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An extended multi-attribute decision making method based on novel information measures and its application

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Abstract: Information measures and decision-making methods are key tools in addressing multi-attribute group decision-making (MAGDM) problems. This paper first introduces an uncertainty measure for q -rung orthopair fuzzy sets (q -ROFSs), supported by an axiomatic framework. Compared with existing entropy measures, the proposed measure considers both fuzziness and hesitation simultaneously, enabling more precise quantification of uncertainty information. Secondly, a new score function is developed that incorporates hesitation degrees, improving the ability to distinguish among different q -ROFSs. Based on these two measures, we propose a MAGDM method under a q -ROF environment, integrating the Criteria Importance Through Intercriteria Correlation (CRITIC) model, an optimization model, and the Weighted Aggregated Sum Product Assessment (WASPAS) method. Q -ROFSs allow more flexible expression of membership, non-membership, and hesitation, enhancing their capacity to handle uncertain information in real-world decision-making. In the proposed framework, the CRITIC method and the new score function are used to determine expert weights, accounting for dependence and hesitation of experts. Attribute weights are derived by combining the uncertainty measure and optimization model, considering both objective and subjective factors. The WASPAS method is then employed to rank alternative solutions. Finally, a case study on the selection of food waste treatment technologies (FWTTs) is conducted to verify the universality and superiority of the established framework and the effectiveness and robustness are further validated through comparative analysis and sensitivity analysis.

Keywords: Q -rung orthopair fuzzy sets, Uncertainty measure, Score function, CRITIC model, WASPAS method, Muti-attribute group decision making (MAGDM).

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1 Introduction

Multi-attribute group decision making (MAGDM) is an important tool for addressing complex problems in modern society. Its essence lies in selecting the best option among multiple alternatives based on various attributes. With the widespread use of big data and artificial intelligence technology, people's decision-making environment has become increasingly complex, so the study of MAGDM methods is particularly important. In recent years, MAGDM approaches have caught the eye of numerous researchers and have been employed to assess diverse kinds of MAGDM issues. These application areas include medical waste treatment [1], supplier selection [2], environmental science [3], emergency plan selection [4], renewable energy development [5] and traffic system management [6], etc. In the early stages, studies on MAGDM primarily concentrated on the realm of exact mathematics. Nevertheless, as society has developed rapidly and information has become more complex, people often encounter fuzziness and uncertainty when describing information. To address this issue, the concept of fuzzy sets (FSs) was first introduced by Zadeh [7]. Fuzzy sets theory effectively addresses the ambiguity in human cognition by utilizing fuzzy numbers, which represent the uncertain thought processes of decision-makers (DMs). This approach employs membership degrees to quantify and express knowledge. Since the introduction of fuzzy sets theory, many scholars have further extended it and defined various types of fuzzy sets, such as type-2 fuzzy sets [8], hesitant fuzzy sets (HFSs) [9], probabilistic hesitant fuzzy sets [10], hesitant fuzzy linguistic term sets (HFLTSs) [11], picture fuzzy sets [12] and so on. These fuzzy sets have been widely applied in various fields and have achieved remarkable results in solving various decision-making challenges.

However, the FSs theory has two common flaws. Firstly, the fuzzy sets only concentrates on the membership degree of elements to a specific set. Secondly, the reliability of these sets is questionable [13]. To overcome the aforementioned issues, Yager [14] proposed the concept of q -rung orthopair fuzzy sets (q -ROFSs). The characteristic of q -ROFSs is that the sum of the q th powers of the membership degree (η) and the non-membership degree (σ) is less than or equal to 1, such that $0 \leq \eta^q + \sigma^q \leq 1$, where q is an integer greater than or equal 1. Specifically, when $q = 1$, q -ROFSs degenerate into intuitionistic fuzzy sets (IFSs) [15]; when $q = 2$, q -ROFSs degenerate into Pythagorean fuzzy sets (PFSs) [16]; when $q = 3$, q -ROFSs degenerate into Fermatean fuzzy sets (FFSs) [17], as shown in Fig. 1. Compared with IFSs, PFSs and FFSs, since q is a variable parameter, q -ROFSs provide decision-makers with greater flexibility to describe uncertain and fuzzy information. Consequently, q -ROFSs have drawn the interest of numerous scholars in subsequent research and have been successfully applied in multiple industry fields [18–21]. Therefore, this paper utilizes q -ROFSs to handle the fuzzy and uncertain information present in MAGDM problems.

In MAGDM problems, owing to the various in the professional fields, educational backgrounds and social statuses of different experts, their weights also vary. Therefore, determining the weights of experts is another significant challenge faced in MAGDM problems. To address this issue, several researchers assign explicit weight values to DMs within the $[0, 1]$ interval according to their knowledge background, and experience under certain conditions. However, other researchers tend to use the information provided by experts and some appropriate methods to determine the weights, such as the method based on aggregation operators (AO) [22], the CRITIC method [23], the Best-Worst Method (BWM) [24], the TOPSIS method [25], and the transformative method [26]. To comprehensively account for the mutual influence among experts, this study employs the CRITIC method to assign the weights to each expert. When implementing this method, To effectively handle the information, other literature often uses the

score function defined by Wei et al. [27]. However, this score function exhibits certain limitations, particularly its failure to rank all q-ROFNs with equal membership and non-membership degrees. Therefore, in Section 4, this paper introduces a new score function to enhance the credibility of the scoring and make it more advantageous in handling such situations.

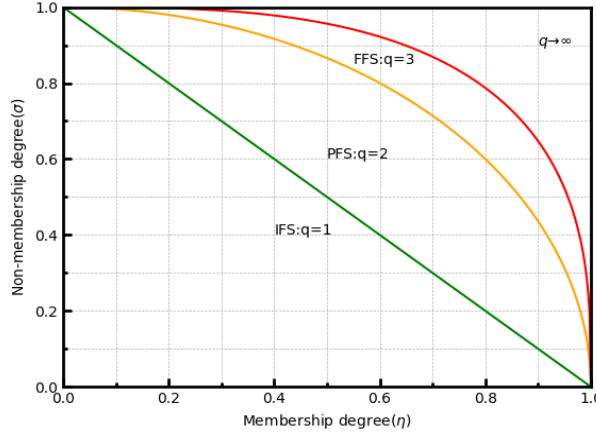


Fig. 1. The q-ROFS with different parameters q represents different decision making spaces.

Within the realm of fuzzy MAGDM, entropy measures serve as a tool for quantifying uncertain information, have extensive applications and significant theoretical value. Peng and Liu [28] were the first to define the entropy measure of q-ROFSs and its axiomatic properties. Ganie [29] defined a series of entropy measures for q-ROFSs based on t-norm and t-conorm, and used the new entropy measure to determine attribute weights and combined the inferior ratio method to select the best alternative. Alkan Kahraman [30] developed a MAGDM approach by integrating an entropy measure with the TOPSIS model, and applied this method to the selection of treatment strategies for COVID-19. Despite numerous studies on entropy measures, a shared limitation among them is the neglect of the hesitation factor inherent in q-ROFNs. To overcome this problem, an uncertainty measure of q-ROFSs is proposed in Section 3. Secondly, While the use of entropy measures for determining attribute weights is straightforward, it solely relies on objective data, overlooking the importance of incorporating subjective weight information, that can compromise the decision-making process. Although some scholars have explored the integration of subjective and objective methods [1] for the determination of attribute weights, the assignment of coefficients to subjective and objective components has been somewhat arbitrary. Consequently, this paper introduce a more scientific method, which determines the attribute weights by comprehensively considering objective information and subjective information.

The classic methods used for ranking alternatives in MAGDM problems mainly include TOPSIS [30], VIKOR [1], MARCOS [2], CoCoSo [31], EDAS [32], TODIM [33] and WASPAS [34]. Compared with these methods, the WASPAS method has the following advantages: (1) a simple calculation process; (2) it combines the WSM and the WPM models and adjusts the proportion of the two through parameters, which is more flexible and superior; (3) it can select the most appropriate aggregation operator to process the evaluation information. Therefore, the WASPAS model has been applied in various fields to solve MAGDM problems. [35] evaluated health-care waste disposal alternatives in the IFS environment by combining the Archimedean t-norm and t-conorm with the WASPAS method. Xiao et al. [36] proposed

an extended WASPAS method and utilized it for supplier selection. Arslan [37] extended the WASPAS method to decomposed fuzzy sets and employed it in the selection of companies within the cold-food supply chain. Deveci [38] prioritized safety E-scooter operation alternatives by combining the Einstein and WASPAS models. To better fuse information, Farid [39] proposed q-rung orthopair fuzzy Aczel-Alsina aggregation operators. Compared with other aggregation operators, it can more effectively convey fuzzy information and manage the interaction between input parameters. However, there is currently no research that combines q-ROF Aczel-Alsina aggregation operators and WASPAS model to resolve MAGDM problems. Based on the previous discussions and analyses, the main motivations of this study are as follows:

(1) The score function serves as a crucial tool in handling MAGDM problems. Currently, many scholars have studied the score function under q-ROF environment. However, the existing score functions always have some limitations, such as counter-intuitiveness, failure to consider the hesitation, and inability to distinguish two different q-ROFNs. Therefore, inventing a new score function for q-ROFSs is necessary.

(2) Entropy measures can effectively quantify the uncertain information. Currently, many scholars have studied entropy measures under q-ROF environment. However, the existing entropy measures always have some deficiencies, such as not considering the hesitation and the division-by-zero problem. Therefore, inventing a new tool to quantify uncertain information for q-ROFSs is crucial.

(3) In MAGDM problems, it is often necessary to invite multiple experts to evaluate the information of the schemes. Due to differences in knowledge background, social status, work experience, etc., the importance of each expert varies. Moreover, experts are not independent of each other. When making decisions, they communicate with one another and thus influence each other. At the same time, experts themselves may hesitate when dealing with matters. Therefore, it is very important to take into account both the non-independence and hesitation of experts when determining their weights. However, many previous methods for determining expert weights have not taken into account the interaction among experts. Therefore, it is essential to identify an effective method for calculating the weights of various experts.

(4) When choosing the best alternative, multiple attributes are usually considered comprehensively. Therefore, determining the attribute weights reasonably is another challenge in MAGDM problems. The existing methods for determining attribute weights mainly fall into three categories: objective weight method, subjective weight method, and the combination of objective and subjective weight methods. Furthermore, the previous combination of objective and subjective weight methods merely linearly combines the existing objective and subjective methods, and the determination of coefficients is too arbitrary. Therefore, it is urgent to propose a scientific method for determining attribute weights.

(5) The q-ROF Aczel-Alsina aggregation operator and the WASPAS model both have their unique advantages in dealing with MAGDM. But currently, there has no study that combines the two and applies them in the q-ROFSs environment for multi-attribute group decision making.

Therefore, these research gaps have prompted us to develop an extended MAGDM method in the q-ROFSs environment to select the optimal solution. The innovations of this study are as follows:

(1) The uncertainty measure of q-ROFSs is defined to quantify the uncertain information, and its axiomatic definition is defined and proved. Compared with the existing entropy measures, it takes into account both the fuzziness and hesitation of q-ROFSs simultaneously, and can quantify the uncertainty of information more precisely, demonstrating certain superiority and reliability.

(2) A novel score function for q-ROFSs is introduced, which effectively overcomes the limitations of existing score functions and can more precisely rank and distinguish between two q-ROFNs.

(3) The CRITIC method combined with a new score function is used to determine the weights of experts. Its advantage lies in considering the social attributes of experts in the decision-making process, that is, experts are not completely independent; they influence and communicate with each other. At the same time, it takes into account the experts' own hesitation towards information. Therefore, it makes the decision-making process more scientific and reasonable.

(4) The attribute weights are determined by combining the uncertainty measure and the optimization model. This approach considers both the subjective and objective information of attribute weights simultaneously, and the importance of the two types of information is not arbitrarily assigned. Compared with previous methods, it is more scientific and comprehensive.

(5) The WASPAS method is extended by using q-ROFSs, and on this basis, an integrated MAGDM method, namely the q-ROFSs information measures-based WASPAS method is proposed. Finally, the proposed method is applied to the selection of FWTTs, and the effectiveness and stability of the method are verified through comparative analysis and sensitivity analysis.

