Lens sensitivity to aberration: scaling rules

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Abstract We show that aberration sensitivities are independent of lens scaling when the perturbation amount is kept the same. This behavior is in agreement with the results of actual calculation. Further, we point out that aberration sensitivities scale in the same way as aberration scale as a function of the lens field of view and aperture. © *2022 Society of Photo-Optical Instrumentation Engineers (SPIE)* [DOI: 10.1117/1.OE.61.10.105102]

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1 Introduction

Optical system performance as a function of the optical aperture scale is a critical issue in lens design. It has long been understood that as aperture increases for a given optimized lens design, field of view must be scaled down[1] to maintain performance. Conventional system design has attempted to counteract this scaling law by increasing lens complexity as lens scale increases.

Over the past decade, however, an alternative strategy based on parallel microcamera arrays has become increasingly popular.[2] Array cameras accept decreased field of view within each microcamera as scale increases and restore field of view through multiaperture image fusion. The emergence of neural processing strategies, analogous to the biological visual cortex, render parallel processing cameras increasingly attractive.[3]

Lens scale is a critical factor in array camera design. The overall goal is to select a microcamera aperture that matches desired instantaneous field of view targets while selecting a microcamera field of view that minimizes system cost per pixel. Over a wide range of resolution targets, this goal can be achieved using a common sensor size with microcamera field of view decreasing linearly as a function of the instantaneous field of view.[4] Although this approach works well from a design perspective, one may reasonably wonder whether or not manufacturing tolerances must be tightened as focal length increases and how lens sensitivities scale.

Lens design optimization often includes finding lens configurations with low sensitivity to manufacturing errors. The subject of lens sensitivity to aberration has become of increasing interest because it affects lens cost and because calculation power is available for lens analysis. There have been several discussions[5–14] that address lens sensitivity and approaches to mitigate it. However, the general nature of lens sensitivity has not been addressed; this is how lens sensitivity behaves with lens size, field of view, and optical speed.

Using structural aberration coefficients, we show that lens sensitivities are independent of lens scaling and that they follow the same field and aperture dependences that aberrations follow. This holds when the sensitivities are calculated using fixed amounts of surface error. We also present sensitivity analyses of some simple lens systems and provide for lens comparison purposes what can be called characteristic lens sensitivities. Because of the behavior of aberration sensitivities, one can use them to characterize a given lens form. Then lens form comparison can be carried through the characteristics sensitivities. Our sensitivity analysis applies to the monochromatic primary aberrations and has been verified with direct sensitivity computation within a lens design program. The results presented are helpful for lens tolerancing analysis.

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2 Primary Aberration Lens Sensitivities

Consider an axially symmetric lens system that is perturbed from its nominal design by small changes in radii, surface spacings, and surface tilts. The main effects from the system perturbation are small changes in the optical power and in the primary aberrations of spherical W_{040}, coma W_{131}, astigmatism W_{222}, field curvature W_{220}, and distortion W_{311}. In addition, and because the loss of axial symmetry, there can be uniform astigmatism W_{UA}, anamorphic distortion W_{AD}, uniform coma W_{UC}, linear astigmatism W_{LA}, field tilt W_{FT}, and smile W_{SD} and keystone W_{KD} distortion.[15,16]

The aberration coefficients[15] of a lens system can be written as a function of the Lagrange invariant , the system optical power Φ, the first-order marginal ray height y_P at the principal planes, and the structural coefficients σ as shown in Table 1. The structural aberration coefficients σ are independent of lens system scale. The use of sigma σ in the structural coefficients should not be confused with the standard deviation of the wavefront σ_W error.

Under the system perturbation, aberration values will change. For example, spherical aberration will change to $W_{040} = \frac{1}{32} y_P^4 \Phi^3 \sigma'_{\mathrm{SA}}$, where σ'_{SA} is the new value of the structural aberration coefficient. To first order of approximation, we can write $\sigma'_{\mathrm{SA}} \cong \sigma_{\mathrm{SA}} + \frac{d\sigma_{\mathrm{SA}}}{dP} \Delta P$, where P is the perturbed parameter. Then a sensitivity ΔW_{040} to aberration change can be written as $\Delta W_{040} = \frac{1}{32} y_P^4 \Phi^3 \frac{d\sigma_{\mathrm{SA}}}{dP} \Delta P$.

