is:

$$\text{Conv}(k \cdot I)_{i,j} = k \cdot \sum_{m,n} I_{i-m,j-n} \cdot W_{m,n} = k \cdot \text{Conv}(I)_{i,j}.$$

Similarly, the relationship $\text{ReLU}(k \cdot I)_{i,j} = k \cdot \text{ReLU}(I)_{i,j}$ also holds. Thus, the claim is established. $\square$

Building on this, the entire residual block, including the skip connection, preserves this property as well.

Lemma 2 (Homogeneity of Residual Block). *The ‘Conv-ReLU-Conv’ residual block is first-order homogeneous.*

Proof. Given Lemma 1, we have:

$$\begin{aligned} \text{RB}(k \cdot I) &= \text{Conv}(\text{ReLU}(\text{Conv}(k \cdot I))) + k \cdot I \\ &= k \cdot (\text{Conv}(\text{ReLU}(\text{Conv}(I))) + I) = k \cdot \text{RB}(I). \end{aligned}$$

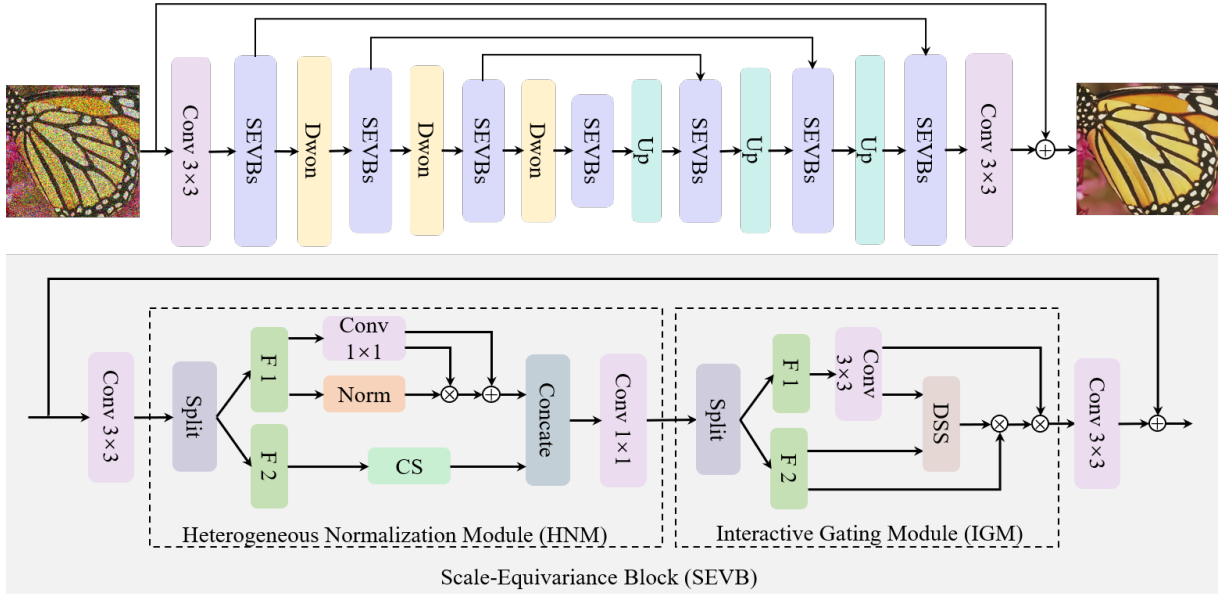


Figure 2: Overview of the proposed SEVNet with scale-equivariant design. CS represents Constant Scaling. DSS involves the dual-signal scaling term. The split operation divides the feature map into equally shaped sub-maps along the channel dimension.

Method	In-Distribution	Out-Of-Distribution		
	Gaussian $\sigma = 20$	Speckle $\sigma = 90$	Poisson $\alpha = 6$	Mixture
Uformer	32.89/0.9450	23.47/0.7510	22.92/0.7253	20.07/0.5812
NAFNet	32.28/0.9402	22.69/0.7047	22.30/0.6901	20.15/0.5995
GRL	32.77/0.9419	<u>24.15/0.8021</u>	<u>23.65/0.7868</u>	<u>21.19/0.6990</u>
CGNet	32.85/0.9454	22.77/0.6999	22.32/0.6829	20.02/0.5850
Xformer	32.99/0.9456	21.80/0.6386	21.15/0.6096	17.89/0.4682
MHNet	32.87/0.9452	21.70/0.6474	21.03/0.6150	17.56/0.4552
AKDT	32.54/0.9417	22.58/0.6774	22.08/0.6568	19.52/0.5468
CRWKV	32.39/0.9377	20.17/0.5845	19.60/0.5594	16.57/0.4299
Baseline	32.85/0.9447	24.49/0.8286	24.04/0.8183	21.67/0.7554

Table 1: Quantitative comparisons in PSNR/SSIM on the Urban100 dataset. The mixture noise is composed of Speckle and Poisson. The best results are highlighted in **bold**, while the second-best ones are underlined.

The property holds for the entire block. $\square$

As reported in Tab. 1, this simple baseline outperforms those sophisticated but non-equivariant models on OOD tasks while remaining competitive on ID cases. The comparison corroborates our theoretical analysis.

Designing a Scale-Equivariant Network

Our first-principles validation demonstrated that even a simple equivariant network can outperform sophisticated yet non-compliant ones. To further advance performance, we now introduce several novel modules designed to solve key challenges in constructing a high-performance, scale-equivariant denoiser. These components collectively define our SEVNet architecture, as illustrated in Fig. 2. Due to the page limit, formal proofs of equivariance for all modules are provided in the appendix.

Constant Scaling. Modern networks often rely on normalization operations, *e.g.*, Layer Norm, to stabilize training. While effective, these operations are scale-independent and thus conflict with our equivariant objective. To resolve this, we propose Constant Scaling (CS) as an equivariant stabilizer applied to the feature map $F \in \mathbb{R}^{c \times h \times w}$:

$$\text{CS}(F) = \frac{F}{\sqrt{c}} \odot \eta, \quad (4)$$

where $\eta \in \mathbb{R}^c$ is a learnable, per-channel parameter initialized to 1. This operation normalizes the initial feature magnitude (preventing uncontrolled variance growth), while allowing η to adapt channel-wise amplitudes during training.

Normalized Self-Modulator. While CS ensures global stability, it does not address local feature distortions caused by noise with spatial amplitude variations. To counteract this, we propose the Normalized Self-Modulator (NSM), a dynamic pixel-wise module designed to provide localized feature correction while preserving scale equivariance. The core idea is to perform a temporary local normalization, followed by an adaptive rescaling that restores critical magnitude in an equivariant manner. NSM performs self-modulation using affine parameters derived from the token’s original pre-normalized state. This can be formulated as:

$$\begin{aligned} [\gamma, \beta] &\leftarrow \text{Conv}_{1 \times 1}(F), \\ \text{NSM}(F) &= \gamma \odot \frac{X - \mu(F)}{\sqrt{\sigma^2(F)}} + \beta, \end{aligned} \quad (5)$$

where μ and σ^2 are the per-token mean and variance, respectively. Since the affine parameters $\gamma \in \mathbb{R}^{c \times h \times w}$ and $\beta \in \mathbb{R}^{c \times h \times w}$ are derived via a linear transformation of the input, they are first-order homogeneous. This ensures that the entire NSM remains strictly scale-equivariant, providing adaptive feature correction without violating our principle.

Method	In-Distribution	Out-Of-Distribution					
	Gaussian	Speckle	Poisson	Mixture	Variant 1	Variant 2	Variant 3
Kodak24							
SwinIR (Liang et al. 2021)	33.50/0.9029	27.55/0.7931	27.23/0.7820	22.79/0.5887	25.99/0.7457	26.31/0.7541	25.63/0.7313
Uformer (Wang et al. 2022)	33.62/0.9062	27.20/0.7656	26.86/0.7506	22.27/0.5102	25.42/0.6950	25.82/0.7102	25.07/0.6750
Restormer (Zamir et al. 2022)	33.70/0.9076	26.12/0.6978	25.74/0.6807	20.11/0.4279	23.19/0.6078	24.01/0.6280	22.87/0.5809
NAFNet (Chen et al. 2022)	33.42/0.9048	25.39/0.6681	25.15/0.6580	22.16/0.5422	23.49/0.6087	23.99/0.6227	23.33/0.5967
GRL (Li et al. 2023)	33.52/0.9025	27.58/0.7981	27.29/0.7904	23.66/0.6789	26.19/0.7643	26.47/0.7684	25.87/0.7560
CGNet (Ghasemabadi et al. 2024)	33.65/0.9075	25.43/0.6564	25.11/0.6416	21.74/0.4981	23.32/0.5953	23.88/0.6106	23.04/0.5714
Xformer (Zhang et al. 2024)	33.66/0.9069	25.29/0.6388	24.82/0.6165	18.91/0.3507	22.56/0.5433	23.15/0.5582	22.10/0.5143
MHNet (Gao et al. 2025)	33.64/0.9074	24.88/0.6390	24.43/0.6163	18.67/0.3462	21.91/0.5361	22.72/0.5573	21.64/0.5121
AKDT (Brateanu et al. 2025)	33.48/0.9050	25.57/0.6565	25.20/0.6391	20.94/0.4493	23.39/0.5887	23.94/0.6018	23.05/0.5637
ESC (Lee, Yun, and Ro 2025)	33.51/0.9028	26.52/0.7017	26.16/0.6843	21.58/0.4691	24.44/0.6261	24.94/0.6402	24.03/0.6021
CRWKV (Chen et al. 2025)	33.50/0.9038	23.89/0.5978	23.43/0.5763	17.68/0.3380	20.73/0.5026	21.48/0.5186	20.26/0.4726
Ours	<u>33.68/0.9069</u>	27.94/0.8164	27.66/0.8106	24.23/0.7368	26.64/0.7908	26.91/0.7932	26.34/0.7858
Urban100							
SwinIR (Liang et al. 2021)	32.71/0.9415	25.53/0.8338	25.19/0.8238	20.78/0.6490	23.80/0.7858	26.09/0.8529	23.75/0.7831
Uformer (Wang et al. 2022)	32.89/0.9450	25.03/0.8055	24.71/0.7936	20.07/0.5812	23.11/0.7408	25.60/0.8331	23.10/0.7374
Restormer (Zamir et al. 2022)	33.12/0.9469	24.31/0.7547	24.00/0.7439	18.56/0.5142	21.65/0.6759	24.27/0.7797	21.64/0.6722
NAFNet (Chen et al. 2022)	32.28/0.9402	23.87/0.7430	23.64/0.7356	20.15/0.5995	22.05/0.6832	22.44/0.7000	22.04/0.6824
GRL (Li et al. 2023)	32.77/0.9419	<u>25.53/0.8362</u>	<u>25.21/0.8279</u>	21.19/0.6990	23.91/0.7973	26.08/0.8541	<u>23.85/0.7952</u>
CGNet (Ghasemabadi et al. 2024)	32.85/0.9454	<u>24.13/0.7449</u>	<u>23.85/0.7361</u>	20.02/0.5850	22.14/0.6835	22.56/0.7012	22.09/0.6790
Xformer (Zhang et al. 2024)	<u>32.99/0.9456</u>	23.74/0.7138	23.37/0.6996	17.89/0.4682	21.24/0.6326	21.69/0.6467	21.19/0.6276
MHNet (Gao et al. 2025)	32.87/0.9452	23.51/0.7224	23.14/0.7073	17.56/0.4552	20.81/0.6290	21.41/0.6505	20.84/0.6262
AKDT (Brateanu et al. 2025)	32.54/0.9417	24.05/0.7304	23.76/0.7197	19.52/0.5468	22.06/0.6697	22.46/0.6841	22.01/0.6657
ESC (Lee, Yun, and Ro 2025)	32.97/0.9439	24.38/0.7463	24.07/0.7351	19.42/0.5445	22.25/0.6793	22.64/0.6917	22.18/0.6747
CRWKV (Chen et al. 2025)	32.39/0.9377	22.00/0.6544	21.67/0.6410	16.57/0.4299	19.42/0.5743	19.94/0.5889	19.34/0.5675
Ours	32.94/0.9453	26.14/0.8644	25.84/0.8590	21.94/0.7698	24.59/0.8371	26.66/0.8771	24.54/0.8357