The subsequent sections of this paper are structured as follows: Section 2 reviews the relevant theoretical basis of q-ROFSs. Section 3 proposes the uncertainty measure of q-ROFSs and proves its effectiveness and superiority by comparing it with existing entropy measures. Section 4 presents a new score function for q-ROFSs and verifies its reliability and rationality by comparing it with existing score functions. Section 5 elaborates on the framework of an extended multi-attribute decision-making method (MAGDM) in the q-ROFSs environment. Section 6 applies the proposed method to a practical case of food waste treatment technology selection and verifies its effectiveness, superiority, and stability through sensitivity and comparative analyses. Finally, section 7 concludes the entire paper.

2 Preliminaries

In this section, we review several concepts associated with q-rung orthopair fuzzy sets (q-ROFSs).

Definition 1 [14]. Given a fixed set X , a q-ROFS Q on X is defined as follows:

$$Q = \{x, \langle \eta_Q(x), \sigma_Q(x) \rangle | x \in X\},$$

where $\eta_Q(x) : X \rightarrow [0, 1]$ and $\sigma_Q(x) : X \rightarrow [0, 1]$ represent the membership degree and non-membership degree with the element $x \in X$ to Q , respectively. Furthermore $\eta_Q(x)$ and $\sigma_Q(x)$ satisfy the condition of $0 \leq (\eta_Q(x))^q + (\sigma_Q(x))^q \leq 1$, where $q \geq 1$. In addition, $\pi_Q(x) = \sqrt[q]{1 - (\eta_Q(x))^q - (\sigma_Q(x))^q}$ denotes the hesitancy degree of $x \in X$. For convenience, $Q = (\eta_Q(x), \sigma_Q(x))$ is considered as a q-rung orthopair fuzzy number (q-ROFN) and written as $Q = (\eta_Q, \sigma_Q)$.

Definition 2 [14]. Let $\alpha = (\eta_1, \sigma_1)$, $\beta = (\eta_2, \sigma_2)$ be two q-ROFNs, then some operational rules with q-ROFNs are as follows:

$$(1) \alpha \oplus \beta = (((1 - \eta_1^q)(1 - \eta_2^q))^{\frac{1}{q}}, \sigma_1 \sigma_2);$$

- (2) $\alpha \otimes \beta = (\eta_1 \eta_2, ((1 - \sigma_1^q)(1 - \sigma_2^q))^{\frac{1}{q}})$;
- (3) $\lambda \alpha = ((1 - (1 - \eta_1^q)^\lambda)^{\frac{1}{q}}, \sigma_1^\lambda)$;
- (4) $\beta^\lambda = (\eta_1^\lambda, (1 - (1 - \sigma_1^q)^\lambda)^{\frac{1}{q}})$;
- (5) $\alpha^c = (\sigma_1, \eta_1)$.

Definition 3 [27]. Let $\vartheta = (\eta, \sigma)$ be a q-ROFN, then its score function $S(\vartheta)$ and accuracy function $A(\vartheta)$ are defined as follows:

$$S(\vartheta) = \frac{1 + \eta^q - \sigma^q}{2}, \quad (2.1)$$

$$A(\vartheta) = \eta^q + \sigma^q, \quad (2.2)$$

where $0 \leq S(\vartheta) \leq 1$, $0 \leq A(\vartheta) \leq 1$.

Suppose $\alpha = (\eta_1, \sigma_1)$, $\beta = (\eta_2, \sigma_2)$ are two q-ROFNs, we can compare them by the following rules:

- (1) If $S(\alpha) > S(\beta)$, then $\alpha > \beta$;
- (2) If $S(\alpha) < S(\beta)$, then $\alpha < \beta$;
- (3) If $S(\alpha) = S(\beta)$, then
 - (i) $A(\alpha) > A(\beta)$, then $\alpha > \beta$;
 - (ii) $A(\alpha) < A(\beta)$, then $\alpha < \beta$;
 - (iii) $A(\alpha) = A(\beta)$, then $\alpha = \beta$.

Definition 4 [40]. Let $\alpha_i = (\eta_i, \sigma_i) (i = 1, 2, 3, \dots, n)$ be a set of n q-ROFNs with weight vectors $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ that satisfy the conditions $\sum_{i=1}^n \omega_i = 1$ and $\omega_i \geq 0$. In the case, the weighted average (q-ROFWA) and the weighted geometric (q-ROFWG) aggregation operators can be performed as follows:

$$q-ROFWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \bigoplus_{i=1}^n \omega_i \alpha_i = ((1 - \prod_{i=1}^n (1 - \eta_i^q)^{\omega_i})^{\frac{1}{q}}, \prod_{i=1}^n \sigma_i^{\omega_i}), \quad (2.3)$$

$$q-ROFWG(\alpha_1, \alpha_2, \dots, \alpha_n) = \bigotimes_{i=1}^n \omega_i \alpha_i = (\prod_{i=1}^n \eta_i^{\omega_i}, (1 - \prod_{i=1}^n (1 - \sigma_i^q)^{\omega_i})^{\frac{1}{q}}). \quad (2.4)$$

Definition 5 [28]. Suppose X is a fixed set, A and B are two q-ROFSs on X , then a mapping $E: q-ROFS(X) \rightarrow [0, 1]$ is called an entropy measure $E(A)$, which satisfies the following axioms:

- (E1) $0 \leq E(A) \leq 1$;
- (E2) $E(A) = 0$ iff A is a crisp set;
- (E3) $E(A) = 1$ iff $\eta_A(x) = \sigma_A(x)$;
- (E4) $E(A) \leq E(B)$ if A is less fuzzy than B , i.e., $\eta_A(x) \leq \eta_B(x) \leq \sigma_B(x) \leq \sigma_A(x)$ or $\sigma_A(x) \leq \sigma_B(x) \leq \eta_B(x) \leq \eta_A(x)$;
- (E5) $E(A) = E(A^c)$.

3 An uncertainty measure for q-ROFSs

This section first reviews several entropy measures of q-ROFSs previously proposed and analyzes in detail their limitations and deficiencies. Then, we proposes an uncertainty measure

of q-ROFSs. Finally a brief comparison between the proposed uncertainty measure and the existing entropy measures to prove the feasibility and advantages of the proposed uncertainty measure.

3.1 Some existing entropy measures for q-ROFSs

Entropy measure is an important tool for quantifying the uncertainty of fuzzy sets, therefore, in order to quantify the information uncertainty of q-ROFSs, [28] initially defined the five axiomatic properties of entropy measures and proposed a series of entropy measures for q-ROFSs. After that, [29] developed a series of novel entropy measures for q-ROFSs based on t-conorms. These entropy measures are presented in Table 1.

From Table1, we notice that the entropy measures proposed in [28, 29] only take into

Table 1: Some existing entropy measures for q-ROFSs.

Authors	Entropy measures
Peng et al. [28]	$E_{P1}(A) = \frac{1}{ X } \sum_{x \in X} \frac{\pi_A^q(x)+1- \eta_A^q(x)-\sigma_A^q(x) }{\pi_A^q(x)+1- \eta_A^q(x)+\sigma_A^q(x) }$ $E_{P2}(A) = 1 - \frac{1}{ X } \sum_{x \in X} \eta_A^q(x) - \sigma_A^q(x) $ $E_{P3}(A) = \frac{\sum_{x \in X} \eta_A^q(x) \wedge \sigma_A^q(x)}{\sum_{x \in X} \eta_A^q(x) \vee \sigma_A^q(x)}$ $E_{P4}(A) = \frac{\sum_{x \in X} 0.5\pi_A^q(x) + (\eta_A^q(x) \wedge \sigma_A^q(x))}{\sum_{x \in X} 0.5\pi_A^q(x) + (\eta_A^q(x) \vee \sigma_A^q(x))}$ $E_{P5}(A) = \frac{1}{(\sqrt{2}-1) X } \sum_{x \in X} \left(\sin \frac{1+\eta_A^q(x)-\sigma_A^q(x)}{4} + \sin \frac{1-\eta_A^q(x)+\sigma_A^q(x)}{4} \pi - 1 \right)$ $E_{P6}(A) = 1 - \frac{1}{ X } \sum_{x \in X} \tan\left(\frac{\pi}{4} - \frac{ \eta_A^q(x)-\sigma_A^q(x) }{4(1+\pi_A^q(x))}\pi\right)$
Ganie et al. [29]	$E_{G1}(A) = 1 - \frac{1}{ X } \sum_{x \in X} \frac{2(\eta_A^q(x)-\sigma_A^q(x) - \eta_A^q(x)-\sigma_A^q(x) ^2)}{1- \eta_A^q(x)-\sigma_A^q(x) }$ $E_{G2}(A) = 1 - \frac{1}{ X } \sum_{x \in X} 2(\eta_A^q(x) - \sigma_A^q(x) - \eta_A^q(x) - \sigma_A^q(x) ^2)$ $E_{G3}(A) = 1 - \frac{1}{ X } \sum_{x \in X} \min(1, 2 \eta_A^q(x) - \sigma_A^q(x))$ $E_{G4}(A) = 1 - \frac{1}{ X } \sum_{x \in X} \frac{2 \eta_A^q(x)-\sigma_A^q(x) }{1+ \eta_A^q(x)-\sigma_A^q(x) ^2}$

account the fuzziness of q-ROFSs, but not consider its hesitancy $\pi_A^q(x)$. For example, given two q-ROFSs $A = (0.5, 0.5)$ and $B = (0.2, 0.2)$, we can find that the entropy measures of A and B are equal and both equal to 1, but from a intuitive perspective, this is not reasonable, because the hesitant degree of A is smaller than that of B , therefore, the uncertainty information conveyed by A should be less than that in B . Besides the relationship between fuzziness, hesitancy, and entropy measure is not yet clear, especially the monotonic relationship.

In addition, the entropy measures E_{P3} and E_{G1} may encounter a situation where the divisor is zero during the calculation process. That is to say, for some q-ROFSs, their entropy cannot be calculated. Therefore, these two formulas are not universal.

Finally, there are some entropy measures cannot distinguish the entropies of two q-ROFS

in certain specific circumstances. For instance, there are two q-ROFSs, $A = (0.5, 0.4)$, $B = (0.2, 0.3)$, when $q = 1$, $E_{P2}(A) = E_{P2}(B)$. Obviously, this is unreasonable. More specific circumstances will be discussed in Section 3.3.

3.2 The uncertainty measure for q-ROFSs

In order to more precisely quantify the uncertain information of q-ROFSs and overcome the limitations of the above mentioned entropy measures, in this section, we propose the uncertainty measure of q-ROFSs and the axiomatic properties it satisfies.

Let X be a nonempty set, for a q-ROFS A defined on X , its uncertainty measure ought to possess several intuitive properties. From a rational standpoint, an uncertainty measure must be a non-negative function of both membership and non-membership values, and its values should no more than 1. The uncertainty measure of a q-ROFS should be equivalent to that of its complement. The uncertainty measure should decrease monotonically as information clarity $|\eta_A^q(x) - \sigma_A^q(x)|$ increases. Since crisp set convey the highest level of information, when a q-ROFS is simplified to a crisp set, the uncertainty measure will reach its lowest value of 0. Conversely, when $\eta_A(x) = \sigma_A(x) = 0$, it indicates that we possess no information regarding the situation, thereby the uncertainty measure reaches its largest value of 1. For the case that $\eta_A(x) = \sigma_A(x) \neq 0$, a higher hesitant degree $\pi_A^q(x)$ indicates a greater level of uncertainty. Considering these intuitive characteristics, we can formally define the axiomatic properties that characterize the uncertainty measure of q-ROFSs.

Definition 6. Let X be a fixed set, A be a q-ROFS on X , a mapping $U: \text{q-ROFS}(X) \rightarrow [0, 1]$ is called an uncertainty measure $U(A)$, which satisfies the following axioms:

- (U1) $0 \leq U(A) \leq 1$;
- (U2) $U(A) = 0$ iff A is a crisp set;
- (U3) $U(A) = 1$ iff $\eta_A(x) = \sigma_A(x) = 0$;
- (U4) $U(A)$ is decreasing with $\Delta_A^q(x) = |\eta_A^q(x) - \sigma_A^q(x)|$ and increasing with $\pi_A^q(x)$;
- (U5) $U(A) = U(A^c)$.