If the lens system is scaled by a factor K, we have

$$P_K = KP, \tag{13}$$

Table 1 Wave coefficients in terms of structural aberration coefficients σ (pupils located at the principal planes).

Spherical aberration	$W_{040} = \frac{1}{32} y_P^4 \Phi^3 \sigma_{\mathrm{SA}}$	(1)
Linear coma	$W_{131} = \frac{1}{4} Ж y_P^2 \Phi^2 \sigma_{\mathrm{LC}}$	(2)
Quadratic astigmatism	$W_{222} = \frac{1}{2} Ж^2 \Phi \sigma_{\mathrm{QA}}$	(3)
Field curvature	$W_{220} = \frac{1}{4} Ж^2 \Phi (\sigma_{\mathrm{QA}} + \sigma_{\mathrm{FC}})$	(4)
Cubic distortion	$W_{311} = \frac{Ж^2 \sigma_{\mathrm{CD}}}{y_p^2}$	(5)
Uniform astigmatism	$W_{\mathrm{UA}} = \frac{1}{2} y_P^2 \Phi \sigma_{\mathrm{UA}}$	(6)
Anamorphic distortion	$W_{\mathrm{AD}} = 2Ж \sigma_{\mathrm{AD}}$	(7)
Uniform coma	$W_{\mathrm{UC}} = \frac{1}{4} y_P^3 \Phi^2 \sigma_{\mathrm{UC}}$	(8)
Linear astigmatism	$W_{\mathrm{LA}} = Ж y_p \Phi \sigma_{\mathrm{LA}}$	(9)
Field tilt	$W_{\mathrm{FT}} = \frac{1}{2} Ж y_p \Phi \sigma_{\mathrm{FT}}$	(10)
Smile distortion	$W_{\mathrm{SD}} = \frac{1}{2} \frac{Ж^2}{y_p} \sigma_{\mathrm{SD}}$	(11)
Keystone distortion	$W_{\mathrm{KD}} = \frac{1}{2} \frac{Ж^2}{y_p} \sigma_{\mathrm{KD}}$	(12)

Table 2 Aberration coefficient functional dependence with the field of view $\vec{H}$ and the system aperture $\vec{\rho}$. The direction of aberration symmetry is indicated by the vector $\vec{i}$.

Spherical aberration $W_{040}(\vec{\rho}\cdot\vec{\rho})^2$	Linear coma $W_{131}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	Quadratic astigmatism $W_{222}(\vec{H}\cdot\vec{\rho})^2$	Field curvature $W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$
Cubic distortion $W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})$	Uniform astigmatism $W_{\mathrm{UA}}(\vec{i}\cdot\vec{\rho})^2$	Anamorphic distortion $W_{\mathrm{AD}}(\vec{i}\cdot\vec{H})(\vec{i}\cdot\vec{\rho})$	Uniform coma $W_{\mathrm{UC}}(\vec{i}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$
Linear astigmatism $W_{\mathrm{LA}}(\vec{H}\cdot\vec{\rho})(\vec{i}\cdot\vec{\rho})$	Field tilt $W_{\mathrm{FT}}(\vec{i}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$	Smile distortion $W_{\mathrm{SD}}(\vec{H}\cdot\vec{H})(\vec{i}\cdot\vec{\rho})$	Keystone distortion $W_{\mathrm{KD}}(\vec{i}\cdot\vec{H})(\vec{i}\cdot\vec{\rho})$

$$y_{PK} = K y_P, \tag{14}$$

$$\Phi_K = \Phi/K, \tag{15}$$

$$dP_K = K dP, \tag{16}$$

$$\frac{d\sigma_{\mathrm{SA}}}{dP_K} = \frac{d\sigma_{\mathrm{SA}}}{dP}\frac{dP}{dP_K}\Delta P. \tag{17}$$

Then the scaled system sensitivity to aberration is

$$\Delta W_{040K} = \frac{1}{32} y_{PK}^4 \Phi_K^3 \frac{d\sigma_{\mathrm{SA}}}{dP_K}\Delta P = \frac{1}{32} y_{PK}^4 \Phi_K^3 \frac{d\sigma_{\mathrm{SA}}}{dP}\frac{dP}{dP_K}\Delta P = \frac{1}{32} y_P^4 \Phi^3 \frac{d\sigma_{\mathrm{SA}}}{dP}\Delta P. \tag{18}$$