Table 2: Quantitative denoising comparisons (PSNR/SSIM) on the different datasets. The average results across 3 noise levels for Speckle and Poisson noise are reported. The best results are marked in **bold**, while the second-best ones are underlined.

Heterogeneous Normalization Module. While individual CS and NSM modules provide stabilization, they offer a trade-off between efficiency and expressive power. To get the best of both worlds, we combine them into a single Heterogeneous Normalization Module (HNM). The design is simple yet effective: we partition the feature channels and process one subset with the lightweight CS for baseline stability, while the other subset is processed by the more powerful NSM for dynamic, fine-grained correction. This heterogeneous strategy creates a robust, multi-level stabilization mechanism that achieves adaptive feature modulation.

Interactive Gating Module. Although ReLU satisfies the requirement of scale equivariance, its limited expressiveness may hinder performance. In contrast, more sophisticated activations such as GELU and Swish offer stronger nonlinear modeling capabilities but inherently violate scale equivariance, introducing a potential bottleneck in equivariant design. To bridge this gap, we propose the Interactive Gating Module (IGM), a novel activation function that is both highly expressive and strictly scale-equivariant by construction. Specifically, given the input feature map $F \in \mathbb{R}^{c \times h \times w}$, we first split it along the channel dimension into two equal parts $F_1 \in \mathbb{R}^{\frac{c}{2} \times h \times w}$ and $F_2 \in \mathbb{R}^{\frac{c}{2} \times h \times w}$, and then feed them into a gating unit. We define the IGM as follows:

$$\begin{aligned}
[F_1, F_2] &\leftarrow \text{Split}(F), F_v \leftarrow \text{Conv}_{3 \times 3}(F_1), F_g \leftarrow F_2, \\
\text{IG}(F_v, F_g) &= \frac{F_v \odot F_g}{\sqrt{\sigma^2(F_v) + \sigma^2(F_g)}}, \quad (6)
\end{aligned}$$

where σ^2 stands for the per-token variance. A brilliance of our IGM is the dual-signal scaling term, which not only ensures the entire module remains scale-equivariant but also enables the gating strength to be co-determined by both the feature F_v and the gating signal F_g . This is achieved by transforming the gating signal’s variance into a relative importance weight $\frac{\sigma^2(F_g)}{\sigma^2(F_v) + \sigma^2(F_g)}$. Beyond its mathematical compliance, the IGM is interactive. The denominator normalizes the output based on the combined energy (variance) of both the value and the gate. This prevents the output from exploding when both signals are strong, allowing the gate’s influence to be modulated by its own magnitude relative to the value signal. This stable interaction enhances the network’s feature selection capability beyond a simple ReLU, providing the desired equivariant non-linearity.

Experiments

Our SEVNet is built on the principle of scale equivariance. Together, these components create a network that is theoretically sound for OOD generalization. We now turn to the experimental results to verify these claims.

Datasets We train models on the DIV2K (Timofte et al. 2017) dataset with Gaussian noise of various levels ($\sigma \in [5, 20]$) for synthetic OOD noise removal, and ISP noise (Guo et al. 2019) for handling real-world cases. We evaluate models on signal-dependent noise (Poisson, Speckle, and their mixtures). To enhance the non-uniformity of the

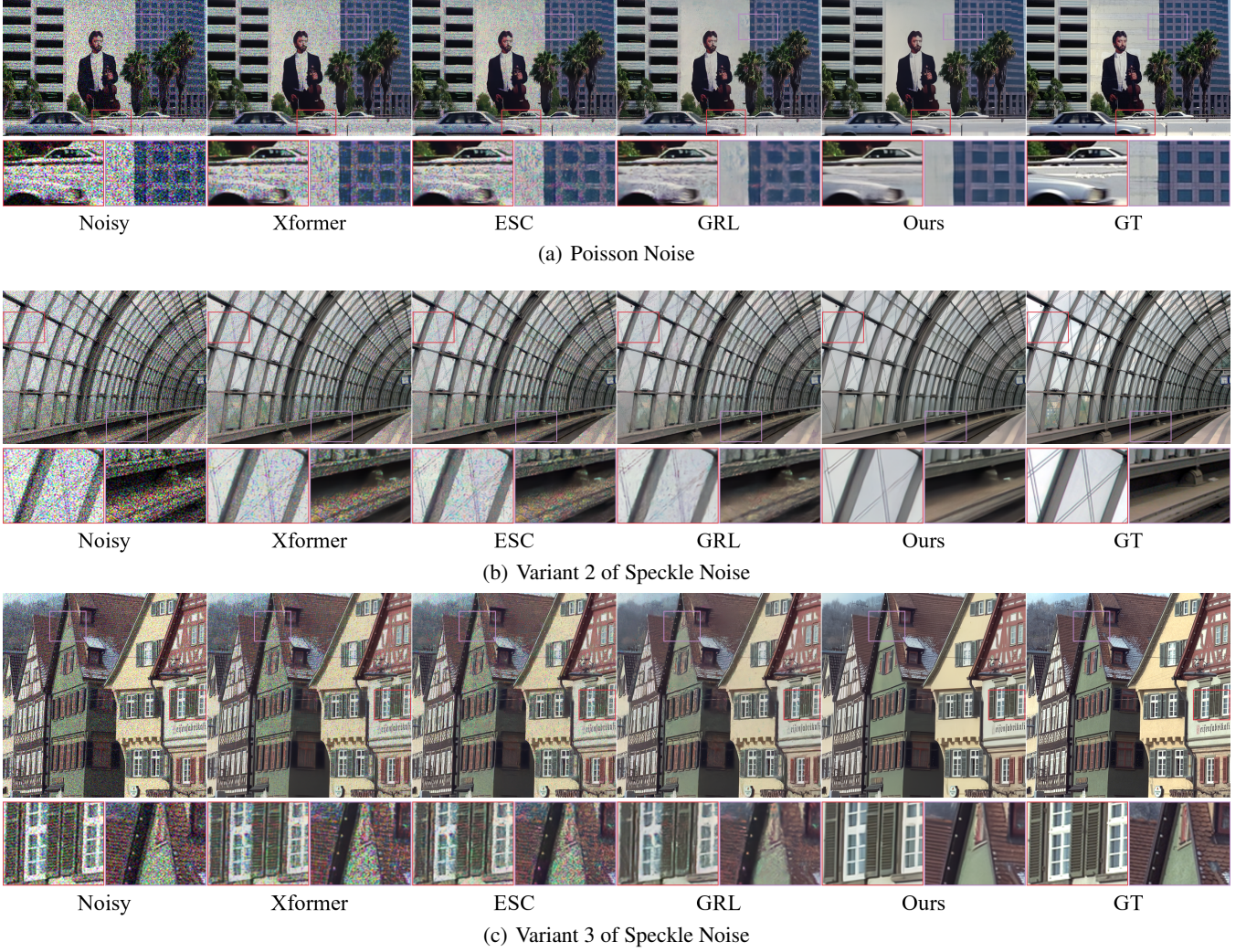


Figure 3: Qualitative comparisons on OOD noises. Please refer to the appendix to see more visual results.

level map, variants (Variant 1-3) of Speckle noise are synthesized by spatial functions (Yue et al. 2024): sine-cosine, peaks, and Gaussian kernels. The evaluation is conducted on standard test sets (Kodak24, CBSD68, Urban100) and real-world benchmarks (CC, PolyU, HighISO) (Ha et al. 2024). Denoising quality is quantified by the Peak Signal-to-Noise Ratio (PSNR) and the Structural Similarity Index (SSIM).

Method	CC PSNR/SSIM	PolyU PSNR/SSIM	HighISO PSNR/SSIM
SwinIR	33.61/0.8583	36.02/0.9151	35.53/0.8797
Uformer	34.30/0.8766	36.18/0.9182	35.96/0.8891
Restormer	33.71/0.8576	36.02/0.9146	35.57/0.8788
NAFNet	34.88/0.8952	36.30/0.9230	36.38/0.9002
GRL	33.92/0.8638	36.14/0.9181	35.75/0.8842
CGNet	33.69/0.8580	36.05/0.9155	35.58/0.8802
Xformer	33.65/0.8567	36.01/0.9141	35.54/0.8786
MHNet	33.83/0.8634	36.05/0.9156	35.70/0.8826
AKDT	34.48/0.8838	36.17/0.9197	36.05/0.8921
ESC	34.08/0.8697	36.30/0.9208	35.94/0.8880
CRWKV	31.13/0.7507	31.98/0.7718	31.66/0.7571
Ours+NT	36.95/0.9486	37.60/0.9557	39.36/0.9675

Table 4: Quantitative comparisons on real-world noise.