From Definition 6, it is evident that the uncertainty measure of q-ROFSs proposed in this paper enhances the third and fourth properties outlined in Definition 5. Specifically, the third and fourth properties in Definition 5 focus solely on the fuzziness of q-ROFSs, neglecting their hesitation. As a result, this paper refines these two properties to make them more scientifically rigorous and comprehensive.

Definition 7. For a q-ROFS A on X , an uncertainty measure for A is defined as follows:

$$U(A) = 1 - \frac{1}{2|X|} \sum_{x \in X} (|\eta_A^q(x) - \sigma_A^q(x)| + \eta_A^q(x) + \sigma_A^q(x))(\eta_A^q(x) + \sigma_A^q(x)). \quad (3.1)$$

Using the $\Delta_A^q(x)$ and $\pi_A^q(x)$ expression, Eq.(3.1) can be rewrote as follows:

$$U(A) = 1 - \frac{1}{2|X|} \sum_{x \in X} (\Delta_A^q(x) + 1 - \pi_A^q(x))(1 - \pi_A^q(x)). \quad (3.2)$$

Theorem 1. The uncertainty measure $U(A)$ for q-ROFSs given by Eq.(3.1) satisfies the five axiomatic properties listed in Definition 6.

Proof. The proofs for (U1)-(U5) are given as follows:

- (U1) It is obvious based on the properties of q-ROFSs.
- (U2) For a crisp set, we have $\Delta_A^q(x) = 1$ and $\pi_A^q(x) = 0$, then we can get $U(A) = 0$.

Considering $0 \leq \Delta_A^q(x) \leq 1$, $0 \leq \pi_A^q(x) \leq 1$, $U(A) = 0$ implies $(\Delta_A^q(x) + 1 - \pi_A^q(x))(1 - \pi_A^q(x)) = 2$, then we have $\Delta_A^q(x) = 1$ and $\pi_A^q(x) = 0$, which indicates A is a crisp set.

(U3) $U(A) = 1$ implies $(\Delta_A^q(x) + 1 - \pi_A^q(x))(1 - \pi_A^q(x)) = 0$, then we have $\pi_A^q(x) = 1$ or $\Delta_A^q(x) + 1 - \pi_A^q(x) = 0$, so we can get $\eta_A(x) = \sigma_A(x) = 0$.

Obviously, when $\eta_A(x) = \sigma_A(x) = 0$, we have $U(A) = 1$.

(U4) To prove the fourth property, it is necessary to construct the subsequent function:

$$f(y, z) = -(y + 1 - z)(1 - z),$$

The partial derivatives of f with respect to y and z can be obtained as follows:

$$\frac{\partial f}{\partial y} = z - 1, \quad \frac{\partial f}{\partial z} = y + 2 - 2z,$$

If $y, z \in [0, 1]$, we have $\frac{\partial f}{\partial y} \leq 0$ and $\frac{\partial f}{\partial z} \geq 0$, which indicates that $f(y, z)$ is decreasing with y and increasing with z . Given $0 \leq \Delta_A^q(x) \leq 1$ and $0 \leq \pi_A^q(x) \leq 1$ for a fixed $x \in X$, $f(\Delta_A^q(x), \pi_A^q(x))$ is decreasing with $\Delta_A^q(x)$ and increasing with $\pi_A^q(x)$. So the uncertainty measure $U(A) = 1 - \frac{1}{2|X|} \sum_{x \in X} (\Delta_A^q(x) + 1 - \pi_A^q(x))(1 - \pi_A^q(x))$ has the same monotony as $f(\Delta_A^q(x), \pi_A^q(x))$. Thus the fourth property is satisfied.

(U5) It is obvious based on the properties of q-ROFSs. $\square$

We analyse the uncertainty measure of q-ROFSs, which consists of fuzziness and hesitancy. Fuzziness is controlled by $\Delta_A^q(x)$, while hesitancy is determined by $\pi_A^q(x)$, which we refer to as the fuzzy part and hesitant part.

3.3 Comparative study

To assess the effectiveness and advantages of the proposed uncertainty measure, Example 1 presents a comparative analysis with the existing entropy measures.

Example 1. Eight q-ROFSs defined on $X = \{x\}$ are given as: $A_1 = \langle x, 0.4, 0.4 \rangle$, $A_2 = \langle x, 0.5, 0 \rangle$, $A_3 = \langle x, 0.5, 0.2 \rangle$, $A_4 = \langle x, 0.5, 0.5 \rangle$, $A_5 = \langle x, 0.6, \sqrt{0.15} \rangle$, $A_6 = \langle x, 0.8, 0.1 \rangle$, $A_7 = \langle x, 0, 1 \rangle$, $A_8 = \langle x, 0, 0 \rangle$.

Next, Table 2 presents some calculation results of the explored uncertainty measure and existing entropy measures using the q-ROFSs from Example 1. From Table 2, the limitations and defects of the existing entropy measurements are summarized and analyzed in detail. Here we take $q = 2$.

(1) From Table 2, we can see E_{P_2} , E_{P_4} , E_{P_5} , E_{G_1} , E_{G_2} , E_{G_3} and E_{G_4} cannot distinguish the uncertainty between A_3 and A_5 . Since the entropy measures E_{P_2} , E_{P_4} , E_{G_1} , E_{G_2} , E_{G_3} and E_{G_4} are defined based on $|\eta_A^q(x) - \sigma_A^q(x)|$. That is to say, for two different q-ROFSs A and B , if $|\eta_A^q(x) - \sigma_A^q(x)| = |\eta_B^q(x) - \sigma_B^q(x)|$, then their uncertainties are equal, which is unreasonable.

(2) We observe that the membership and non-membership degrees for A_1 and A_4 are identical in Example 1, so the existing entropy measures in Table 2 assign them the maximum uncertainty of 1, which is unreasonable. Because from an intuitive perspective, the hesitancy degree of A_1 is greater than the hesitancy degree of A_4 , so A_1 conveys more uncertainty than A_4 .

(3) We observe that E_{P_3} and E_{G_3} assign zero uncertainty to A_2 and A_6 , respectively. Additionally, it can be noted that for any q-ROFS A , if $\eta_A = 0$ or $\sigma_A = 0$, its entropy measure E_{P_3} will be zero. Moreover, for any q-ROFS A , if the condition $|\eta_A^q(x) - \sigma_A^q(x)| \geq 0.5$ holds, its entropy measure E_{G_3} will also be zero, which is counterintuitive.

Table 2: The calculation results regarding Example 1.

Method	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
E_{P1}	1	0.750	0.781	1	0.753	0.364	0	1
E_{P2}	1	0.750	0.790	1	0.790	0.370	0	1
E_{P3}	1	0	0.160	1	0.417	0.016	0	NAL
E_{P4}	1	0.6	0.653	1	0.653	0.227	0	1
E_{P5}	1	0.934	0.954	1	0.954	0.591	0	1
E_{P6}	1	0.798	0.823	1	0.800	0.445	0	1
E_{G1}	1	0.600	0.653	1	0.653	0.227	NAL	1
E_{G2}	1	0.625	0.668	1	0.668	0.534	0	1
E_{G3}	1	0.500	0.580	1	0.580	0	0	1
E_{G4}	1	0.529	0.598	1	0.598	0.098	0	1
U	0.898	0.938	0.928	0.875	0.816	0.584	0	1

(4) For both the entropy measures E_{P3} and E_{G1} , a situation occurred in which the denominator was zero.

After the above analysis, all existing entropy measures would lead to unreasonable results. Next, we analyze the proposed uncertainty measure, for convenience, the uncertainty measure of q-ROFSs is denoted by $U(A)$.

For all the existing entropy measures, which also assign the maximum value of 1 to A_1 , A_4 and A_8 , whose membership degree is equal to non-membership degree, this is illogical. Since the hesitancy degrees of A_1 , A_4 and A_8 are different, their uncertainty should satisfy $U(A_4) < U(A_1) < U(A_8)$. Additionally, E_{P2} , E_{P4} , E_{G1} , E_{G2} , E_{G3} and E_{G4} assign the same value to A_3 and A_5 . This is due to the characteristic of these entropy measures, which focus solely on $|\eta_A^q(x) - \sigma_A^q(x)|$. We observe that $|\eta_{A_3}^q(x) - \sigma_{A_3}^q(x)| = |\eta_{A_5}^q(x) - \sigma_{A_5}^q(x)|$, but A_3 has a higher hesitancy degree. As a result, A_3 should exhibit greater uncertainty than A_5 . It is evident that the uncertainty measure we propose effectively capture these relationships: $U(A_4) < U(A_1) < U(A_8)$ and $U(A_5) < U(A_3)$.

Hence, the analysis above shows that the uncertainty measure we proposed is more appropriate for quantifying q-rung fuzzy uncertainty. Furthermore, it provides a more accurate representation of the relationships between different q-ROFSs.

4 A novel score function for q-ROFSs

In this section, we first review several existing score functions and point out their limitations. Then, we propose a novel score function for q-ROFSs. Finally, by briefly comparing it with the previous score function, we demonstrate the effectiveness of the proposed score function and its significant advantages over traditional score functions.

4.1 Some existing score function for q-ROFSs

The score function can map a two-dimensional q-ROFN to a single real value, so the score function is often used to rank different q-ROFNs. As an important tool for ranking q-ROFNs, it is increasingly receiving widespread attention from scholars. Through literature search, we obtained some score functions for q-ROFNs, among which 12 score functions are presented

in Table 3 and assigned S_1 to S_{12} . However, most of the existing score functions have some weaknesses, mainly in the following three aspects.

(I) Some score functions have a range that includes negative values, such as the ranges of score functions S_1 , S_5 , S_6 and S_8 , which are $[-1, 1]$. In the real number field, negative numbers cannot be taken to even roots ($\sqrt[n]{S}$, n is even), but this operation is very important to us, especially when we use weighted product method to aggregate score information, and we always encounter operations involving even roots. For example, suppose there have a score value set $S = \{s_1, s_2, \dots, s_n\}$ with a weight vector $W = \{w_1, w_2, \dots, w_n\}$, the weighted product of the score values in S can be calculated as $S = \prod_{i=1}^n s_i^{w_i}$, when $s_i = -0.5$, $w_i = 0.25$, the value $(-0.5)^{0.25}$ is meaningless under real number field.

(II) Moreover, the score functions S_1 , S_2 , S_3 , S_4 , S_9 , S_{11} and S_{12} fail to take into account the hesitation π_A^q or accuracy $\eta_A^q + \sigma_A^q$, which is a crucial measure of uncertain information in q-ROFSs theory. As a result, the reliability of these score functions is debatable.

(III) In some specific cases, there exist some score functions that cannot correctly rank the q-ROFNs. For example, S_1 , S_2 , S_4 and S_6 can not distinguish the two q-ROFNs, such as $A = (0.4, 0.4)$ and $B = (0.5, 0.5)$. More unreasonable situations will be discussed in Section 4.3.

Table 3: Some existing score functions for q-ROFSs.

Authors	Score function	Range	Weaknesses		
			I	II	III
Liu and Wang [40]	$S_1(A) = \eta_A^q - \sigma_A^q$	$[-1, 1]$	✓	✓	✓
Wei et al [27]	$S_2(A) = \frac{1+\eta_A^q-\sigma_A^q}{2}$	$[0, 1]$		✓	✓
Mi et al. [41]	$S_3(A) = \frac{2+\eta_A^q-\sigma_A^q}{2-\eta_A^q+\sigma_A^q \times 2 - (\eta_A^q - \sigma_A^q)}$	$[\frac{1}{3}, 3]$		✓	✓
Li et al. [42]	$S_4(A) = \frac{\sqrt[q]{1-\sigma_A^q}}{\sqrt[q]{(1-\sigma_A^q)^q+1-\sigma_A^q} + \sqrt[q]{(1-\eta_A^q)^q+1-\eta_A^q}}$	$[0, 1]$		✓	✓
Peng and Huang [43]	$S_5(A) = \eta_A^q - \sigma_A^q - \ln(1 + \pi_A^q)$	$[-1, 1]$	✓		✓
Garg [44]	$S_6(A) = (\eta_A^q - \sigma_A^q)(1 + \pi_A^q)$	$[-1, 1]$	✓		✓
Rani and Mishra [45]	$S_7(A) = \eta_A^q(1 + \pi_A^q)$	$[0, 2]$			✓
Aydemir et al. [46]	$S_8(A) = \eta_A^q - \sigma_A^q - \frac{\pi_A^q \log_2(1+\pi_A^q)}{100}$	$[-1, 1]$	✓		
Peng et al. [47]	$S_9(A) = \frac{1}{2}(\eta_A^2 + \sqrt[q]{1 - \sigma_A^q})^2$	$[0, 1]$		✓	✓
Xiao et al. [36]	$S_{10}(A) = \frac{1+\eta_A^q-\sigma_A^q+\log_2 \sqrt[10]{\frac{1+\eta_A^q+\sigma_A^q}{2}}}{2}$	$[0, 1]$			✓
Zhang and Wang [48]	$S_{11}(A) = \sqrt{\frac{\eta^{2q} + (1-\sigma^q)^2}{2(2-\eta^q-\sigma^q)}}$	$[0, 1]$		✓	
Mishra et ai. [49]	$S_{12} = \frac{1}{3} \left((u^q - v^q) + \left(\frac{\cos(1-u^q)\frac{\pi}{2} + \cos\frac{v^q\pi}{2}}{2} + 1 \right) \right)$	$[0, 1]$		✓	

4.2 A novel score function for q-ROFSs

In order to address the limitations of previous score functions of q-ROFSs, this section proposes a novel score function for q-ROFSs and elaborates on its relevant properties.