This shows that if the parameter change ΔP is the same for the scaled and unscaled system, then the sensitivity to aberration is independent from lens scaling. This result applies to other aberration sensitivities and indicates that sensitivities to aberration follow the same dependence with respect to field of view and optical speed that aberrations follow. Thus if the optical speed is doubled, then the spherical aberration sensitivity increases by a factor of sixteen; if the field of view is doubled, then the coma sensitivity doubles, and the sensitivity to astigmatism quadruples. This behavior agrees with actual calculation of aberration sensitivities when the lens changes are specified with surface sag in Newton rings, surface axial spacings, and indicator run out for surface tilt. Table 2 provides the functional dependence of each aberration with respect to the field of view vector $\vec{H}$ and the system aperture vector $\vec{\rho}$.

3 Thin Lens Sensitivities

Since the structural aberration coefficients of a thin lens are known, we can calculate analytically the aberration sensitivities. We treat the case of spherical aberration, linear coma, and quadratic astigmatism. We have that for spherical aberration, linear coma, and quadratic astigmatism the structural coefficients are

$$\sigma_{\mathrm{SA}} = AX^2 - BXY + CY^2 + D, \tag{19}$$

$$\sigma_{\mathrm{LC}} = EX - FY, \tag{20}$$

$$\sigma_{\mathrm{QA}} = 1, \tag{21}$$

where $X = -\frac{r_1+r_2}{r_1-r_2}$ and $Y = \frac{1+m}{1-m}$ are, respectively, the shape factor and conjugate factor in which the thin lens works, r is the surface radius of curvature, m is the transverse magnification, and the coefficients A, B, C, D, E, and F depend only on the lens material index of refraction.[15]

For errors on the shape factor X of the thin lens, the spherical aberration sensitivity is

$$\Delta W_{040} = \frac{1}{32} y_P^4 \Phi^3 \frac{d\sigma_{\mathrm{SA}}}{dP}\Delta P = \frac{1}{32} y_P^4 \Phi^3 (2AX - BY)\Delta X. \tag{22}$$

To the first order of approximation, there is minimum sensitivity when $2AX - BY = 0$; this coincides with the minimum amount of spherical aberration for a given shape factor X and conjugate factor Y. Thus aberration sensitivity decreases as the amount of aberration decreases. This also suggests that reducing the amount of aberration propagating in a lens system reduces the lens sensitivity to aberration.[17]

The linear coma sensitivity is $\Delta W_{131} = \frac{1}{4}\text{Ж}y_P^2\Phi^2 E\Delta X$; this indicates that it is independent of the lens shape factor. The sensitivity to quadratic astigmatism is $\Delta W_{222} = 0$; this indicates that there is no change of astigmatism for errors on the shape of the thin lens when the stop aperture is located at the thin lens.

4 Aplanatic Doublets

To illustrate the calculation of sensitivities, consider two aplanatic doublets as shown in Fig. 1, one cemented and the other split. Tables 3 and 4 provide the sensitivity analysis for spherical aberration, linear coma, quadratic astigmatism, uniform coma, linear astigmatism, and field tilt. Uniform astigmatism is not included because as it depends on the square of surface tilt, it is typically negligible. Distortion aberration does not affect image sharpness, and it is not included in the analysis.

The analysis in Tables 3 and 4 provides surface by surface the sensitivities for surface sag errors of three Newton rings at 633 nm, then for surface axial spacings errors of 0.025 mm, and finally for surface tilts expressed as half the lens total indicator run-out (TIR) of 0.0075 mm. The TIR is the maximum thickness difference of a lens when measured at the lens edge. The root sum squares (RSS) of the surface sensitivities for each category are also given as well as the expected wavefront change $\Delta\sigma_W$ calculated using the RSS values.