Quantitative Comparison. As shown in Tab. 2, our scale-equivariant network exhibits superior OOD robustness without compromising ID performance. Despite incorporating advanced modules and popular components, these existing networks violate scale equivariance, leading to severe performance degradation in OOD scenarios. More results and efficiency comparisons can be found in the appendix.

Qualitative Comparison. As shown in Fig. 3, even in severely degraded conditions, our network effectively removes various OOD noises while preserving clear edges and some textures. In contrast, the denoising results of competing methods are poor and exhibit different degradation patterns, *e.g.*, residual noise and artifacts.

Real-World Extension. The applicability of our scale-equivariant network, which assumes independent noise, can be successfully extended to real-world data by integrating a lightweight, SEVB-based noise translator (Ha et al. 2024). The strong performance of this approach (Tab. 4), surpassing complex yet inefficient models, highlights a highly promising path for synthetic-to-real generalization that involves shifting the focus from building ever-more-complex denoisers to combining simple, principled networks, *i.e.*, those that

Setting	Speckle $\sigma = 90$	Poisson $\alpha = 60$	Mixture ($\sigma = 90, \alpha = 6$)	Variant 1	Variant 2	Variant 3
	PSNR/SSIM	PSNR/SSIM	PSNR/SSIM	PSNR/SSIM	PSNR/SSIM	PSNR/SSIM
Heterogeneous Normalization Module (HNM)						
(1a) Ours	26.86/0.7933	26.42/0.7838	24.23/0.7368	26.64/0.7908	26.91/0.7932	26.34/0.7858
(1b) HNM $\rightarrow$ Full CS	26.78/0.7888	26.35/0.7793	24.17/0.7317	26.57/0.7858	26.83/0.7887	26.27/0.7812
(1c) w/ CS + w/o NSM	26.74/0.7885	26.30/0.7791	24.16/0.7322	26.52/0.7855	26.79/0.7885	26.23/0.7810
(1d) w/ HNM, IGM $\rightarrow$ ELU	26.77/0.7894	26.33/0.7799	24.16/0.7325	26.55/0.7866	26.82/0.7893	26.25/0.7818
(1e) w/ HNM, IGM $\rightarrow$ GELU	26.74/0.7872	26.29/0.7775	24.13/0.7291	26.52/0.7844	26.79/0.7871	26.23/0.7794
(1f) w/o HNM, IGM $\rightarrow$ ELU	22.79/0.5859	21.51/0.5205	16.63/0.2782	21.56/0.5565	22.42/0.5896	20.84/0.5186
(1g) w/o HNM, IGM $\rightarrow$ GELU	25.73/0.7241	25.17/0.7007	22.41/0.5495	25.40/0.7119	25.74/0.7228	24.97/0.6938
(1h) Scale-independent affine	26.54/0.7778	26.09/0.7666	23.85/0.7101	26.32/0.7747	26.59/0.7778	26.01/0.7684
Interactive Gating Module (IGM)						
(2a) w/o IGM	26.75/0.7887	26.31/0.7789	24.15/0.7317	26.53/0.7858	26.80/0.7886	26.24/0.7811
(2b) IGM $\rightarrow$ ReLU	26.80/0.7892	26.36/0.7797	24.17/0.7317	26.59/0.7865	26.85/0.7891	26.28/0.7814
(2c) Dual $\rightarrow$ Single scaling	26.83/0.7916	26.39/0.7821	24.20/0.7347	26.62/0.7891	26.88/0.7916	26.31/0.7840
(2d) HNM $\rightarrow$ LN, w/ IGM	26.39/0.7684	25.90/0.7545	23.46/0.6744	26.13/0.7636	26.41/0.7676	25.83/0.7562
(2e) HNM $\rightarrow$ LN, IGM $\rightarrow$ ReLU	25.53/0.6979	24.82/0.6612	21.88/0.5057	25.15/0.6898	25.54/0.7034	24.68/0.6640
(2f) HNM $\rightarrow$ LN, IGM $\rightarrow$ GELU	26.24/0.7583	25.78/0.7426	23.07/0.5991	25.96/0.7485	26.28/0.7576	25.65/0.7388

Table 3: Ablation studies on the Kodak24 dataset.

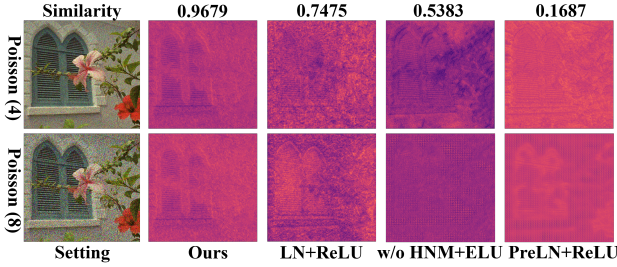


Figure 4: Comparison in visual and cosine similarities of feature maps under different noise level maps.

are scale-equivariant, with noise adaptation methods.

Ablation Study. In Tab. 3, we validate the effectiveness of the HNM and IGM, as well as the role of scale equivariance:

1) *Impact of HNM.* Applying CS to the entire feature mapping (1b) or bypassing NSM (1c) leads to a performance drop, confirming the necessity of combining both components. Interestingly, HNM allows the network to maintain comparable generalization even when using exponential activations like ELU (1d) and GELU (1e). Without HNM, however, these activations (1f-1g) severely impair OOD capability, likely because HNM shifts features to intervals where the exponential effect is less pronounced. For the NSM’s design, generating its affine parameters from the normalized features (1h), also degrades performance, though the decline is moderate since the NSM operates on only a portion of the features. Collectively, these results (1e-1h) demonstrate that scale equivariance is critical for robust denoising and that our HNM can partially suppress the adverse effects of violating this principle.

2) *Impact of IGM.* Removing IGM (2a) or replacing it with ReLU (2b) both yield suboptimal results, demonstrating IGM’s superior nonlinear capabilities. compared with the scaling term obtained from gating (2c), the dual-signal scaling term further enhances OOD robustness through its

feature selection effect. Under the scale-independent Layer Norm (LN) setting, our scaling term (2d) enables the network to perceive the scale but does not give accurate equivariance, thereby retaining weak generalization. This resilience is not found in standard activations like ReLU (2e) and GELU (2f). More importantly, without the scaling term, IGM becomes a second-order homogeneous operation, and the resulting quadratic growth in feature variance causes training to collapse, making it impossible to report the corresponding ablation results. Ultimately, these results (2d-2f) demonstrate that a scale-equivariant design is key for OOD denoising and that our IGM partially mitigates the performance degradation caused by scale-invariant operations.

Feature Analysis. To further explain the advantage of scale equivariance, we visualize feature maps (Fig. 4) from the final decoder stage. Under vastly different corruption degrees, our scale-equivariant network produces remarkably consistent feature maps. Conversely, designs that violate this principle heighten the network’s sensitivity to variations in the level map. For instance, a network with scale-independence (middle and preceding Layer Norm) or exponential activation (ELU) yields dissimilar feature pairs. In short, Scale equivariance indeed forces the network to learn robust representations that are decoupled from the noise level map.

Conclusion

We have demonstrated that scale equivariance is a key architectural principle for OOD denoising, allowing networks to generalize from uniform to non-uniform noise level maps. Guided by this principle, we develop several network components. Specifically, HNM stabilizes and corrects feature distributions. IGM provides powerful nonlinear representations and refines the transmission of information flow. Our work not only delivers a superior OOD denoiser but also advocates for a new perspective in designing deep networks for robust image restoration. Future work could explore this principle for more spatially non-uniform restoration tasks.

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Appendix

Theoretical Analysis

Following the design principle of scale equivariance, we propose the Constant Scaling (CS), Normalized Self-Modulator (NSM), and Interactive Gating Module (IGM) to construct a more powerful OOD denoiser. The proofs of equivariance for the main steps within these network components are provided below. To facilitate the proofs, we uniformly represent the input as vectors, as the key operations are performed on pixel feature vectors.

Lemma 3 (Homogeneity of CS). *The Constant Scaling operation is first-order homogeneous.*

Proof. Given an input $\mathbf{v} \in \mathbb{R}^c$ and a scalar $k > 0$, we need to show that $\text{CS}(k \cdot \mathbf{v}) = k \cdot \text{CS}(\mathbf{v})$.

Since the learnable parameter $\eta \in \mathbb{R}^c$ is independent of the input $\mathbf{v}$, it remains unchanged when the input is scaled. We proceed as follows:

$$\text{CS}(k \cdot \mathbf{v}) = \frac{k \cdot \mathbf{v}}{\sqrt{c}} \odot \eta = k \cdot \text{CS}(\mathbf{v}).$$

This completes the proof. $\square$

Lemma 4 (Homogeneity of NSM). *The Normalized Self-Modulator (NSM) operation is first-order homogeneous.*

Proof. Given an input $\mathbf{v} \in \mathbb{R}^c$ and a scalar $k > 0$, we need to show that $\text{NSM}(k \cdot \mathbf{v}) = k \cdot \text{NSM}(\mathbf{v})$.

First, we analyze the components:

- **Affine Parameters:** The 1×1 convolution is a linear transformation and thus satisfies first-order homogeneity:

$$[k \cdot \beta, k \cdot \gamma] \leftarrow \text{Conv}_{1 \times 1}(k \cdot \mathbf{v}).$$

- **Mean and Standard Deviation:** The mean and variance of the scaled vector $k \cdot \mathbf{v}$ are:

$$\begin{aligned} \mu(k \cdot \mathbf{v}) &= \frac{1}{c} \sum_{i=1}^c (k \cdot \mathbf{v}_i) = k \cdot \mu(\mathbf{v}). \\ \sigma^2(k \cdot \mathbf{v}) &= \frac{1}{c} \sum_{i=1}^c (k \cdot \mathbf{v}_i - k \cdot \mu(\mathbf{v}))^2 \\ &= \frac{k^2}{c} \sum_{i=1}^c (\mathbf{v}_i - \mu(\mathbf{v}))^2 = k^2 \cdot \sigma^2(\mathbf{v}). \end{aligned}$$

Therefore, the standard deviation is $\sigma(k \cdot \mathbf{v}) = k \cdot \sigma(\mathbf{v})$.