Definition 8. For a q-ROFN $A = (\eta_A, \sigma_A)$, the novel score function for A is defined as follows:

$$S(A) = \frac{\eta_A^q - \sigma_A^q}{2} + \frac{e^{(\eta_A^q + \sigma_A^q - 1)} + 2}{6}. \quad (4.1)$$

Proposition I. The score function S increases monotonically with the membership degree η_A and decreases monotonically with the non-membership degree σ_A .

Proof. Based on Eq.(4.1), we take the partial derivative of S with η_A ,

$$\frac{\partial S}{\partial \eta_A} = \frac{q\eta_A^{q-1}}{6}(3 + e^{(\eta_A^q + \sigma_A^q - 1)}),$$

Similarly, we take the partial derivative of S with σ_A ,

$$\frac{\partial S}{\partial \sigma_A} = \frac{q\sigma_A^{q-1}}{6}(-3 + e^{(\eta_A^q + \sigma_A^q - 1)}).$$

Based on some basic properties of q-ROFS, we have $q \geq 1$, $0 \leq \eta_A, \sigma_A \leq 1$ and $0 \leq \eta_A^q + \sigma_A^q \leq 1$. Then we can get $\frac{\partial S}{\partial \eta_A} = \frac{q\eta_A^{q-1}}{6}(3 + e^{(\eta_A^q + \sigma_A^q - 1)}) \geq 0$ and $\frac{\partial S}{\partial \sigma_A} = \frac{q\sigma_A^{q-1}}{6}(-3 + e^{(\eta_A^q + \sigma_A^q - 1)}) \leq 0$, therefore, the proposition I is proved. $\square$

The graphical representations of the relationships between the partial derivatives $\frac{\partial S}{\partial \eta_A}$ and $\frac{\partial S}{\partial \sigma_A}$ with respect to the variables η_A and σ_A , for $q = 1, 2, 3, \dots, 10$ are shown in Fig. 2(a) and Fig. 2(b), respectively. It is evident from these figures that $\frac{\partial S}{\partial \eta_A} \geq 0$ and $\frac{\partial S}{\partial \sigma_A} \leq 0$.

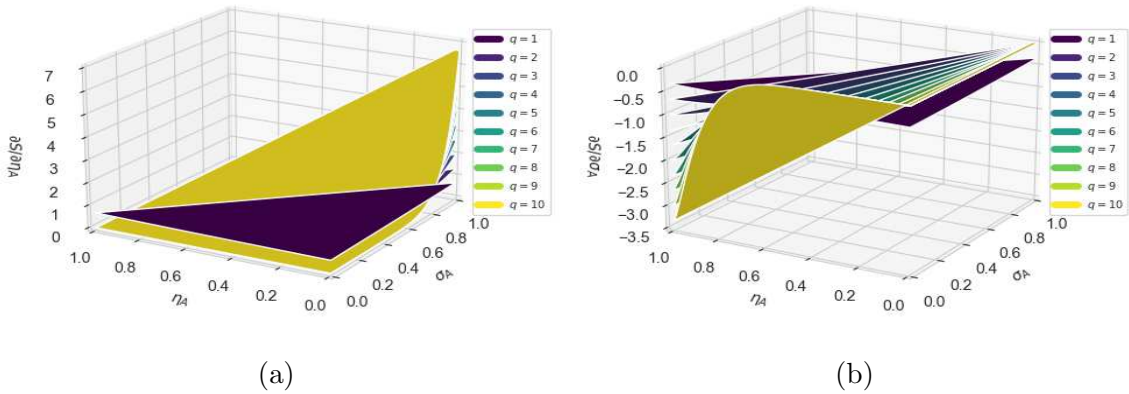


Fig. 2. The relationship between partial derivative $\frac{\partial S}{\partial \eta_A}$ and $\frac{\partial S}{\partial \sigma_A}$ and the variables η_A and σ_A .

Proposition II. For a q-ROFN $A = (\eta_A, \sigma_A)$, its score function should satisfy the following three properties:

- (1) $S(A) = 1$ iff $A = (1, 0)$;
- (2) $S(A) = 0$ iff $A = (0, 1)$;
- (3) $0 \leq S(A) \leq 1$.

Proof. Based on **Proposition I** we know that the score function S increases monotonically with the membership degree η and decreases monotonically with the non-membership degree σ , that is, S attains its maximum value at $A = (1, 0)$ and its minimum value at $A = (0, 1)$.

When A is equal to $(1, 0)$, the score function $S(A)$ is calculated as follows:
 $S(A) = \frac{1-0}{2} + \frac{e^{1+0-1}+2}{6} = 1$;

When A is equal to $(0, 1)$, the score function $S(A)$ is calculated as follows:
 $S(A) = \frac{0-1}{2} + \frac{e^{0+1-1}+2}{6} = 0$.

Therefore, the maximum and minimum values of $S(A)$ are 1 and 0, respectively, i.e., $0 \leq S(A) \leq 1$.

To offer a clearer and more intuitive illustration of **Proposition I** and **Proposition**

II, Fig. 3 presents graphical examples of the score function S for $q = 1, 3, 5$ and 7 . These visual representations validate that the score functions in these scenarios comply with both **proposition I** and **proposition II**.

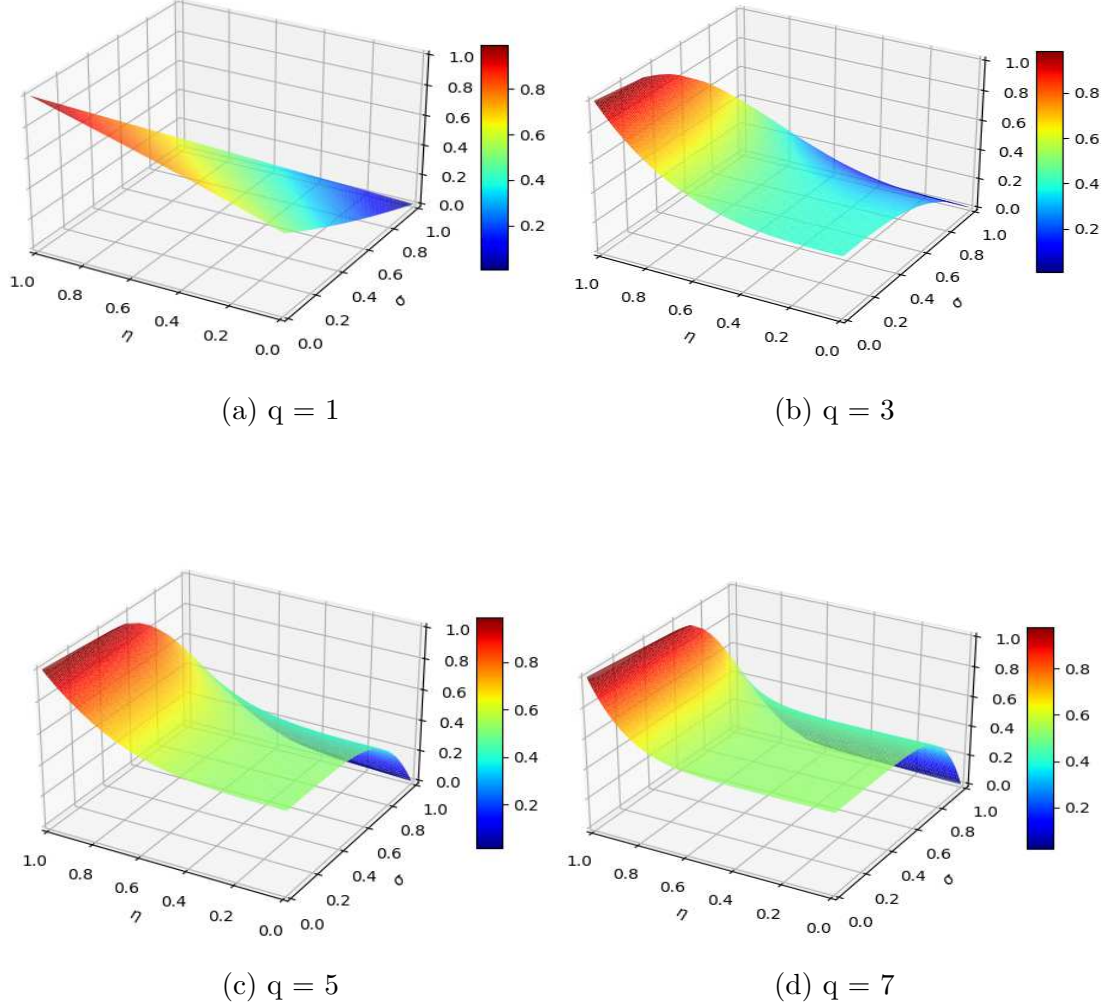


Fig. 3. The distributions of score function S under $q = 1, 3, 5, 7$.

Proposition III. Let $A = (\eta_A, \sigma_A)$ and $B = (\eta_B, \sigma_B)$ be two q -ROFNs, then we have

- (1) If $\eta_A \geq \eta_B$ and $\sigma_A \leq \sigma_B$, then $S(A) \geq S(B)$;
- (2) If $\eta_A = \sigma_A$, $\eta_B = \sigma_B$ and $\pi_A \geq \pi_B$, then $S(A) \leq S(B)$.

Proof. It is evident from **Proposition I** and the expression of the score function that the aforementioned statements are self-evident.

4.3 Comparative study

This section presents the validation and superiority of the proposed score function, which was confirmed through a comparative analysis with the 12 existing score functions detailed in Table 3. The outcomes of this comparison are displayed in Table 4.

Table 4 demonstrates that the developed score function exhibits greater practicality and discriminability compared to existing score functions. In certain scenarios where other approaches are ineffective, this proposed score function is capable of addressing ranking

problems. The following observations can be made from Table 4.

Table 4: Comparison results of the proposed score function and the existing ones.