The variance of the wavefront is given by

$$\begin{aligned}\sigma_W^2 = &\frac{1}{2}\left(W_{020} + \frac{1}{2}W_{\text{UA}} + W_{040} + \left(W_{\text{FT}} + \frac{1}{2}W_{\text{LA}}\right)H + \left(W_{220} + \frac{1}{2}W_{222}\right)H^2\right)^2 \\ &+ \frac{1}{4}\left(\frac{2}{3}W_{\text{UC}} + \frac{2}{3}W_{131}H + W_{311}H^3\right)^2 + \frac{1}{180}(W_{040})^2 + \frac{1}{72}(W_{\text{UC}} + W_{131}H)^2 \\ &+ \frac{1}{24}(W_{\text{UA}} + W_{\text{LA}}H + W_{222}H^2)^2, \end{aligned} \tag{23}$$

where H is the normalized field height, $H = 1$ at full field.

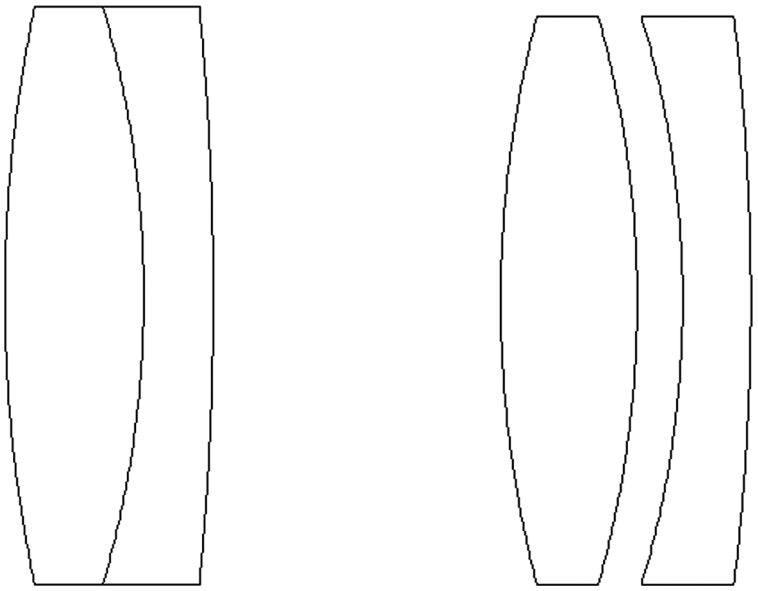

Fig. 1 A cemented doublet and a split doublet.

Table 3 Aplanatic doublet cemented. Sensitivities at $f = 100$ mm, $F/4$, FOV ± 1 deg, glasses BK7 and SF5, nominal $\sigma_W = 0.147$.

Sag (3 rings)	ΔW_{040}	ΔW_{131}	ΔW_{222}	$\Delta\sigma_W$
1	0.017	0.000	0.000	
2	0.023	−0.002	0.000	
3	−0.039	0.007	0.000	
RSS	0.049	0.008	0.000	0.005
Thickness (0.025 mm)	ΔW_{040}	ΔW_{131}	ΔW_{222}	
1	0.003	0.000	0.000	
2	−0.006	0.002	0.000	
3	0.003	−0.001	0.000	
RSS	0.007	0.002	0.000	0.001
Tilt (0.0075 mm)	ΔW_{UC}	ΔW_{LA}	ΔW_{FT}	
1	0.013	0.009	0.004	
2	0.043	−0.002	−0.001	
3	−0.109	0.016	0.002	
RSS	0.118	0.019	0.005	0.016

Table 4 Split aplanatic doublet. Sensitivities at $f = 100$ mm, $F/4$, FOV ± 1 deg, glasses FK51 and LASF43, nominal $\sigma_W = 0.113$.

Sag (3 rings)	ΔW_{040}	ΔW_{131}	ΔW_{222}	$\Delta\sigma_W$
1	0.025	0.000	0.000	
2	−0.101	0.009	0.000	
3	0.129	−0.010	0.000	
4	−0.048	0.008	0.000	
RSS	0.173	0.016	0.000	0.016
Thickness (0.025 mm)	ΔW_{040}	ΔW_{131}	ΔW_{222}	
1	0.008	−0.001	0.000	
2	0.175	−0.026	0.001	
3	−0.185	0.028	−0.001	
4	0.002	−0.002	0.000	
RSS	0.255	0.038	0.001	0.030
Tilt (0.0075 mm)	ΔW_{UC}	ΔW_{LA}	ΔW_{FT}	
1	0.023	0.010	0.005	
2	−0.305	0.022	0.004	
3	0.283	−0.011	−0.002	
4	−0.0135	0.019	0.003	
RSS	0.438	0.032	0.007	0.083

The inner surfaces of the split doublet have relatively strong curvatures and can be independently misaligned because they are not physically in contact; thus the split doublet is much more sensitive than the cemented doublet.