- **Normalized Vector:** The normalized vector part of NSM is scale-independent (zero-order homogeneous):

$$\frac{k \cdot \mathbf{v} - \mu(k \cdot \mathbf{v})}{\sigma(k \cdot \mathbf{v})} = \frac{\mathbf{v} - \mu(\mathbf{v})}{\sigma(\mathbf{v})}.$$

Now, we combine these results to analyze the full NSM operation:

$$\begin{aligned} \text{NSM}(k \cdot \mathbf{v}) &= k \cdot \gamma \odot \frac{k \cdot \mathbf{v} - \mu(k \cdot \mathbf{v})}{\sigma(k \cdot \mathbf{v})} + k \cdot \beta \\ &= k \cdot \text{NSM}(\mathbf{v}). \end{aligned}$$

This completes the proof. $\square$

Lemma 5 (Homogeneity of IGM). *The Interactive Gating Module (IGM) is a first-order homogeneous system.*

Proof. Given an input $\mathbf{v} \in \mathbb{R}^c$ and a scalar $k > 0$, we need to show that $\text{IGM}(k \cdot \mathbf{v}) = k \cdot \text{IGM}(\mathbf{v})$.

We will analyze the homogeneity of the numerator and the denominator of the IGM formula separately. Here, the convolution and split operations are omitted

- **Numerator Analysis:** The numerator is a Hadamard product. Its homogeneity is determined as follows:

$$(k \cdot \mathbf{v}_v) \odot (k \cdot \mathbf{v}_g) = k^2 \cdot (\mathbf{v}_v \odot \mathbf{v}_g).$$

Thus, the numerator is second-order homogeneous.

- **Denominator Analysis:** The denominator is the square root of the sum of variances. The variance of a scaled vector $\sigma^2(k \cdot \mathbf{v})$ is second-order homogeneous. Applying this property to the denominator, we get:

$$\sigma^2(k \cdot \mathbf{v}_v) + \sigma^2(k \cdot \mathbf{v}_g) = k^2 \cdot (\sigma^2(\mathbf{v}_v) + \sigma^2(\mathbf{v}_g)).$$

Combining the results for the numerator and the denominator, we analyze the full IGM operation:

$$\begin{aligned} \text{IGM}(k \cdot \mathbf{v}_v, k \cdot \mathbf{v}_g) &= \frac{(k \cdot \mathbf{v}_v) \odot (k \cdot \mathbf{v}_g)}{\sqrt{\sigma^2(k \cdot \mathbf{v}_v) + \sigma^2(k \cdot \mathbf{v}_g)}} \\ &= k \cdot \text{IGM}(\mathbf{v}_v, \mathbf{v}_g). \end{aligned}$$

This completes the proof. $\square$

Experiments

Implementation details. We extract 128×128 size of patches for training. The data augmentation is conducted on input patches with random horizontal flips and rotations. The batch size is set to 4. For synthetic OOD noises, we adopt the Adam solver with $\beta_1 = 0.9$ and $\beta_2 = 0.999$ for 1000 epochs. The learning rate starts from 2×10^{-4} and decays by a factor of 0.5 every 200 epochs. For real-world cases, we first freeze the pre-trained blind denoiser. Then, we set the epoch and initial learning rate to 250 and 1×10^{-4} , respectively, to train the lightweight noise translator based on our SEVB. The corresponding training process and loss function refer to this noise adaptive method (Ha et al. 2024). All the experiments are conducted with PyTorch and NVIDIA GeForce RTX 3090 GPUs. Our network and others are trained under the blind denoising task. To facilitate image processing, data centralization and its corresponding inverse operation are applied at the start and end of all networks, respectively.

Synthetic OOD Noise. For the OOD scenarios, we synthesize several pixel-dependent noises, and the corresponding noise level maps are spatially non-uniform. 1) *Poisson noise*: $Y = X + \alpha \cdot N_p$, where Y , X , and N_p respectively represent the observed image, the clear image, and the Poisson noise component. The synthesis of N_p depends on the pixels of X . $\alpha \in \{4, 5, 6\}$ represents the scaling factor of the noise level. 2) *Speckle noise*: $Y = X + \sigma \cdot \sqrt{X} \odot N$, where N is derived from the standard Gaussian distribution. The noise level at each position is affected by the corresponding pixel, while $\sigma \in \{60, 80, 90\}$ controls the global damage intensity. 3)

Method	Speckle Noise			Poisson Noise		
	$\sigma = 60$	$\sigma = 80$	$\sigma = 90$	$\alpha = 4$	$\alpha = 5$	$\alpha = 6$
Kodak24						
SADNet (Chang et al. 2020)	28.58/0.8236	26.76/0.7703	25.97/0.7409	28.20/0.8138	26.77/0.7711	25.53/0.7227
SwinIR (Liang et al. 2021)	29.11/0.8368	27.22/0.7866	26.33/0.7560	28.71/0.8272	27.21/0.7859	25.78/0.7328
Uformer (Wang et al. 2022)	28.97/0.8326	26.83/0.7567	25.81/0.7075	28.53/0.8202	26.82/0.7563	25.22/0.6752
Restormer (Zamir et al. 2022)	28.21/0.7837	25.71/0.6839	24.45/0.6257	27.74/0.7670	25.75/0.6847	23.74/0.5904
NAFNet (Chen et al. 2022)	26.75/0.7182	25.06/0.6563	24.36/0.6297	26.38/0.7040	25.07/0.6557	23.99/0.6144
GRL (Li et al. 2023)	29.08/0.8360	<u>27.23/0.7900</u>	<u>26.42/0.7682</u>	28.68/0.8265	<u>27.22/0.7897</u>	<u>25.96/0.7549</u>
CGNet (Ghasemabadi et al. 2024)	27.08/0.7213	25.02/0.6411	24.19/0.6068	26.61/0.7017	25.01/0.6383	23.72/0.5848
Xformer (Zhang et al. 2024)	27.84/0.7554	24.74/0.6145	23.28/0.5466	27.25/0.7298	24.74/0.6132	22.46/0.5065
MHNet (Gao et al. 2025)	27.11/0.7399	24.43/0.6212	23.10/0.5559	26.56/0.7157	24.42/0.6183	22.32/0.5149
AKDT (Brateanu et al. 2025)	27.49/0.7389	25.12/0.6382	24.09/0.5923	26.99/0.7175	25.11/0.6362	23.51/0.5637
ESC (Lee, Yun, and Ro 2025)	28.46/0.7851	26.08/0.6850	25.01/0.6351	27.98/0.7656	26.10/0.6834	24.41/0.6040
CRWKV (Chen et al. 2025)	26.50/0.7053	23.29/0.5742	21.87/0.5139	25.86/0.6785	23.33/0.5721	21.11/0.4782
Ours	29.35/0.8463	27.61/0.8096	26.86/0.7933	28.97/0.8385	27.60/0.8094	26.42/0.7838
Urban100						
SADNet (Chang et al. 2020)	26.06/0.8468	23.95/0.7910	23.08/0.7617	25.68/0.8380	24.03/0.7932	22.67/0.7458
SwinIR (Liang et al. 2021)	27.37/0.8794	25.10/0.8264	24.11/0.7955	26.90/0.8699	25.11/0.8266	23.56/0.7750
Uformer (Wang et al. 2022)	27.05/0.8699	24.56/0.7956	23.47/0.7510	26.58/0.8583	24.62/0.7971	22.92/0.7253
Restormer (Zamir et al. 2022)	26.54/0.8294	23.82/0.7419	22.56/0.6927	26.08/0.8172	23.94/0.7470	21.98/0.6675
NAFNet (Chen et al. 2022)	25.43/0.7915	23.49/0.7328	22.69/0.7047	25.07/0.7817	23.55/0.7351	22.30/0.6901
GRL (Li et al. 2023)	27.34/0.8787	25.09/0.8277	<u>24.15/0.8021</u>	26.87/0.8690	<u>25.11/0.8279</u>	<u>23.65/0.7868</u>
CGNet (Ghasemabadi et al. 2024)	25.92/0.8019	23.69/0.7330	22.77/0.6999	25.49/0.7901	23.75/0.7352	22.32/0.6829
Xformer (Zhang et al. 2024)	26.26/0.8078	23.16/0.6950	21.80/0.6386	25.71/0.7906	23.26/0.6987	21.15/0.6096
MHNet (Gao et al. 2025)	25.81/0.8121	23.01/0.7076	21.70/0.6474	25.30/0.7961	23.09/0.7108	21.03/0.6150
AKDT (Brateanu et al. 2025)	25.98/0.7973	23.59/0.7164	22.58/0.6774	25.53/0.7835	23.66/0.7187	22.08/0.6568
ESC (Lee, Yun, and Ro 2025)	26.53/0.8206	23.86/0.7307	22.74/0.6876	26.05/0.8059	23.96/0.7338	22.21/0.6655
CRWKV (Chen et al. 2025)	24.41/0.7439	21.42/0.6347	20.17/0.5845	23.88/0.7253	21.54/0.6383	19.60/0.5594
Ours	27.83/0.8944	25.74/0.8584	24.84/0.8405	27.40/0.8877	25.76/0.8589	24.36/0.8305
CBSD68						
SADNet (Chang et al. 2020)	27.02/0.8109	25.24/0.7532	24.50/0.7243	26.70/0.8011	25.31/0.7556	24.15/0.7093
SwinIR (Liang et al. 2021)	<u>28.05/0.8367</u>	<u>26.21/0.7864</u>	25.36/0.7580	<u>27.67/0.8272</u>	<u>26.21/0.7861</u>	24.86/0.7376
Uformer (Wang et al. 2022)	27.74/0.8274	25.67/0.7559	24.72/0.7124	27.33/0.8145	25.69/0.7552	24.19/0.6843
Restormer (Zamir et al. 2022)	26.74/0.7652	24.32/0.6738	23.19/0.6258	26.28/0.7488	24.38/0.6757	22.59/0.5984
NAFNet (Chen et al. 2022)	25.04/0.6973	23.41/0.6347	22.77/0.6084	24.66/0.6823	23.40/0.6330	22.41/0.5917
GRL (Li et al. 2023)	28.00/0.8344	26.18/0.7848	<u>25.40/0.7614</u>	27.62/0.8245	26.19/0.7849	<u>24.97/0.7476</u>
CGNet (Ghasemabadi et al. 2024)	25.81/0.7191	23.83/0.6454	23.03/0.6132	25.39/0.7032	23.85/0.6450	22.61/0.5943
Xformer (Zhang et al. 2024)	26.52/0.7474	23.66/0.6260	22.37/0.5674	25.96/0.7251	23.68/0.6262	21.66/0.5336
MHNet (Gao et al. 2025)	25.72/0.7292	23.20/0.6241	23.20/0.6241	25.22/0.7085	23.22/0.6234	21.29/0.5317
AKDT (Brateanu et al. 2025)	26.09/0.7295	23.82/0.6379	22.87/0.5962	25.62/0.7108	23.84/0.6377	22.36/0.5721
ESC (Lee, Yun, and Ro 2025)	27.13/0.7766	24.93/0.6908	23.96/0.6488	26.70/0.7598	24.96/0.6902	23.43/0.6229
CRWKV (Chen et al. 2025)	24.17/0.6607	21.14/0.5363	19.89/0.4813	23.49/0.6323	21.11/0.5322	19.17/0.4461
Ours	28.17/0.8407	26.44/0.7978	25.70/0.7786	27.81/0.8320	26.46/0.7980	25.31/0.7679

Table 5: Quantitative denoising comparisons (PSNR/SSIM) on the different datasets. The best results are marked in **bold**, while the second-best ones are underlined.