Score function	Comparison results			Hesitancy or accuracy	$F = (0, 1)$ $\sqrt[n]{S(F)}, n \in N^*$
	Case 1	Case 2	Case 3		
	$F = (0.3, 0.3)$ $Q = (0.4, 0.4)$	$F = (0.5, 0.5)$ $Q = (0.3, 0.1)$	$F = (0, 0.4)$ $Q = (0, 0.5)$		
S_1	$\mathbf{F} = \mathbf{Q}$	$F < Q$	$F > Q$	$\times$	$\times$
S_2	$\mathbf{F} = \mathbf{Q}$	$F < Q$	$F > Q$	$\times$	$\checkmark$
S_3	$F < Q$	$F < Q$	$F > Q$	$\times$	$\checkmark$
S_4	$\mathbf{F} = \mathbf{Q}$	$F < Q$	$F > Q$	$\times$	$\checkmark$
S_5	$F < Q$	$\mathbf{F} > \mathbf{Q}$	$F > Q$	$\checkmark$	$\times$
S_6	$\mathbf{F} = \mathbf{Q}$	$F < Q$	$F > Q$	$\checkmark$	$\times$
S_7	$F < Q$	$\mathbf{F} > \mathbf{Q}$	$\mathbf{F} = \mathbf{Q}$	$\checkmark$	$\checkmark$
S_8	$F < Q$	$F < Q$	$F > Q$	$\checkmark$	$\times$
S_9	$\mathbf{F} > \mathbf{Q}$	$F < Q$	$F > Q$	$\times$	$\checkmark$
S_{10}	$F < Q$	$\mathbf{F} > \mathbf{Q}$	$F > Q$	$\checkmark$	$\checkmark$
S_{11}	$F < Q$	$\mathbf{F} > \mathbf{Q}$	$F > Q$	$\times$	$\checkmark$
S_{12}	$F < Q$	$F < Q$	$F > Q$	$\times$	$\checkmark$
$S_N(\text{proposed})$	$F < Q$	$F < Q$	$F > Q$	$\checkmark$	$\checkmark$

For case 1, where $F = (0.3, 0.3)$, $Q = (0.4, 0.4)$, and $q \geq 1$, the score values of the q-RFNs F and Q , calculated using S_1 , S_2 , S_4 and S_6 are equal, implying that $F = Q$. However, this result is unreasonable, as the hesitation degree of F is greater than that of Q . Therefore, the score functions S_1 , S_2 , S_4 and S_6 fail to distinguish between F and Q . Additionally, in case 1, it was observed that the ranking results obtained using S_9 differed from those obtained with other score functions. Therefore, in this case, it is not appropriate to use the score function S_9 for ranking q-ROFNs.

For case 2, where $F = (0.5, 0.5)$, $Q = (0.3, 0.1)$ and $q = 1$. The ranking results obtained using score functions S_5 , S_7 and S_{10} differ from those produced by other score functions. Therefore, in this case, it is not suitable to use the score function S_5 , S_7 , S_{10} and S_{11} for ranking q-ROFNs.

For case 3, where $F = (0, 0.4)$, $Q = (0, 0.5)$ and $q \geq 1$. We observed that S_7 cannot distinguish between F and Q . Specially, for any two q-ROFNs F and Q , as long as their membership degrees are both 0, then $S_7(F) = S_7(Q) = 0$, which implies $F = Q$. This is clearly unreasonable.

Moreover, the score functions S_1 , S_2 , S_3 , S_4 , S_9 , S_{11} and S_{12} fail to take into account the hesitation π_A^q or accuracy $\eta_A^q + \sigma_A^q$, which is a crucial measure of uncertain information in q-ROFSs theory.

Consider $F = (0, 1)$, $q = 1$. In this case $S_1(F) = S_5(F) = S_6(F) = S_8(F) = -1$, However in the real number domain, $\sqrt[n]{-1}$ ($n \in N^*$) is meaningless. Therefore, when weighted product aggregation of score information is required, score functions S_1 , S_5 , S_6 and S_8 are no longer applicable.

Based on the analysis above and the comparison results in Table 4, it can be concluded that the proposed score function effectively addresses the limitations of most existing functions, demonstrating both a wider range of applications and greater discriminatory power.

5 An extended q-ROFSs MAGDM method

This section proposes an extended MAGDM approach that incorporates a developed uncertainty measure and a novel score function. This approach is structured into four distinct stages. Firstly, the linguistic scale based on q-ROFNs is used to obtain the experts' evaluation information on each alternative under each attribute, so as to generate the individual decision matrix under the q-ROF background. Secondly, considering the mutual influence among experts, the CRITIC method [50] is used to determine expert weights, and the q-ROFWA aggregation operator is utilized to construct the group decision matrix. Thirdly, an optimization model is developed using the uncertainty measure and experts' subjective opinions to determine attribute weights. The advantage of this approach is that it simultaneously accounts for both objective and subjective aspects of attribute information. Finally, the Aczel-Alsina WASPAS approach is employed to prioritize the alternatives and identify the optimal one. Correspondingly, the flowchart illustrating the developed framework is presented in Fig. 4.

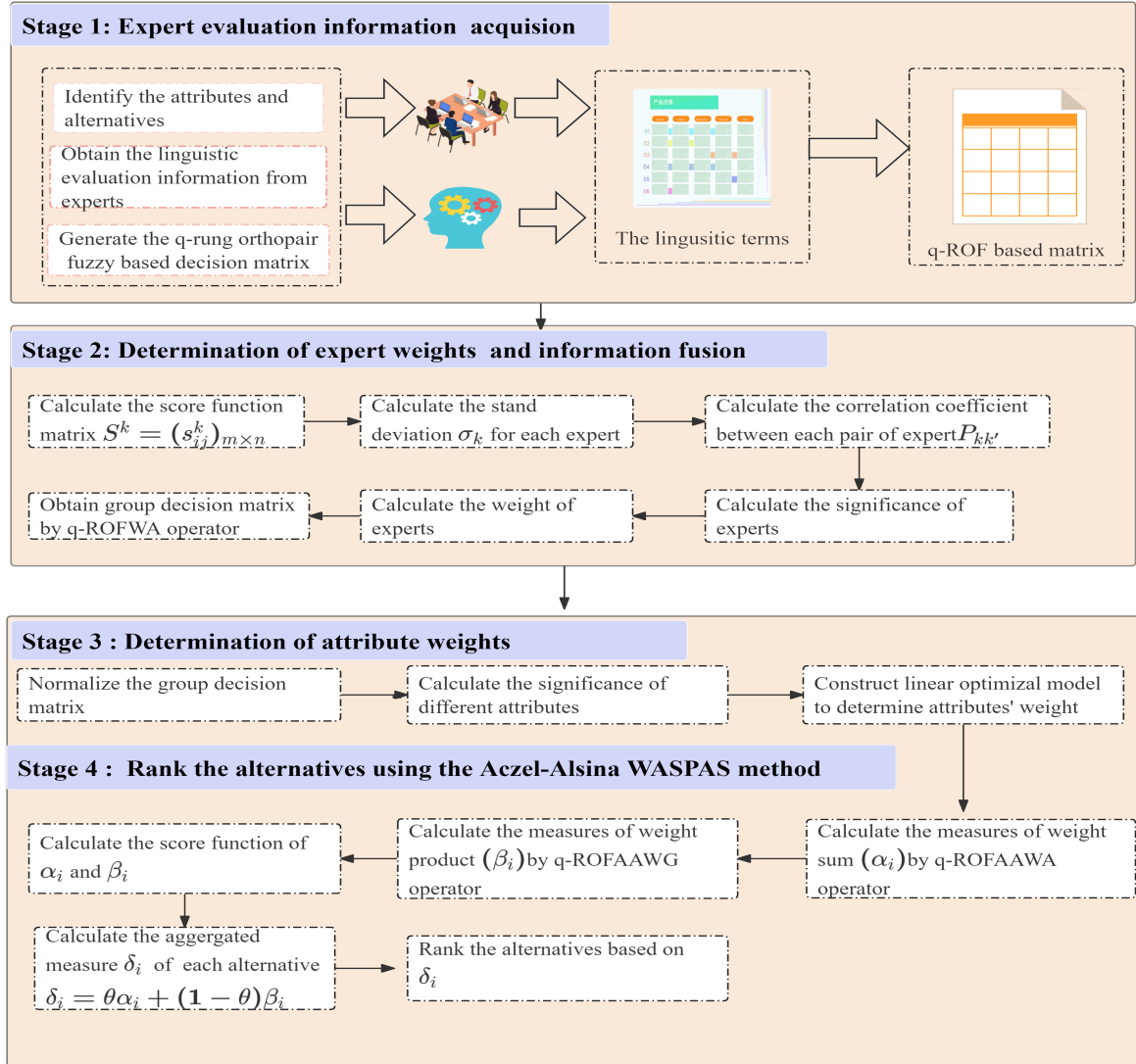


Fig. 4. Flowchart of the proposed MAGDM approach.

Stage 1: Expert evaluation information acquisition.

For a multi-attribute decision-making problem, let $A = \{A_1, A_2, \dots, A_m\}$ be the set of alternatives, $D = \{D_1, D_2, \dots, D_t\}$ be the set of experts with a weighting vector $\tau = \{\tau_1, \tau_2, \dots, \tau_m\}$ such that each τ^k lies within the interval $[0, 1]$ and the sum of τ^k over all k from 1 to t is equal to 1. Additionally, let $C = \{C_1, C_2, \dots, C_n\}$ be a set of attributes with a weighting vector $W = \{\omega_1, \omega_2, \dots, \omega_t\}$ where each ω_j is within the range $[0, 1]$ and the sum of ω_j for all j from 1 to n equals 1.

Step 1.1: The evaluation data for each alternative, corresponding to every attribute, is provided by experts in accordance with the linguistic scale presented in Table 5.

Step 1.2: Utilizing the linguistic scale from Table 5, the matrix of linguistic evaluation provided by the experts is converted into a q-ROFN evaluation matrix, denoted as $D^{(k)} = (d_{ij}^k)_{m \times n}$, where d_{ij}^k indicates the evaluation information for i th alternative under j th attribute by k th expert. The result is shown in Table 6.

Stage 2: Determination of Expert weights and information fusion.

In MAGDM problems, it is common practice to invite multiple experts to evaluate various alternatives. However, in practical applications, each expert has distinct knowledge backgrounds, varying levels of expertise and different attitudes toward the subject matter. As a result, determining appropriate weights for experts becomes a critical task. Unfortunately, previous research on expert weight determination has often overlooked the mutual influence among experts. To address this gap, this paper employs the CRITIC method [50], utilizing the proposed score function to determine expert weights. The specific steps are as follows:

Table 5: Linguistic scale as q-ROFN of alternatives [30].

Linguistic terms	q-ROFN for alternatives	
	η	σ
Certainly low importance(CLI)	0.11	0.99
Very low importance(VLI)	0.22	0.88
Low importance(LI)	0.33	0.77
Under average importance(UAI)	0.44	0.66
Average importance(AI)	0.55	0.55
Above average importance(AAI)	0.66	0.44
High importance(HI)	0.77	0.33
Very high importance(VHI)	0.88	0.22
Certainly high importance(CHI)	0.99	0.11

Table 6: The q-ROFN decision matrix $D^{(k)}$ given by expert k .

Alternatives	attributes			
	C_1	C_2	$\dots$	C_n
A_1	$(\eta_{11}^k, \sigma_{11}^k)$	$(\eta_{12}^k, \sigma_{12}^k)$	$\dots$	$(\eta_{1n}^k, \sigma_{1n}^k)$
A_2	$(\eta_{21}^k, \sigma_{21}^k)$	$(\eta_{22}^k, \sigma_{22}^k)$	$\dots$	$(\eta_{2n}^k, \sigma_{2n}^k)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$(\eta_{m1}^k, \sigma_{m1}^k)$	$(\eta_{m2}^k, \sigma_{m2}^k)$	$\dots$	$(\eta_{mn}^k, \sigma_{mn}^k)$

Step 2.1: Calculate the score function matrix of each expert according to Eq.(4.1):

$$S^{(k)} = [s_{ij}^k]_{n \times m} = [S(d_{ij}^k)]_{n \times m}, \quad (5.1)$$

where $S(d_{ij}^k) = \frac{(\eta_{ij}^k)^q - (\sigma_{ij}^k)^q}{2} + \frac{e^{(\eta_{ij}^k)^q + (\sigma_{ij}^k)^q - 1} + 2}{6}$.

Step 2.2: Calculate the standard deviation for each expert:

$$\sigma_k = \sqrt{\frac{\sum_{i=1}^m \sum_{j=1}^n (s_{ij}^k - \bar{s}^k)^2}{mn}}, \quad (5.2)$$

where $\bar{s}^k = \frac{\sum_{i=1}^m \sum_{j=1}^n (s_{ij}^k)}{mn}$.

Step 2.3: Calculate the correlation coefficient between each pair of experts:

$$P_{kk'} = \frac{\sum_{i=1}^m \sum_{j=1}^n (s_{ij}^k - \bar{s}^k)(s_{ij}^{k'} - \bar{s}^{k'})}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n (s_{ij}^k - \bar{s}^k)^2 (s_{ij}^{k'} - \bar{s}^{k'})^2}}, \quad (5.3)$$

where $k, k' = 1, 2, \dots, t$.