5 Cooke Triplet Lens Characteristic Sensitivities

Since for a given lens perturbation amount the lens sensitivities are independent of lens scaling, characterizing lens forms by their aberration sensitivities can be useful for analysis. Table 5 provides the aberration sensitivities for a Cooke triplet lens shown in Fig. 2 working at $F/4$ and a field of view of ± 20 deg. The perturbations used are three Newton rings at 633 nm for surface sag, 0.025 mm for surface spacings, and 0.0075 mm for half the TIR. In Table 5, two values are given for each entry; one for a focal length of 5 mm, and the other for a scaled up lens with a focal length of 50 mm. The entries are essentially the same and the largest difference is 0.003, attributable to numerical calculation and to rounding. This further supports that the aberration sensitivities are independent of scaling for the same amount of perturbation.

The nominal standard deviation of the wavefront at full field is 0.056/0.561 waves and errors on surface tilt impact most the performance. Linear astigmatism aberration W_{LA} can impact most the image quality. As discussed above, these sensitivities apply to any scaled version of the lens

Table 5 Cooke triplet examples. Sensitivities at $f = 5$ mm/50 mm, $F/4$, FOV ± 20 deg, glasses LAK33 and TIFN5, nominal $\sigma_W = 0.056/0.561$.

Sag (3 rings)	ΔW_{040}	ΔW_{131}	ΔW_{222}	$\Delta\sigma_W$
1	0.011/0.011	0.062/0.062	0.033/0.033	
2	−0.018/ − 0.018	0.031/0.032	−0.121/ − 0.122	
3	0.111/0.111	−0.170/ − 0.171	0.080/0.080	
4	−0.071/ − 0.070	−0.133/ − 0.132	−0.063/ − 0.063	
5	0.038/0.038	−0.012/ − 0.012	0.062/0.062	
6	−0.074/ − 0.075	0.056/0.056	0.020/0.020	
RSS	0.157/0.157	0.234/0.234	0.174/0.175	0.110/0.110
Thickness (0.025 mm)	ΔW_{040}	ΔW_{131}	ΔW_{222}	
1	0.004/0.004	0.022/0.023	−0.019/ − 0.019	
2	0.012/0.012	−0.036/ − 0.036	0.153/0.151	
3	−0.025/ − 0.025	0.166/0.166	−0.409/ − 0.409	
4	0.062/0.061	0.083/0.080	0.002/−0.001	
5	−0.018/ − 0.018	−0.103/ − 0.103	0.133/0.112	
6	−0.035/ − 0.035	−0.127/ − 0.129	0.168/0.166	
RSS	0.079/0.078	0.251/0.250	0.481/0.480	0.150/0.150
Tilt (0.0075 mm)	ΔW_{UC}	ΔW_{LA}	ΔW_{FT}	
1	−0.004/ − 0.004	0.017/0.017	−0.025/ − 0.025	
2	0.081/0.081	0.236/0.236	−0.050/ − 0.050	
3	0.134/0.134	−0.515/ − 0.515	−0.192/ − 0.192	
4	−0.174/ − 0.174	−0.266/ − 0.266	−0.141/ − 0.141	
5	−0.096/ − 0.096	0.211/0.211	0.026/0.026	
6	−0.266/0.266	0.269/0.269	0.079/0.079	
RSS	0.367/0.367	0.713/0.713	0.259/0.259	0.240/0.238

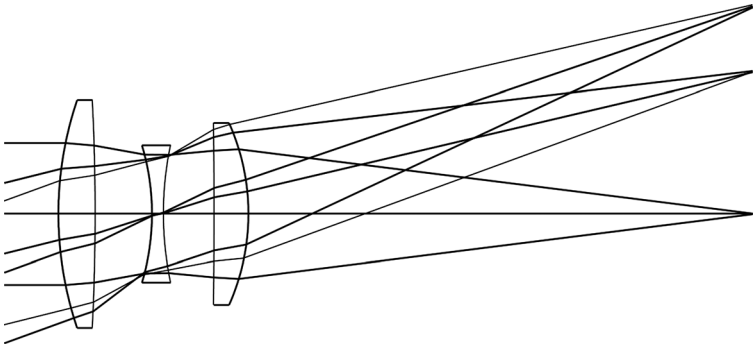

Fig. 2 Cooke triplet lens.