Mixture Noise: $Y = X + \sigma \cdot \sqrt{X} \odot N + \alpha \cdot N_p$, where $\sigma = 90$ and $\alpha = 6$. Its noise level map and base noise are more complex. 4) *Variants of Speckle Noise.* To further enhance the level map non-uniformity of Speckle noise, we adopt spatial functions (sine-cosine, peaks, and Gaussian kernels) to construct its variants (Variant 1-3). The form of the variant is $Y_{ij} = X_{ij} + \phi_{ij} \cdot \sqrt{X_{ij}} \cdot N_{ij}$, where ϕ_{ij} represents the local scaling factor generated by the spatial position (i, j) . The overall value of ϕ is normalized within the range of $[60, 120]$. 5) *Extension of base noise.* We replace the N in the Speckle noise with the N_l sampled from the Laplace distribution, expanding the evaluation of OOD robustness.

Feature Analysis. We provide additional examples of feature comparison. As shown in Fig. 5, our scale-equivariant network consistently captures highly similar features, even when faced with significant differences in global corruption intensity and inconsistent spatial variation patterns in the level map. Conversely, networks that violate this design principle are overly sensitive to such variations. For instance, networks incorporating scale-independent Layer Norm or exponential activations (w/o HNM+ELU) produce markedly dissimilar feature maps. Furthermore, while these non-equivariant designs may yield usable features at low noise levels, they severely distort the network’s representa-

Method	Speckle Noise*			Speckle Noise variant*		
	$\sigma = 60$	$\sigma = 80$	$\sigma = 90$	Variant 1	Variant 2	Variant 3
Kodak24						
SADNet (Chang et al. 2020)	26.83/0.7717	25.15/0.7064	24.41/0.6701	24.23/0.6663	24.45/0.6730	23.92/0.6471
SwinIR (Liang et al. 2021)	27.36/0.7871	25.40/0.7147	24.49/0.6725	24.15/0.6625	24.51/0.6752	23.83/0.6441
Uformer (Wang et al. 2022)	26.92/0.7501	24.74/0.6414	23.81/0.5901	23.51/0.5845	23.86/0.5965	23.24/0.5654
Restormer (Zamir et al. 2022)	25.77/0.6786	23.21/0.5588	22.12/0.5065	21.40/0.5024	21.97/0.5160	21.08/0.4791
NAFNet (Chen et al. 2022)	25.30/0.6619	23.92/0.6122	23.35/0.5919	22.86/0.5785	23.21/0.5906	22.66/0.5665
GRL (Li et al. 2023)	27.29/0.7845	<u>25.52/0.7358</u>	<u>24.78/0.7138</u>	<u>24.59/0.7091</u>	<u>24.82/0.7139</u>	<u>24.32/0.7014</u>
CGNet (Ghasemabadi et al. 2024)	25.13/0.6382	23.55/0.5727	22.94/0.5467	22.44/0.5422	22.82/0.5539	22.17/0.5219
Xformer (Zhang et al. 2024)	24.95/0.6188	21.96/0.4765	20.79/0.4214	20.42/0.4198	20.80/0.4277	20.05/0.4031
MHNet (Gao et al. 2025)	24.41/0.6095	21.75/0.4741	20.63/0.4181	20.00/0.4132	20.53/0.4248	19.70/0.3960
AKDT (Brateanu et al. 2025)	25.23/0.6368	23.16/0.5437	22.34/0.5057	21.95/0.5040	22.33/0.5125	21.66/0.4875
ESC (Lee, Yun, and Ro 2025)	25.95/0.6752	23.77/0.5722	22.90/0.5282	22.54/0.5227	22.91/0.5323	22.24/0.5075
CRWKV (Chen et al. 2025)	21.95/0.4964	19.36/0.3847	18.37/0.3416	17.90/0.3385	18.30/0.3429	17.58/0.3261
Ours	27.69/0.8088	26.02/0.7730	25.31/0.7579	25.13/0.7557	25.35/0.7576	24.87/0.7509
Urban100						
SADNet (Chang et al. 2020)	24.03/0.7909	22.30/0.7274	21.61/0.6954	21.44/0.6904	21.64/0.6978	21.40/0.6859
SwinIR (Liang et al. 2021)	25.32/0.8264	23.19/0.7531	22.31/0.7150	22.09/0.7076	22.32/0.7174	22.05/0.7053
Uformer (Wang et al. 2022)	24.75/0.7940	22.48/0.6964	21.58/0.6519	21.35/0.6467	21.59/0.6569	21.34/0.6440
Restormer (Zamir et al. 2022)	23.89/0.7386	21.35/0.6349	20.32/0.5882	19.84/0.5822	20.21/0.5942	19.81/0.5783
NAFNet (Chen et al. 2022)	23.68/0.7357	22.06/0.6778	21.41/0.6519	21.07/0.6383	21.32/0.6524	21.04/0.6371
GRL (Li et al. 2023)	25.21/0.8224	<u>23.24/0.7648</u>	<u>22.46/0.7385</u>	<u>22.29/0.7342</u>	<u>22.49/0.7389</u>	<u>22.24/0.7322</u>
CGNet (Ghasemabadi et al. 2024)	23.83/0.7333	22.01/0.6660	21.31/0.6372	20.97/0.6275	21.24/0.6414	20.90/0.6232
Xformer (Zhang et al. 2024)	23.40/0.7013	20.63/0.5836	19.58/0.5366	19.31/0.5342	19.58/0.5425	19.25/0.5305
MHNet (Gao et al. 2025)	23.01/0.7000	20.37/0.5776	19.32/0.5270	18.86/0.5211	19.24/0.5327	18.86/0.5181
AKDT (Brateanu et al. 2025)	23.74/0.7181	21.70/0.6373	20.90/0.6032	20.63/0.5996	20.88/0.6086	20.57/0.5955
ESC (Lee, Yun, and Ro 2025)	23.81/0.7271	21.58/0.6371	20.73/0.5991	20.48/0.5948	20.72/0.6025	20.41/0.5905
CRWKV (Chen et al. 2025)	21.45/0.6315	19.00/0.5289	18.08/0.4880	17.70/0.4843	18.02/0.4921	17.61/0.4783
Ours	25.91/0.8593	24.00/0.8193	23.23/0.8005	23.06/0.7975	23.27/0.8005	23.01/0.7959
CBS68						
SADNet (Chang et al. 2020)	25.31/0.7543	23.80/0.6912	23.16/0.6590	23.02/0.6554	23.19/0.6624	22.94/0.6510
SwinIR (Liang et al. 2021)	<u>26.36/0.7855</u>	24.50/0.7176	23.66/0.6805	23.44/0.6732	23.64/0.6826	23.37/0.6695
Uformer (Wang et al. 2022)	25.79/0.7500	23.77/0.6535	22.91/0.6082	22.69/0.6031	22.91/0.6136	22.63/0.5998
Restormer (Zamir et al. 2022)	24.43/0.6703	22.13/0.5682	21.15/0.5223	20.63/0.5166	21.00/0.5294	20.49/0.5077
NAFNet (Chen et al. 2022)	23.52/0.6344	22.26/0.5809	21.77/0.5593	21.43/0.5500	21.64/0.5603	21.31/0.5434
GRL (Li et al. 2023)	26.26/0.7783	<u>24.56/0.7260</u>	<u>23.85/0.7024</u>	<u>23.72/0.6985</u>	<u>23.86/0.7023</u>	<u>23.64/0.6976</u>
CGNet (Ghasemabadi et al. 2024)	23.97/0.6446	22.42/0.5796	21.82/0.5528	21.49/0.5485	21.71/0.5589	21.32/0.5359
Xformer (Zhang et al. 2024)	23.90/0.6302	21.23/0.5053	20.18/0.4546	19.90/0.4511	20.16/0.4603	19.81/0.4493
MHNet (Gao et al. 2025)	23.21/0.6153	20.74/0.4933	19.70/0.4416	19.21/0.4357	19.56/0.4467	19.14/0.4339
AKDT (Brateanu et al. 2025)	23.97/0.6375	22.04/0.5514	21.28/0.5163	21.01/0.5140	21.24/0.5217	20.88/0.5091
ESC (Lee, Yun, and Ro 2025)	24.87/0.6843	22.88/0.5950	22.08/0.5558	21.84/0.5504	22.07/0.5604	21.67/0.5440
CRWKV (Chen et al. 2025)	21.04/0.5211	18.57/0.4077	17.63/0.3637	17.29/0.3604	17.56/0.3674	17.18/0.3586
Ours	26.55/0.7974	24.93/0.7549	24.25/0.7365	24.13/0.7337	24.27/0.7358	24.05/0.7345