Step 2.4: Calculate the importance of each expert:

$$V_k = \sigma_k \sum_{k'=1}^t P_{kk'}. \quad (5.4)$$

Step 2.5: Calculate the weight τ_k of each expert:

$$\tau_k = \frac{V_k}{\sum_{k=1}^t V_k}. \quad (5.5)$$

Step 2.6: The group decision matrix D is obtained through the q-ROFWA aggregation operator as defined in Eq. (2.3), and the results are presented in Table 7:

$$D = [d_{ij}]_{n \times m} \quad (5.6)$$

where $d_{ij} = (\eta_{ij}, \sigma_{ij}) = \bigoplus_{k=1}^t \tau_k d_{ij}^k = \left(1 - \left(\prod_{k=1}^t (1 - \eta_i^q)^{\tau_k} \right)^{\frac{1}{q}}, \prod_{k=1}^t \sigma_i^{\tau_k} \right)$.

Stage 3: Determination of attributes weights .

Table 7: The group decision matrix D

Alternatives	attributes			
	C_1	C_2	$\dots$	C_n
A_1	(η_{11}, σ_{11})	(η_{12}, σ_{12})	$\dots$	(η_{1n}, σ_{1n})
A_2	(η_{21}, σ_{21})	(η_{22}, σ_{22})	$\dots$	(η_{2n}, σ_{2n})
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	(η_{m1}, σ_{m1})	(η_{m2}, σ_{m2})	$\dots$	(η_{mn}, σ_{mn})

On the basis of the introduced q-ROFSs uncertainty measure and the professional background

of experts, this paper constructs a subjective and objective combined attribute weights determination method, which is specifically described as follows:

Step 3.1: Normalize the group decision matrix $E = [d'_{ij}]_{m \times n}$.

$$d'_{ij} = (\eta_{ij}, \sigma_{ij}) = \begin{cases} (\eta_{ij}, \sigma_{ij}), & \text{if } C_j \in J_b, \\ (\sigma_{ij}, \eta_{ij}), & \text{if } C_j \in J_c, \end{cases} \quad (5.7)$$

where J_b and J_c represent the benefit attribute and cost attribute respectively.

Step 3.2: Calculate the significance of different attributes.

The significance of various attributes can be assessed through the certainty degree of their evaluations, which is derived from uncertainty measure. For the j th attribute, the evaluation of the i th alternative is represented as $d'_{ij} = (\eta_{ij}, \sigma_{ij})$.

Then the certainty degree of j th attribute is determined as follows:

$$cer(C_j) = \sum_{i=1}^m (1 - U(d'_{ij})), \quad (5.8)$$

where the calculation formula of $U(d'_{ij})$ is derived from Eq.(3.1).

By normalizing the degree of certainty, the significance of attribute is assigned as:

$$sg(C_j) = \frac{\sum_{i=1}^m (1 - U(d'_{ij}))}{\sum_{j=1}^n \sum_{i=1}^m (1 - U(d'_{ij}))}. \quad (5.9)$$

Step 3.3: Construct a linear optimization model to determine the attribute weights.

From a subjective perspective, experts' professional experience places more emphasis on the practical context of decision-making, making the assigned weights more aligned with the actual situation. From an objective perspective, the uncertainty associated with the attributes captures the weight information across different decision dimensions, thereby preserving the objectivity of the original data. Therefore, the attribute weight vector $W = (\omega_1, \omega_2, \dots, \omega_n)$ can be determined using the following linear optimization model (M-1).

$$\begin{aligned} \text{Max } \sum_{j=1}^n w_j sg(C_j) &= \text{Max } \sum_{j=1}^n w_j \left(\frac{\sum_{i=1}^m (1 - U(d'_{ij}))}{\sum_{j=1}^n \sum_{i=1}^m (1 - U(d'_{ij}))} \right) \\ \text{s.t. } \begin{cases} \sum_{j=1}^n w_j = 1 \\ W \in \Theta \\ w_j \geq 0, \quad j = 1, 2, \dots, n \end{cases} \end{aligned} \quad (5.10)$$

where $\Theta = \{W = (w_1, w_2, \dots, w_n) | w_j \in [w_j^L, w_j^H], 0 \leq w_j^L \leq w_j^H \leq 1\}$ represents the subjective information of attribute weights provided by experts based on their professional knowledge. The optimal attribute weights $W = (w_1, w_2, \dots, w_n)$ can be solved through the above-mentioned linear optimization model.

Stage 4: Rank the alternatives using the Aczel-Alsina WASPAS method.

In this section, the Aczel-Alsina based WASPAS method is constructed to rank the

alternatives in q-ROFSs domains, the specific steps are as follows.

Step 4.1: Calculate the measures of weighted sum (WSM) α_i for each alternative by using the q-rung orthopair fuzzy Aczel-Alsina weighted average (q-ROFAAWA) operator [39]:

$$q-ROFAAWA = \alpha_i = \left(\sqrt[q]{1 - e^{-\left(\sum_{j=1}^n w_j (-\log(1-\eta_{ij}^q))^\lambda\right)^{\frac{1}{\lambda}}}}, \sqrt[q]{e^{-\left(\sum_{j=1}^n w_j (-\log(\sigma_{ij}^q))^\lambda\right)^{\frac{1}{\lambda}}}} \right) (\lambda \geq 1). \quad (5.11)$$

Step 4.2: Calculate the measures of weighted product (WPM) β_i for each alternative by using the q-rung orthopair fuzzy Aczel-Alsina weighted geometric (q-ROFAAWG) operator [39]:

$$q-ROFAAWG = \beta_i = \left(\sqrt[q]{e^{-\left(\sum_{j=1}^n w_j (-\log(\eta_{ij}^q))^\lambda\right)^{\frac{1}{\lambda}}}}, \sqrt[q]{1 - e^{-\left(\sum_{j=1}^n w_j (-\log(1-\sigma_{ij}^q))^\lambda\right)^{\frac{1}{\lambda}}}} \right) (\lambda \geq 1). \quad (5.12)$$

Step 4.3: Calculate the score functions of WSM $S(\alpha_i)$ and WPM $S(\beta_i)$ for each alternative by using Eq.(4.1), respectively.

Step 4.4: Calculate the aggregated measure δ_i for each alternative.

$$\delta_i = \theta S(\alpha_i) + (1 - \theta) S(\beta_i) \quad (5.13)$$

Step 4.5: Rank the δ_i values in descending order to determine the ranking of the alternatives.

6 A case study of selecting FWTTs

To verify the proposed method, this section presents a practical case of selecting FWTTs. This example is adapted from literature [51].

6.1 Background description of food waste treatment technology

According to the United Nations and the Food and Agriculture Organization of the United Nations, pollution caused by food demand will increase to approximately 10 billion by 2050 [52]. At the same time, the continuous growth of food demand will lead to a large amount of food waste. According to statistics, about 130 to 160 million tons of food are wasted globally each year, of which China accounts for 149.21 million tons. Severe food waste (FW) has a huge negative impact on the environment, social and economic costs, food security and the national economy [53]. Rotten FW initially contributes to the greenhouse effect by emitting methane. Additionally, harmful liquids and microbial pathogens from FW can infiltrate agricultural soils and water systems, leading to substantial safety hazards and environmental contamination. Furthermore, the challenges to food security posed by FW can result in social instability. The issue of FW has sparked significant concerns about food security and sustainable development. To address the negative impacts of food waste, the government needs to select the optimal food waste treatment technology.

Food waste treatment technologies (FWTTs), mainly including composting (A_1), anaerobic digestion (A_2), incineration (A_3) and landfill (A_4). However, these FWTTs have their own advantages and disadvantages, making it difficult for government to choose. The details of the treatment technologies are listed as follows:

Composting (A_1): After pre-treatment of food waste, it is transformed into organic fertilizer through the establishment and maintenance of compost piles for recycling. Composting not only reduces the burden on landfills but also improves soil structure and increases soil fertility, thus having high environmental protection value.

Anaerobic digestion (A_2): It decomposes organic waste in an oxygen-free environment to produce biogas and fertilizer. It can generate renewable energy, reduce greenhouse gas emissions, and produce organic fertilizer.

Incineration (A_3): It is the process of burning food waste at high temperatures to convert it into ash, gas and thermal energy. It can reduce the accumulation of waste and the risk of disease transmission, but at the same time it causes air pollution and is costly.

Landfill (A_4): It is a method of treating food waste by burying it in the soil. It has the advantages of low cost and simple operation, but at the same time it causes waste of resources and environmental pollution.

These FWTTs are evaluated in thirteen attributes namely “Accident rate at a workstation”(C_1), “Occupational risks”(C_2), “Labor intensity”(C_3), “Involvement of the developed scenario to environmental capacity”(C_4), “Noise”(C_5), “Traffic”(C_6), “Time and space obligatory by waste management at household”(C_7), “Percentage of the workforce hired locally”(C_8), “Number of customer satisfaction reviews approved per annum”(C_9), “The invention in FWTT over time”(C_{10}), “Number of green procurement”(C_{11}), “Creation of green jobs”(C_{12}), “Number of stakeholders involved to promote the model”(C_{13}). Among them, C_1 , C_2 , C_5 and C_6 are cost attributes, and the rest are benefit attributes.

Obviously, to effectively evaluate FWTTs, we need to utilize these attributes and also invite decision-making experts from different fields to conduct the assessment, which makes the selection of FWTTs a MAGDM problem. Fig. 5 shows the evaluation framework of FWTTs.

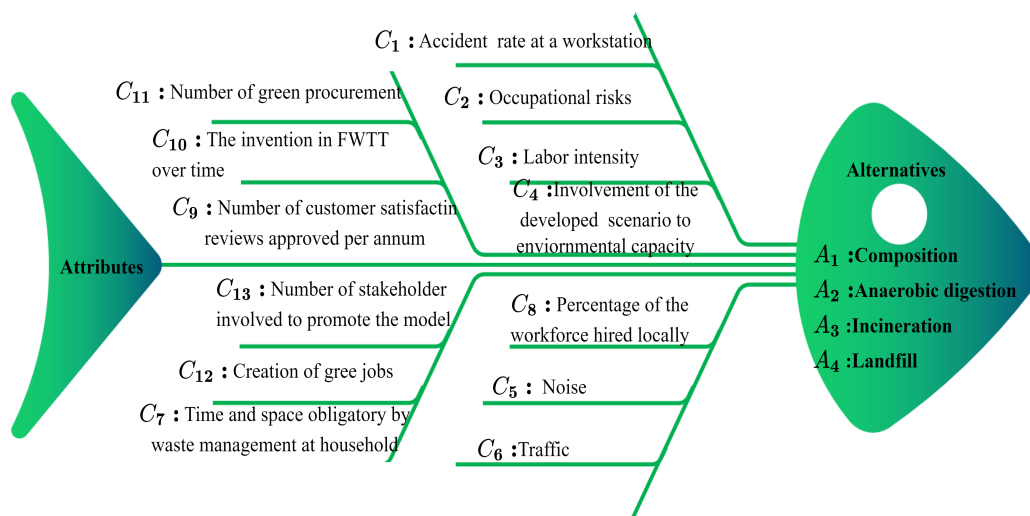


Fig. 5. Hierarchical structure of the problem.

6.2 Decision-making process

This subsection systematically introduces the specific application and implementation process of the proposed MAGDM method in the selection of FWTTs problem.

Stage 1: Expert evaluation information acquisition.

Based on the alternatives and attributes defined above, we invited three experts from different fields to conduct the evaluation to ensure the accuracy and reliability of the assessment information. Specifically, these three experts come from different fields including industries, colleges, and government. Subsequently, we collected the assessment information from three experts, which are described in detail as follows:

Step 1.1: Experts assessed the alternatives using the linguistic scale shown in Table 5, with the results summarized in Table 8.

Step 1.2: The linguistic assessment matrices were converted into a decision matrix using q-ROFNs, in accordance with the conversion rules outlined in Table 5. The resulting data is presented in Tables 9-11.