Table 6 Cooke triplet characteristic sensitivities. At $F/4$ and FOV ± 20 deg, glasses LAK33 and TIFN5.

Sag (3 rings)	ΔW_{040}	ΔW_{131}	ΔW_{222}
RSS	0.157	0.234	0.175
Thickness (0.025 mm)	ΔW_{040}	ΔW_{131}	ΔW_{222}
RSS	0.078	0.250	0.480
Tilt (0.0075 mm)	ΔW_{UC}	ΔW_{LA}	ΔW_{FT}
RSS	0.367	0.713	0.259

provided that the same surface perturbations are applied. For other fields of view or relative apertures, the sensitivities can be scaled according to the functional dependence of the aberration. The RSS of the surface sensitivities serves as an overall figure of merit to characterize the lens sensitivity to a given aberration, and the set of the RSS values characterizes the lens form, in this case a Cooke triplet. Thus the characteristics sensitivities for a Cooke triplet are given in Table 6.

6 Conclusion

We argue that aberration sensitivities are independent of lens scaling when the perturbation amount is kept the same. This behavior is in agreement with the results of actual calculation. Further, we have pointed out that aberration sensitivities scale in the same way as aberration scale as a function of the field of view and aperture. Three examples of calculating surface sensitivities have been presented, and the RSS of the surface sensitivities has been used to characterize the aberration sensitivity of a lens. The analysis of the thin lens supports that aberration sensitivities correlate with the amount of aberration; the larger the amount of aberration generated by a lens, the more sensitive it will be to perturbations.

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References

1. A. W. Lohmann, "Scaling laws for lens systems," *Appl. Opt.* **28**, 4996–4998 (1989).
2. D. J. Brady et al. "Parallel cameras," *Optica* **5**, 127–137 (2018).
3. D. J. Brady, L. Fang, and Z. Ma, "Deep learning for camera data acquisition, control, and image estimation," *Adv. Opt. Photonics* **12**, 787–846 (2020).

4. W. Pang and D. J. Brady, "Distributed focus and digital zoom," *Eng. Res. Express* **2**, 035019 (2020).
5. D. S. Grey, "The inclusion of tolerance sensitivities in the merit function for lens optimization," *Proc. SPIE* **147**, 63 (1978).
6. D. S. Grey, "Tolerance sensitivity and optimization," *Appl. Opt.* **9**, 523–526 (1970).
7. G. Catalan, "Design method of an astronomical telescope with reduced sensitivity to misalignment," *Appl. Opt.* **33**, 1907–1915 (1994).
8. E. I. Betensky, "Aberration correction and desensitization of an inverse-triplet object lens," *Proc. SPIE* **3482**, 264–268 (1998).
9. J. R. Rogers, "Using global synthesis to find tolerance-insensitive design forms," *Proc. SPIE* **6342**, 63420M (2006).
10. M. Isshiki, D. C. Sinclair, and S. Kaneko, "Lens design: global optimization of both performance and tolerance sensitivity," *Proc. SPIE* **6342**, 63420N (2007).
11. A. Yabe, "General method of sensitivity control for manufacturing errors," *Appl. Opt.* **49**, 5175–5182 (2010).
12. I. Powell, "Employment of reverse optimization to relax manufacturing tolerances imposed on system constructional parameters associated with complex optical systems," *Appl. Opt.* **39**, 2174–2183 (2000).
13. Y. Luo, X. Li, and Q. Huang, "A new approach for tolerance sensitivity reduction in lens optimization by controlling the first order derivatives of real ray height on pupil," *Proc. SPIE* **11895**, 1189510 (2021).
14. M. Jeffs, "Reduced manufacturing sensitivity in multi-element lens systems," *Proc. SPIE* **4832**, 104–113 (2002).
15. J. Sasian, *Introduction to Aberrations in Optical Imaging Aberrations*, Cambridge University Press (2013).
16. J. Sasian, *Introduction to Lens Design*, Cambridge University Press (2018).
17. J. Sasian, "Lens desensitizing: theory and practice," *Appl. Opt.* **61**, A62–A67 (2022).

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