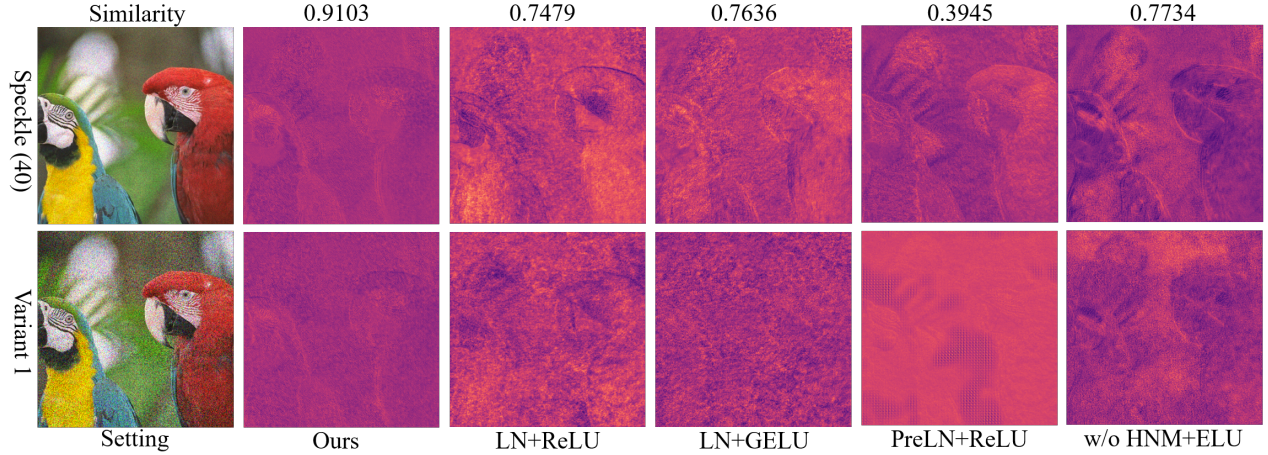
Table 6: Quantitative denoising comparisons (PSNR/SSIM) on the different datasets. The best results are marked in **bold**, while the second-best ones are underlined. "*" indicates that the base noise follows the standard Laplacian distribution.

tions at high levels. particularly, placing Layer Norm at the front of a block (PreLN+ReLU) results in more severe feature distortion under high noise compared to putting it in the middle (LN+ReLU). In short, scale equivariance enables the network to generate aligned feature maps that are decoupled from the noise level map, thereby laying the foundation for robust OOD denoising.

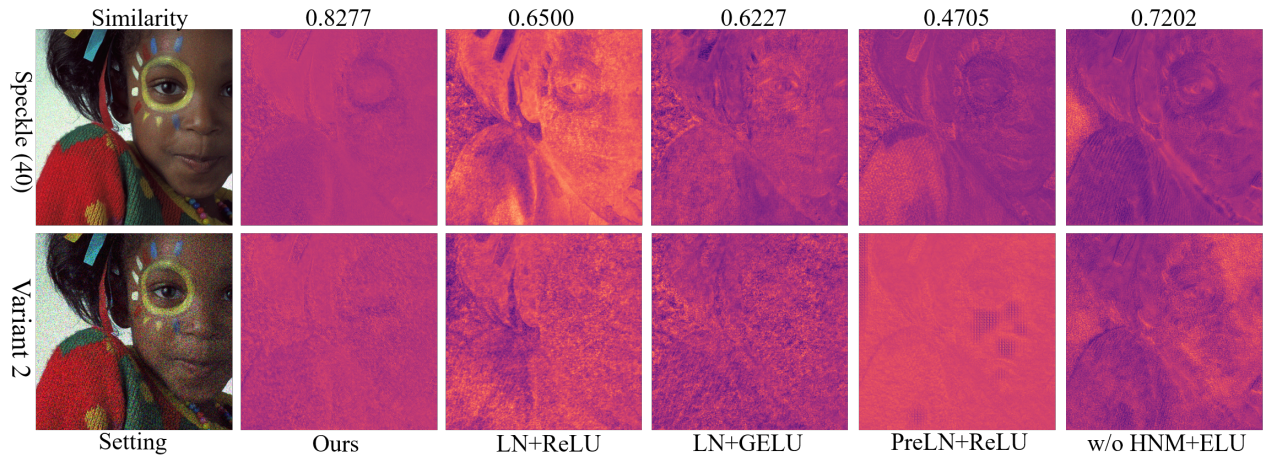
Quantitative Comparison. We provide more detailed quantitative comparisons for Speckle and Poisson noise. As shown in Tab. 5, our scale-equivariant network consistently achieves superior OOD denoising performance across various corruption levels. Conversely, existing methods, which

violate the principle of scale equivariance, suffer from a lack of robustness, even when equipped with advanced modules. Furthermore, when the random distribution used to synthesize Speckle noise is replaced from Gaussian to Laplacian, our scale-equivariant network still demonstrates OOD denoising advantage, as shown in Tab. 6. This indicates its robustness to changes in the base noise distribution.

Qualitative Comparison. As shown in Fig. 6 to Fig. 13, our method produces clean denoised images, restoring clear edges and partial texture information, even at high levels of corruption. In contrast, affected by the non-uniform noise level map, the denoising capabilities of existing methods are



(a) Speckle Noise of $\sigma = 40$ and its variant 1



(b) Speckle Noise of $\sigma = 40$ and its variant 2

Figure 5: Comparison in visual and cosine similarities of feature maps under different noise level maps.

Method	Params (M)	MACs (G)	Times (ms)	PSNR/SSIM
SADNet	4.23	18.97	0.8	25.70/0.7238
SwinIR	3.14	204.99	53.1	25.92/0.7325
Uformer	20.63	41.55	5.1	25.44/0.6844
Restormer	26.13	141.24	4.8	23.67/0.6038
NAFNet	29.16	16.25	3.4	23.92/0.6161
GRL	5.11	316.57	10.4	26.18/0.7593
CGNet	185.38	115.08	5.0	23.75/0.5956
Xformer	25.22	143.12	33.0	24.36/0.5898
MHNet	67.54	127.40	6.7	23.98/0.5878
AKDT	11.43	56.89	0.7	25.08/0.6292
ESC	0.94	51.59	0.5	24.61/0.6206
CRWKV	21.08	113.55	31.2	23.00/0.5585
Ours	31.32	158.69	1.4	26.62/0.7889
Ours(lw)	16.28	76.50	0.7	26.56/0.7857

Table 7: Comparison of different networks in terms of complexity, per-image inference time, and average OOD evaluation on the Kodak24 dataset.

spatially inconsistent and exhibit different patterns of degradation, *e.g.*, coarse results, residual noise, and artifacts.

Real-World Extension. The principle of scale equivari-

ance assumes independently distributed noise, a condition not met by spatially correlated real-world camera noise. To bridge this gap, we construct the SEVB-based noise translator trained on synthetic ISP noise. This translator maps correlated noise to a distribution that our pretrained denoiser can process. As shown in Fig. 14 and Fig. 15, denoising networks trained directly on ISP noise fail to generalize, whereas our approach demonstrates outstanding robustness. This suggests that our scale-equivariant network offers a powerful denoising prior, and combining it with noise adaptation provides an effective path for generalizing from synthetic to real data.

Complexity Comparison According to Tab. 7, our model offers fast inference, though its complexity is not the lowest. We observe that OOD robustness does not correlate with model complexity, as both more and less complex networks perform poorly. To confirm this, we build a lightweight version of our network that retains comparable robustness with only half the complexity. This result confirms that OOD robustness is primarily determined by the scale-equivariant design, not model capacity.

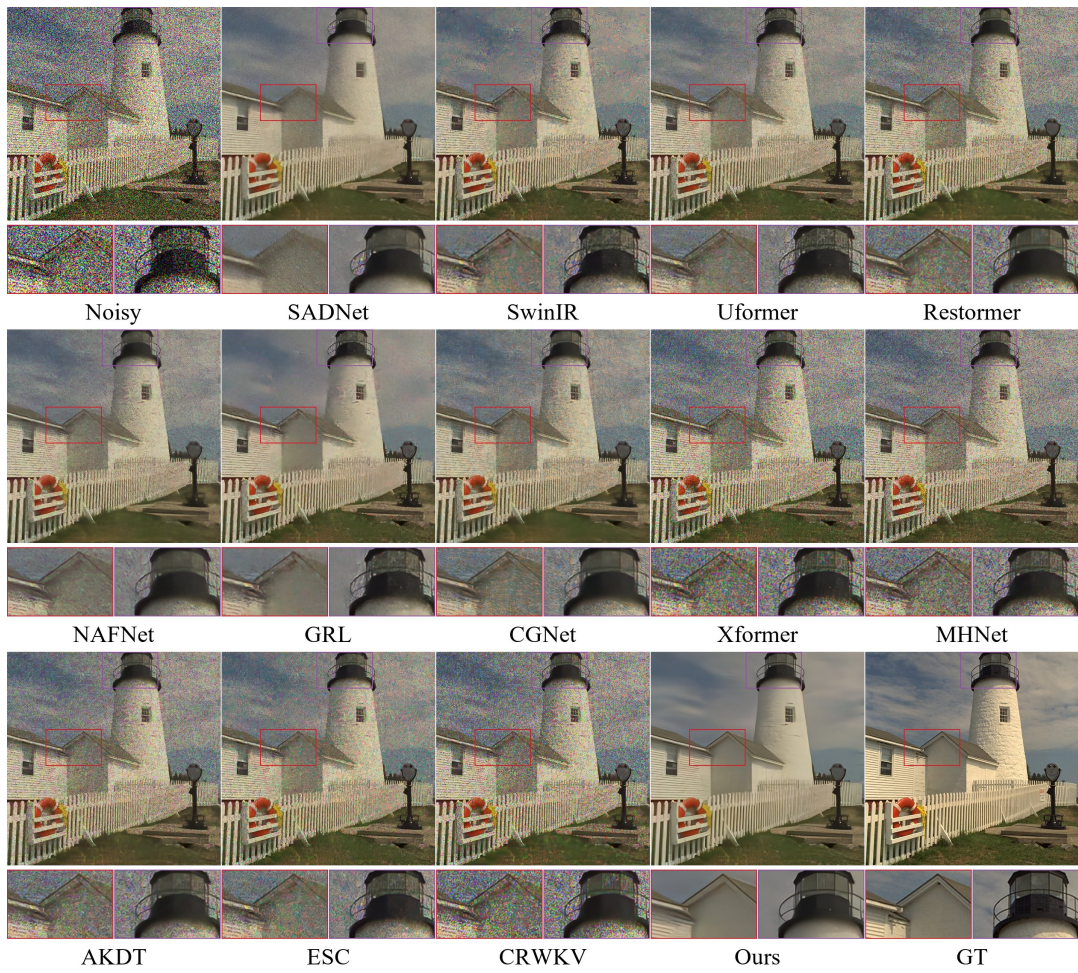


Figure 6: Qualitative comparisons on mixture noise of $\sigma = 90$ and $\alpha = 6$.