Table 8: Linguistic evaluation matrices from three experts.

	e_1				e_2				e_3			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
C_1	UAI	LI	AAI	AAI	VLI	UAI	UAI	AAI	VLI	VLI	AI	AI
C_2	UAI	VLI	UAI	AAI	LI	LI	AI	HI	AI	VLI	AAI	AI
C_3	HI	VHI	AI	VHI	VHI	VHI	AAI	AAI	HI	VHI	AI	VHI
C_4	AI	AI	UAI	VHI	AAI	HI	AAI	AAI	UAI	HI	AI	UAI
C_5	AAI	LI	UAI	LI	HI	UAI	AAI	UAI	AI	VLI	LI	VLI
C_6	LI	LI	HI	LI	LI	VLI	AAI	AAI	VLI	VLI	AI	AI
C_7	AAI	VHI	AAI	VHI	LI	HI	AI	HI	AI	HI	HI	AI
C_8	HI	AAI	AI	AAI	VHI	VHI	AAI	AI	AAI	VHI	LI	LI
C_9	AI	CHI	AAI	VLI	VHI	VHI	HI	UAI	AAI	HI	AI	LI
C_{10}	CHI	HI	UAI	CLI	HI	HI	UAI	UAI	AAI	VHI	AI	LI
C_{11}	LI	LI	AAI	UAI	UAI	LI	AI	LI	VLI	UAI	LI	AI
C_{12}	UAI	LI	VHI	UAI	LI	UAI	AI	AAI	AI	VLI	UAI	VHI
C_{13}	VHI	AAI	AAI	AI	HI	AAI	HI	UAI	HI	HI	VHI	AAI

Stage 2: Determination of experts weight and information fusion.

Step 2.1-2.5: Based on the matrices in Tables 9-11, we initially calculate the score matrices of the three experts using Eq.(4.1), and the outcomes are shown in Table 12. Then, we use Eq.(5.2)-(5.5) to calculate the expert weights, which are expressed as $\tau = (0.349, 0.314, 0.337)$.

Step 2.6: The group decision matrix computed using Eq.(5.6) is shown in Table 13.

Stage 3: Determination of attributes weight .

Step 3.1: Since the 13 attributes mentioned above include both benefit and cost attributes, it is necessary to normalize the matrix. The result is shown in Table 14.

Step 3.2-3.3: Based on Eq.(5.8) and Eq.(5.9), the significance of the attributes is calculated as follows: $sg(C_1) = 0.0691$, $sg(C_2) = 0.0686$, $sg(C_3) = 0.0953$, $sg(C_4) = 0.0678$, $sg(C_5) = 0.0676$, $sg(C_6) = 0.0763$, $sg(C_7) = 0.0796$, $sg(C_8) = 0.0758$, $sg(C_9) = 0.922$, $sg(C_{10}) = 0.0913$, $sg(C_{11}) = 0.0616$, $sg(C_{12}) = 0.0726$, $sg(C_{13}) = 0.0819$. Then, according to the calculation results of attribute importance and the professional knowledge of experts, The optimization model (M-1) is built to determine the attribute weights.

Table 9: The q-ROFN-based evaluation matrix of expert e_1 .

	A_1	A_2	A_3	A_4
C_1	(0.44,0.66)	(0.33,0.77)	(0.66,0.44)	(0.66,0.44)
C_2	(0.44,0.66)	(0.22,0.88)	(0.44,0.66)	(0.66,0.44)
C_3	(0.77,0.33)	(0.88,0.22)	(0.55,0.55)	(0.88,0.22)
C_4	(0.55,0.55)	(0.55,0.55)	(0.44,0.66)	(0.88,0.22)
C_5	(0.66,0.44)	(0.33,0.77)	(0.44,0.66)	(0.33,0.77)
C_6	(0.33,0.77)	(0.33,0.77)	(0.77,0.33)	(0.33,0.77)
C_7	(0.66,0.44)	(0.88,0.22)	(0.66,0.44)	(0.88,0.22)
C_8	(0.77,0.33)	(0.66,0.44)	(0.55,0.55)	(0.66,0.44)
C_9	(0.55,0.55)	(0.99,0.11)	(0.66,0.44)	(0.22,0.88)
C_{10}	(0.99,0.11)	(0.77,0.33)	(0.44,0.66)	(0.11,0.99)
C_{11}	(0.33,0.77)	(0.33,0.77)	(0.66,0.44)	(0.44,0.66)
C_{12}	(0.44,0.66)	(0.33,0.77)	(0.88,0.22)	(0.44,0.66)
C_{13}	(0.88,0.22)	(0.66,0.44)	(0.66,0.44)	(0.55,0.55)

Table 10: The q-ROFN-based evaluation matrix of expert e_2 .

	A_1	A_2	A_3	A_4
C_1	(0.22,0.88)	(0.44,0.66)	(0.44,0.66)	(0.66,0.44)
C_2	(0.33,0.77)	(0.33,0.77)	(0.55,0.55)	(0.77,0.33)
C_3	(0.88,0.22)	(0.88,0.22)	(0.66,0.44)	(0.66,0.44)
C_4	(0.66,0.44)	(0.77,0.33)	(0.66,0.44)	(0.66,0.44)
C_5	(0.77,0.33)	(0.44,0.66)	(0.66,0.44)	(0.44,0.66)
C_6	(0.33,0.77)	(0.22,0.88)	(0.66,0.44)	(0.66,0.44)
C_7	(0.33,0.77)	(0.77,0.33)	(0.55,0.55)	(0.77,0.33)
C_8	(0.88,0.22)	(0.88,0.22)	(0.66,0.44)	(0.55,0.55)
C_9	(0.88,0.22)	(0.88,0.22)	(0.77,0.33)	(0.44,0.66)
C_{10}	(0.77,0.33)	(0.77,0.33)	(0.44,0.66)	(0.44,0.66)
C_{11}	(0.44,0.66)	(0.33,0.77)	(0.55,0.55)	(0.33,0.77)
C_{12}	(0.33,0.77)	(0.44,0.66)	(0.55,0.55)	(0.66,0.44)
C_{13}	(0.77,0.33)	(0.66,0.44)	(0.77,0.33)	(0.44,0.66)

Table 11: The q-ROFN-based evaluation matrix of expert e_3 .

	A_1	A_2	A_3	A_4
C_1	(0.22,0.88)	(0.22,0.88)	(0.55,0.55)	(0.55,0.55)
C_2	(0.55,0.55)	(0.22,0.88)	(0.66,0.44)	(0.55,0.55)
C_3	(0.77,0.33)	(0.88,0.22)	(0.55,0.55)	(0.88,0.22)
C_4	(0.44,0.66)	(0.77,0.33)	(0.55,0.55)	(0.44,0.66)
C_5	(0.55,0.55)	(0.22,0.88)	(0.33,0.77)	(0.22,0.88)
C_6	(0.22,0.88)	(0.22,0.88)	(0.55,0.55)	(0.55,0.55)
C_7	(0.55,0.55)	(0.77,0.33)	(0.77,0.33)	(0.55,0.55)
C_8	(0.66,0.44)	(0.88,0.22)	(0.33,0.77)	(0.33,0.77)
C_9	(0.66,0.44)	(0.77,0.33)	(0.55,0.55)	(0.33,0.77)
C_{10}	(0.66,0.44)	(0.88,0.22)	(0.55,0.55)	(0.33,0.77)
C_{11}	(0.22,0.88)	(0.44,0.66)	(0.33,0.77)	(0.55,0.55)
C_{12}	(0.55,0.55)	(0.22,0.88)	(0.44,0.66)	(0.88,0.22)
C_{13}	(0.77,0.33)	(0.77,0.33)	(0.88,0.22)	(0.66,0.44)

Table 12: The score matrices of three experts.

	e_1				e_2				e_3			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
C_1	0.3274	0.2150	0.5694	0.5694	0.1099	0.3274	0.3274	0.5694	0.1099	0.1099	0.4456	0.4456
C_2	0.3274	0.1099	0.3274	0.5694	0.215	0.215	0.4456	0.6990	0.4456	0.1099	0.5694	0.4456
C_3	0.6990	0.8359	0.4456	0.8359	0.8359	0.8359	0.5694	0.5694	0.6990	0.8359	0.4456	0.8359
C_4	0.4456	0.4456	0.3274	0.8359	0.5694	0.6990	0.5694	0.5694	0.3274	0.6990	0.4456	0.3274
C_5	0.5694	0.2150	0.3274	0.2150	0.6990	0.3274	0.5694	0.3274	0.4456	0.1099	0.2150	0.1099
C_6	0.2150	0.2150	0.6990	0.2150	0.2150	0.1099	0.5694	0.5694	0.1099	0.1099	0.4456	0.4456
C_7	0.5694	0.8359	0.5694	0.8359	0.2150	0.699	0.4456	0.6990	0.4456	0.6990	0.6990	0.4456
C_8	0.6990	0.5694	0.4456	0.5694	0.8359	0.8359	0.5694	0.4456	0.5694	0.8359	0.2150	0.2150
C_9	0.4456	0.9827	0.5694	0.1099	0.8359	0.8359	0.6990	0.3274	0.5694	0.6990	0.4456	0.2150
C_{10}	0.9827	0.6990	0.3274	0.0147	0.6990	0.6990	0.3274	0.3274	0.5694	0.8359	0.4456	0.2150
C_{11}	0.2150	0.2150	0.5694	0.3274	0.3274	0.2150	0.4456	0.2150	0.1099	0.3274	0.2150	0.4456
C_{12}	0.3274	0.2150	0.8359	0.3274	0.2150	0.3274	0.4456	0.5694	0.4456	0.1099	0.3274	0.8359
C_{13}	0.8359	0.5694	0.5694	0.4456	0.6990	0.5694	0.6990	0.3274	0.6990	0.6990	0.8359	0.5694

Table 13: The group decision matrix D .

	A_1	A_2	A_3	A_4
C_1	(0.3195,0.7959)	(0.3426,0.7674)	(0.5687,0.5388)	(0.6286,0.4744)
C_2	(0.4575,0.6515)	(0.261,0.8439)	(0.5649,0.5437)	(0.6739,0.4334)
C_3	(0.8146, 0.2906)	(0.8811, 0.22)	(0.5906, 0.5128)	(0.8377, 0.2735)
C_4	(0.5617, 0.5453)	(0.7151, 0.3944)	(0.5612, 0.5465)	(0.7338, 0.396)
C_5	(0.6739,0.4334)	(0.3426,0.7674)	(0.5049,0.6121)	(0.3426,0.7674)
C_6	(0.2987,0.8054)	(0.265,0.8399)	(0.6785,0.429)	(0.5391,0.5767)
C_7	(0.5495, 0.5655)	(0.8188, 0.2865)	(0.6801, 0.4283)	(0.7769, 0.3403)
C_8	(0.7897, 0.3201)	(0.8319, 0.2802)	(0.5401, 0.5744)	(0.5449, 0.5699)
C_9	(0.7389, 0.3826)	(0.9395, 0.198)	(0.6739, 0.4334)	(0.342, 0.7686)
C_{10}	(0.9176, 0.2478)	(0.8181, 0.2879)	(0.4835, 0.6207)	(0.3246, 0.8009)
C_{11}	(0.3426, 0.7674)	(0.3735, 0.731)	(0.5449, 0.5699)	(0.4575, 0.6515)
C_{12}	(0.4575, 0.6515)	(0.3426, 0.7674)	(0.7118, 0.4248)	(0.7313, 0.4013)
C_{13}	(0.8188, 0.2865)	(0.7051, 0.3993)	(0.7933, 0.3183)	(0.568, 0.5402)

$$\begin{aligned}
\max Z &= 0.0617w_1 + 0.0627w_2 + 0.1175w_3 + 0.0438w_4 + 0.0618w_5 + 0.0803w_6 \\
&+ 0.0708w_7 + 0.0702w_8 + 0.1171w_9 + 0.1233w_{10} + 0.0539w_{11} + 0.0633w_{12} + 0.0754w_{13} \\
\text{s.t.} \quad &\begin{cases} 0.035 \leq w_1 \leq 0.04, & 0.09 \leq w_2 \leq 0.1, & 0.08 \leq w_3 \leq 0.09 \\ 0.045 \leq w_4 \leq 0.05, & 0.13 \leq w_5 \leq 0.15, & 0.045 \leq w_6 \leq 0.05 \\ 0.045 \leq w_7 \leq 0.05, & 0.07 \leq w_8 \leq 0.09, & 0.12 \leq w_9 \leq 0.14 \\ 0.08 \leq w_{10} \leq 0.09, & 0.04 \leq w_{11} \leq 0.05, & 0.04 \leq w_{12} \leq 0.05 \\ 0.09 \leq w_{13} \leq 0.095, & w_j \geq 0, & \sum_{j=1}^{13} w_j = 1 \end{cases}
\end{aligned}$$

By solving the model with Python, we can derive the attribute weights $w_1 = 0.04$, $w_2 = 0.09$, $w_3 = 0.09$, $w_4 = 0.045$, $w_5 = 0.13$, $w_6 = 0.05$, $w_7 = 0.05$, $w_8 = 0.09$, $w_9 = 0.14$, $w_{10} = 0.09$, $w_{11} = 0.04$, $w_{12} = 0.05$, $w_{13} = 0.095$.