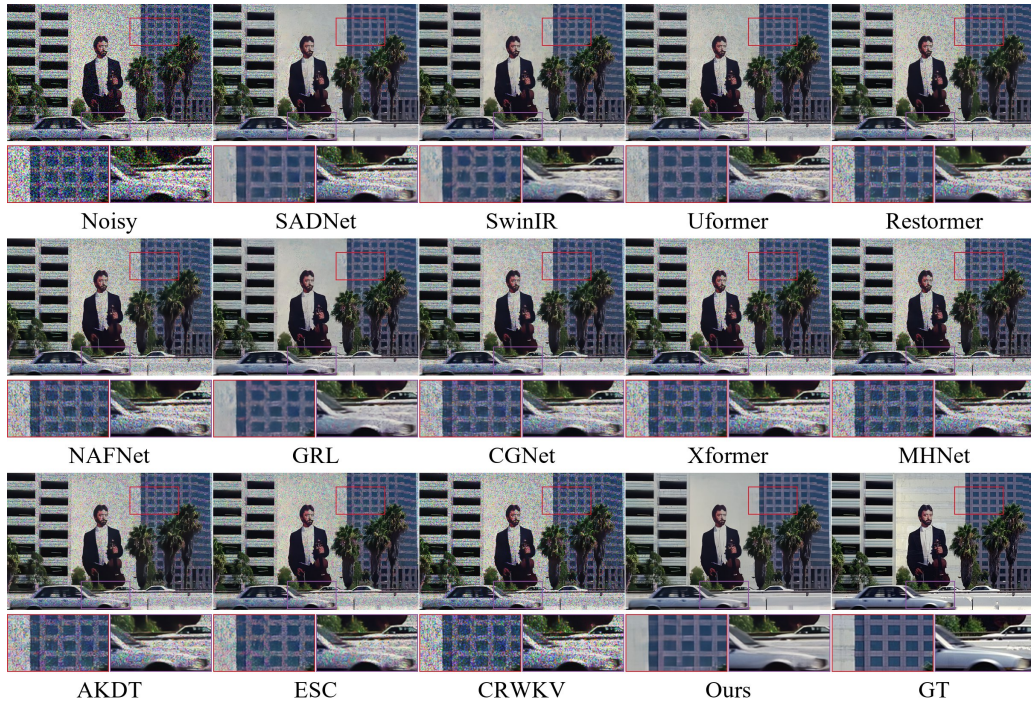


Figure 7: Qualitative comparisons on Poisson noise of $\alpha = 6$.

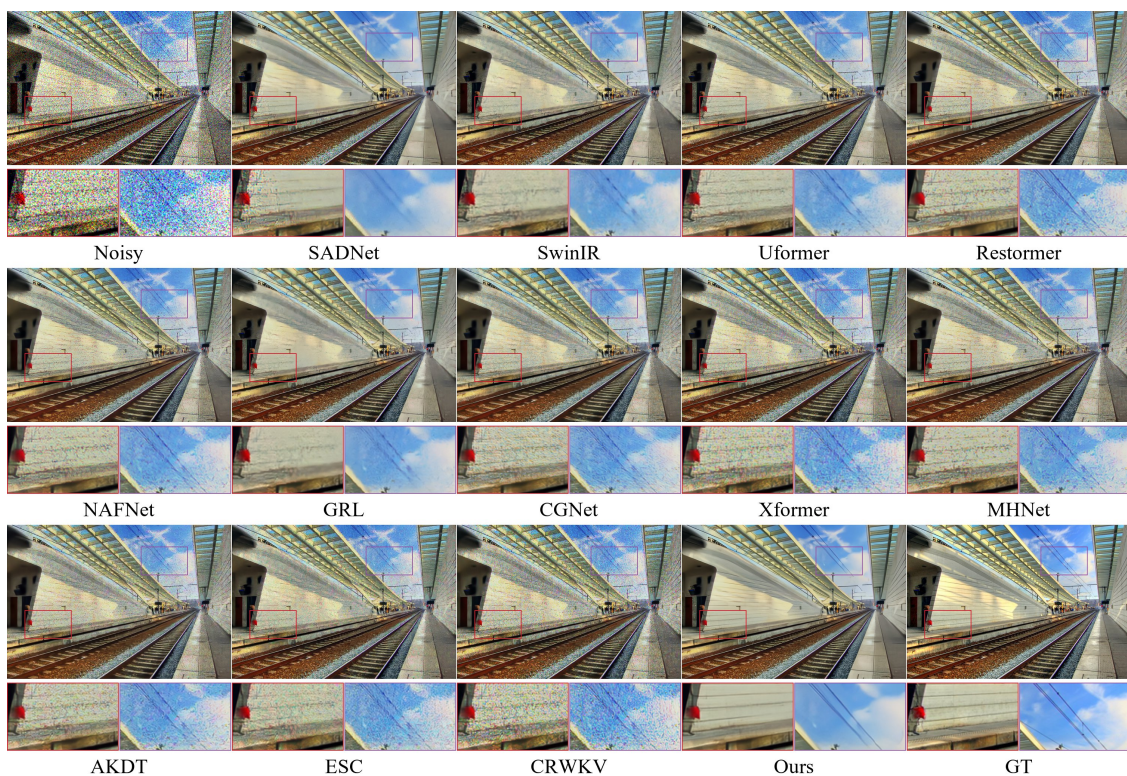


Figure 8: Qualitative comparisons on Poisson noise of $\alpha = 5$.

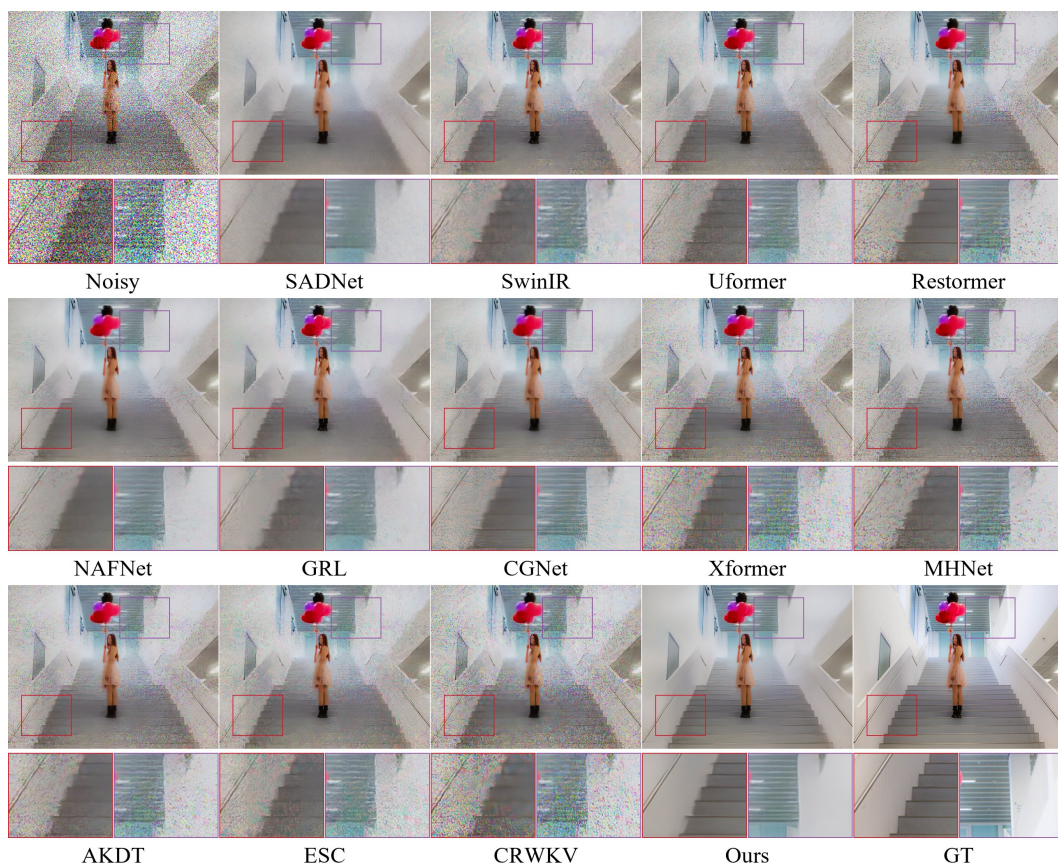


Figure 9: Qualitative comparisons on variant 1 of Speckle noise.

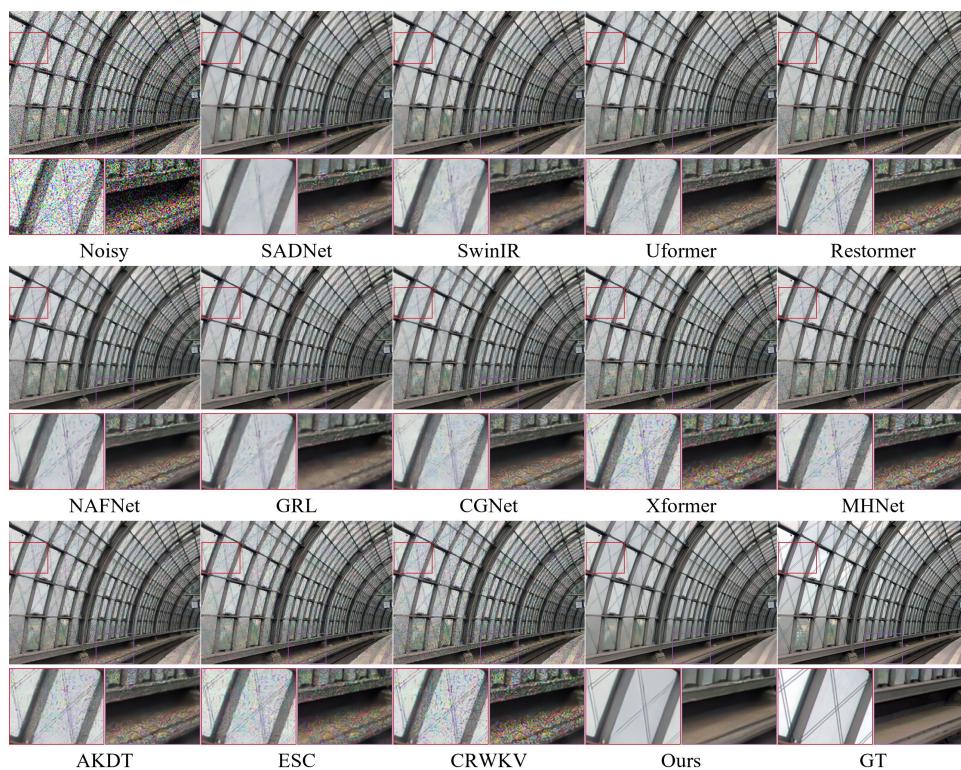


Figure 10: Qualitative comparisons on variant 2 of Speckle noise.

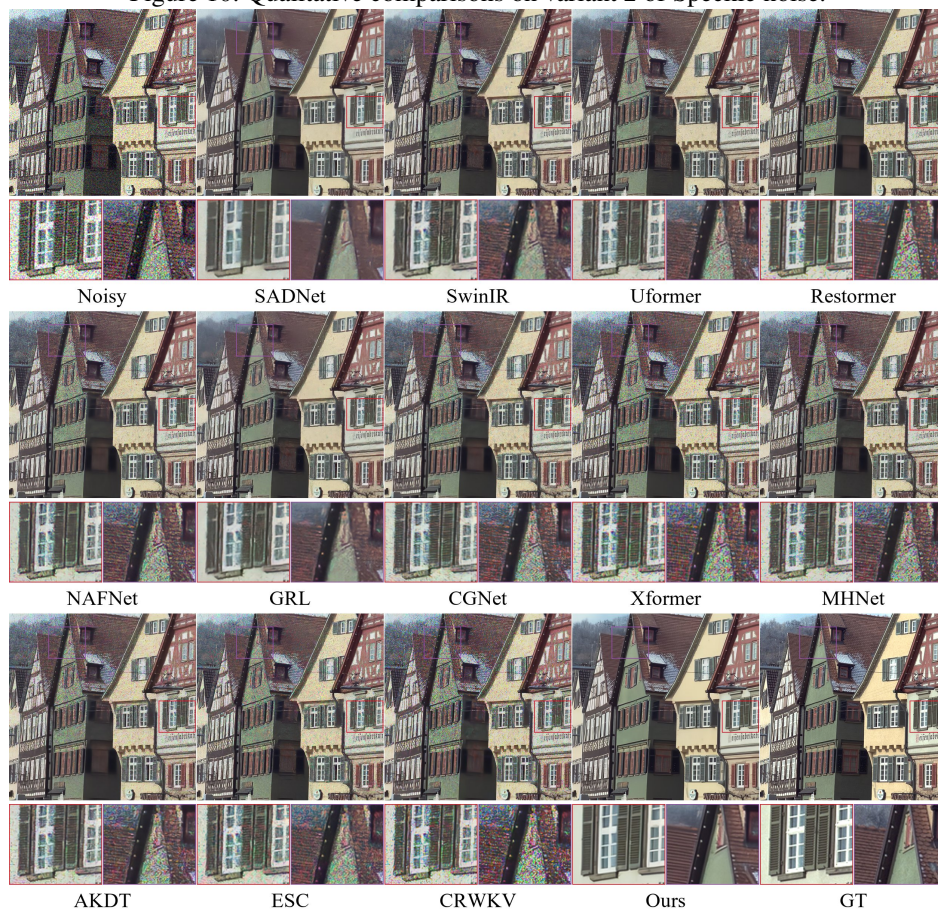


Figure 11: Qualitative comparisons on variant 3 of Speckle noise.

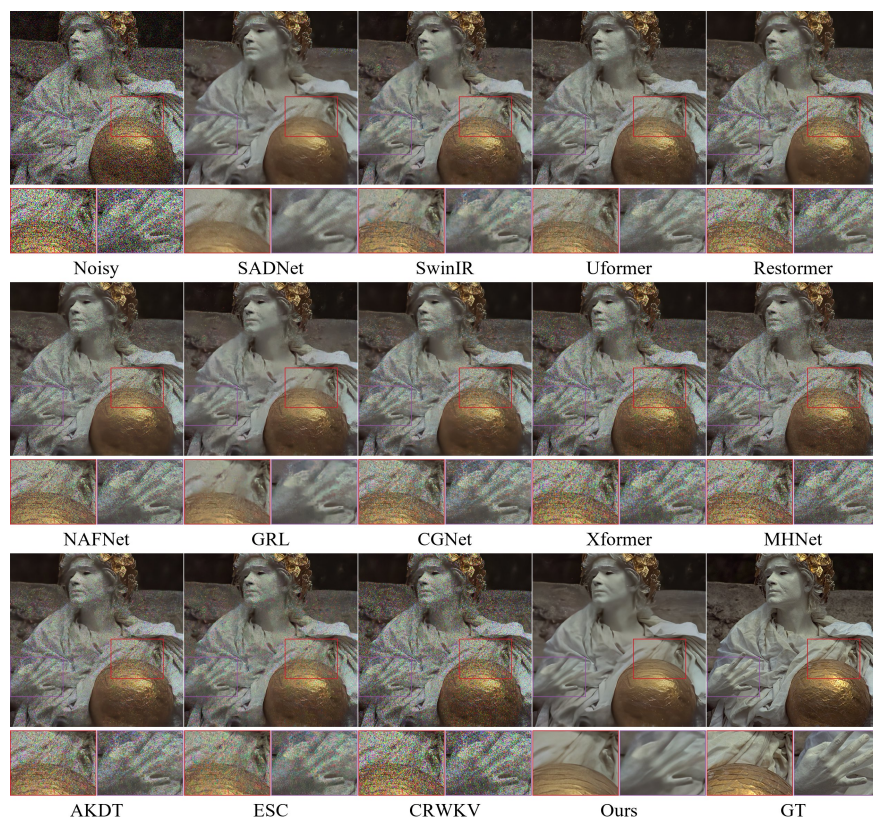


Figure 12: Qualitative comparisons on variant 1 of Speckle noise*.

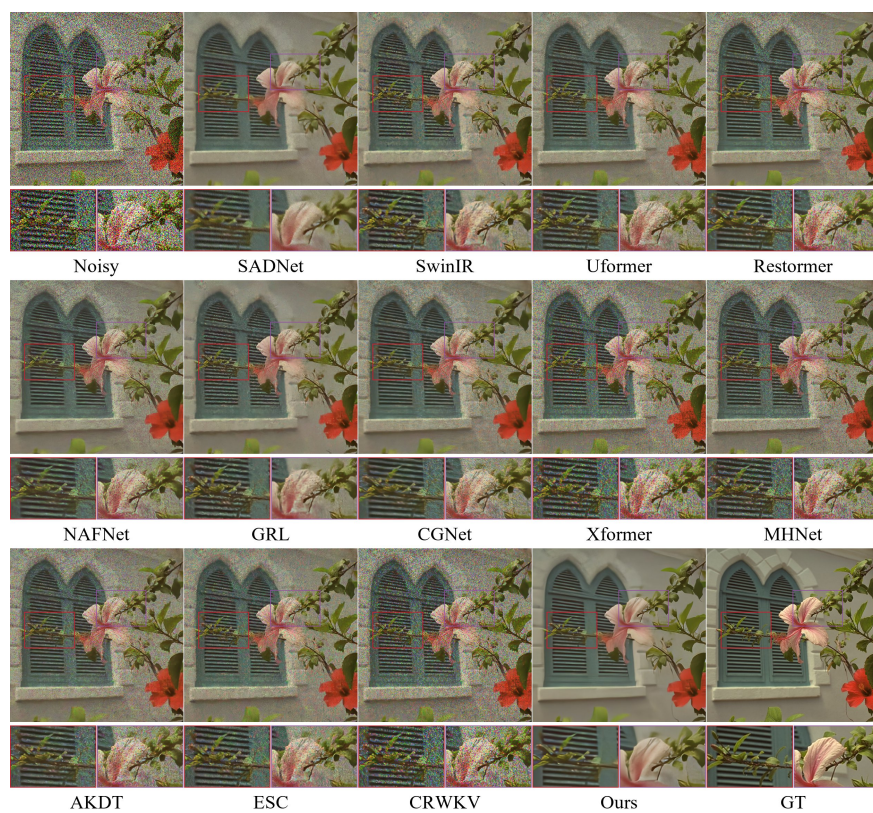


Figure 13: Qualitative comparisons on variant 3 of Speckle noise*.



Figure 14: Qualitative comparisons on CC benchmark.

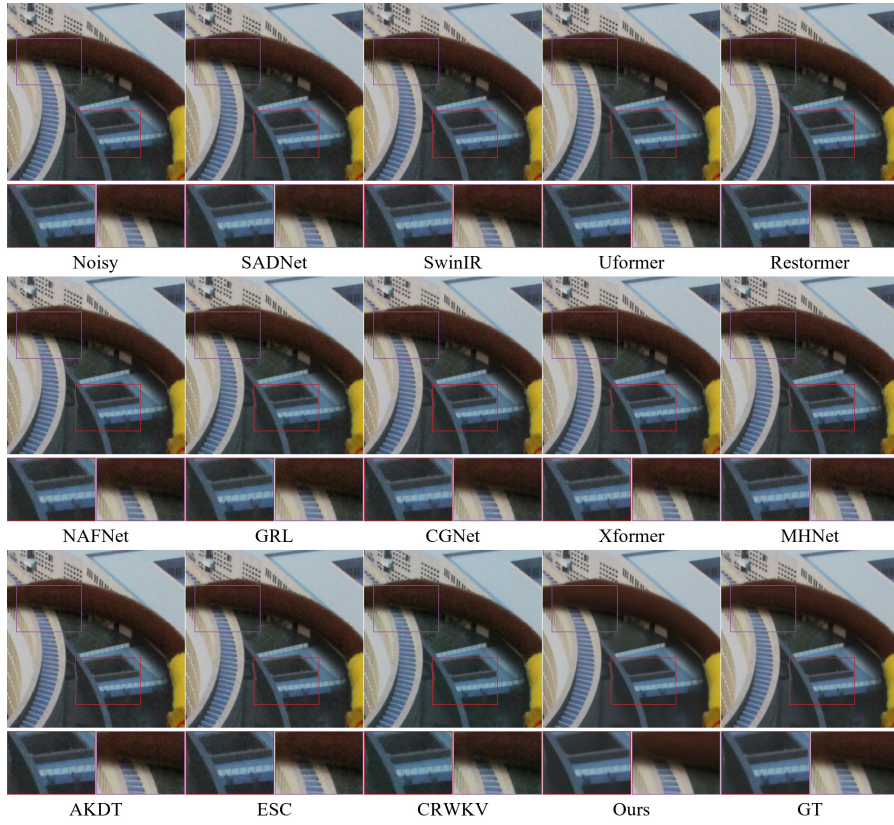


Figure 15: Qualitative comparisons on HighISO benchmark.