Table 14: Normalized group decision matrix E .

	A_1	A_2	A_3	A_4
C_1	(0.7959, 0.3195)	(0.7674, 0.3426)	(0.5388, 0.5687)	(0.4744, 0.6286)
C_2	(0.6515, 0.4575)	(0.8439, 0.261)	(0.5437, 0.5649)	(0.4334, 0.6739)
C_3	(0.8146, 0.2906)	(0.8811, 0.22)	(0.5906, 0.5128)	(0.8377, 0.2735)
C_4	(0.5617, 0.5453)	(0.7151, 0.3944)	(0.5612, 0.5465)	(0.7338, 0.396)
C_5	(0.4334, 0.6739)	(0.7674, 0.3426)	(0.6121, 0.5049)	(0.7674, 0.3426)
C_6	(0.8054, 0.2987)	(0.8399, 0.265)	(0.429, 0.6785)	(0.5767, 0.5391)
C_7	(0.5495, 0.5655)	(0.8188, 0.2865)	(0.6801, 0.4283)	(0.7769, 0.3403)
C_8	(0.7897, 0.3201)	(0.8319, 0.2802)	(0.5401, 0.5744)	(0.5449, 0.5699)
C_9	(0.7389, 0.3826)	(0.9395, 0.198)	(0.6739, 0.4334)	(0.342, 0.7686)
C_{10}	(0.9176, 0.2478)	(0.8181, 0.2879)	(0.4835, 0.6207)	(0.3246, 0.8009)
C_{11}	(0.3426, 0.7674)	(0.3735, 0.731)	(0.5449, 0.5699)	(0.4575, 0.6515)
C_{12}	(0.4575, 0.6515)	(0.3426, 0.7674)	(0.7118, 0.4248)	(0.7313, 0.4013)
C_{13}	(0.8188, 0.2865)	(0.7051, 0.3993)	(0.7933, 0.3183)	(0.568, 0.5402)

Stage 4: The Aczel-Alsina based WASPAS method for ranking alternatives.

Step 4.1-4.2: The weighted sum (WSM) (α_i) for each alternative was computed using the q-ROFAAWA operator, as presented in Eq. (5.11), along with the normalized aggregation matrix from Table 14. The corresponding results are shown in Table 15. Similarly, the weighted product (WPM) (β_i) for each alternative was calculated using the q-ROFAAWG operator from Eq. (5.12) and the normalized aggregation matrix from Table 14, with the results also provided in Table 15.

Step 4.3: Based on Eq.(4.1) and the q-ROFNs in Table 15, the score function of WSM and

Table 15: The q-ROFNs of WSM and WPM for each alternative.

Alternatives	WSM		WPM	
	η	σ	η	σ
A_1	0.7758	0.3769	0.6082	0.5408
A_2	0.8445	0.2928	0.6883	0.4858
A_3	0.6413	0.4853	0.5823	0.5380
A_4	0.6781	0.4711	0.4974	0.6490

WPM for each alternative were calculated respectively. The calculation results of $S(\alpha_i)$ and $S(\beta_i)$ are shown in Table 16.

Step 4.4: By using Eq.(5.13), $S(\alpha_i)$ and $S(\beta_i)$, the WASPAS measure δ_i for each alternative was calculated, and the results are shown in Table 16.

Step 4.5: To determine the ranking of each alternative, the values of (δ_i) are arranged in descending order.

The final ranking of the alternatives is $A_2 > A_1 > A_3 > A_4$. This outcome confirms that “Anaerobic Digestion (A_2)” is the optimal technology for food waste treatment. Throughout the calculation process of the entire case, the parameters $q = 2$, $\lambda = 2$ and $\theta = 0.5$ were set.

Table 16: The score functions of WSM and WPM for each alternative.

Alternatives	$S(\alpha_i)$	$S(\beta_i)$	δ_i	Rank
A_1	0.6923	0.5091	0.4910	2
A_2	0.7834	0.6403	0.5769	1
A_3	0.5383	0.4942	0.4731	3
A_4	0.5735	0.4155	0.3661	4

6.3 Sensitivity analysis

This paper introduces an extended MAGDM approach in which three parameters q , λ and θ are identified subjectively. To thoroughly assess the robustness and effectiveness of the proposed method, a sensitivity analysis is performed in the section. Specifically, three phases of sensitivity analyses will be carried out respectively for these three parameters.

Phase 1: The influence of parameter q on the ultimate results.

In the proposed MAGDM model, the selection of the parameter q is critical, as it significantly influences the decision space. Specifically, different q values correspond to different decision spaces. During the decision-making process, expert may choose a suitable value for q in accordance with their individual preferences and judgement. In this paper, $q \geq 2$. The reason is that the first expert's evaluation value for the first alternative A_1 under the first attribute C_1 is (0.44, 0.66). When $q = 1$, $0.44 + 0.66 > 1$, so expert 1 chose the smallest integer greater than 1, which is $q = 2$. As the q value changes, it may significantly affect the evaluation process and results. Therefore, this phase explores the impact of different q values ($q = 2, 3, \dots, 11$) on the decision-making results.

As shown in Fig. 6, the variation of parameter q exerts a considerable influence on the ultimate WASPAS measure results δ_i of the four alternatives. When q increases from 2 to 3, the WASPAS measure values of all alternative rise; however when q increases from 3 to 11, the WASPAS measure values of all alternatives exhibits a gradual decline. Additionally, the gap between A_1 and A_2 as well as that between A_3 and A_4 gradually narrows, making it more challenging to distinguish the relative superiority or inferiority of each alternative. Therefore, a smaller value of q is more conducive to differentiating the quality of the alternatives.

In addition, as illustrated in Fig. 6, regardless of variations in the parameter q , the order of the four alternatives is stable, maintaining $A_2 > A_1 > A_3 > A_4$. This indicates that the developed model exhibits strong stability.

Phase 2: The influence of parameter λ on the ultimate results.

At this phase, we conducted a sensitivity analysis on parameter λ . Fig. 7 illustrates the impact of varying parameter values ($\lambda = 1, 2, 3, \dots, 10$) on the WASPAS measure results of each alternative.

According to the analysis results in Fig. 7, it is evident that the WASPAS measure(δ_i) of A_1 and A_2 gradually decreases as the parameter λ increases, and the gap between the two gradually narrows, eventually stabilizing at a certain value. In contrast, the results of A_3 and A_4 change relatively little and remain relatively stable. However, regardless of the variations in the value of λ , the ranking of each alternative remains $A_2 > A_1 > A_3 > A_4$. This indicates that the proposed method exhibits highly robustness.

Phase 3: The influence of parameter θ on the ultimate results.

At this stage, we conducted a sensitivity analysis on parameter θ , and the calculation results are shown in Fig. 8. Fig. 8 illustrates the impact of different values of parameter θ within

the range $[0, 1]$ on the WASPAS measure of the four alternatives. According to Fig. 8, when θ varies within the range $[0, 0.7]$, the ranking of the four schemes is $A_2 > A_1 > A_3 > A_4$, and the optimal alternative is A_2 at this time; when θ varies within the range $[0.7, 1]$, the ranking of the four schemes is $A_2 > A_1 > A_4 > A_3$, and the optimal scheme remains A_2 . This analysis reveals that although variations in θ do impact the overall ranking, they do not alter the selection of the optimal alternative. This is because the variation in the parameter θ represents a change in the emphasis placed on the two methods, WSM and WPM. A different choice of method may slightly affect the ranking of the solutions, but it will not impact the ranking of the optimal solution. This suggests that the proposed method demonstrates excellently reliability.

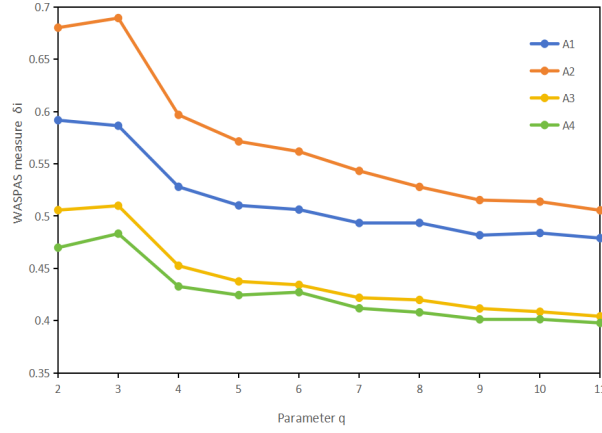


Fig. 6. The influence of parameter q on the ultimate results.

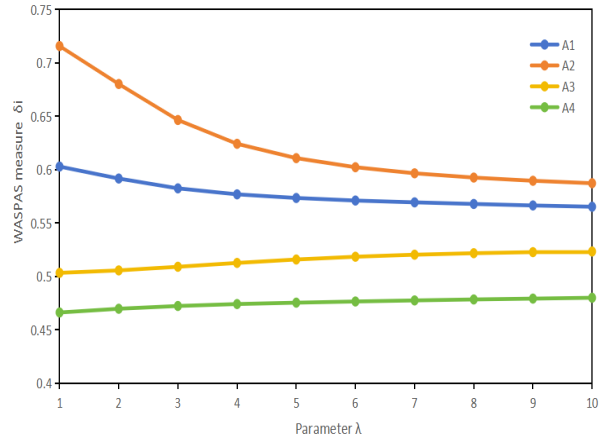


Fig. 7. The influence of parameter λ on the ultimate results.

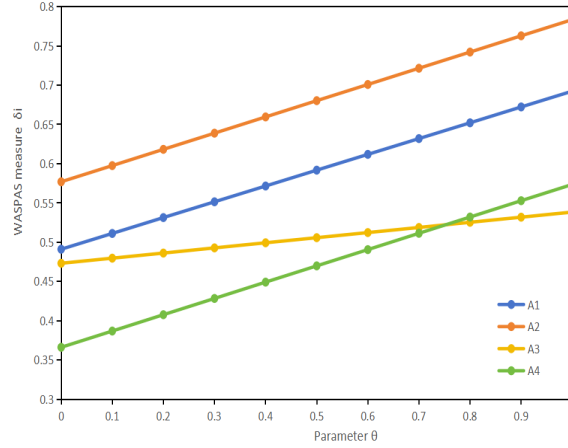


Fig. 8. The influence of parameter θ on the ultimate results.

6.4 Comparative analysis

To verify the effectiveness and robustness of the proposed method, we further calculated the ultimate ranking results of four existing MAGDM methods under the same case from section 6.1. The four existing methods are q-ROFSs TODIM [33], q-ROFSs TOPSIS [54], crisp-number EDAS [55] and single-valued neutrosophic MULTIMOORA [51]. Subsequently, we conducted a detailed comparative analysis of the ranking results we obtained with those from the above-mentioned four MAGDM methods. The results are shown in Table 17.

It can be found from Table 17 that all methods except q-ROFSs TODIM selected A_2 (Anaerobic digestion) as the best food waste treatment technology. This is because in this paper, q is set to 2, but in the TODIM method, the ranking results fluctuate significantly with the variation of q . Moreover, when q increases to be greater than or equal to 6, the optimal solution selected by the TODIM method becomes consistent with that of this paper. This indicates that the method proposed in this paper is more stable and universally applicable compared to the TODIM method. Additionally, it is noteworthy that the rankings of A_3 and A_4 in crisp number EDAS are inconsistent with those of other methods, this difference is due to the adoption of different data types for evaluating the alternatives. The literature [55] uses precise data, compared to q-ROFNs, which cannot fully depict the uncertainty and fuzziness generated by experts during the evaluation process. In contrast, the q-ROFSs chosen in this paper offer stronger expressiveness and a more flexible choice space when evaluating the alternatives. Therefore, the proposed method has certain superiority. In conclusion, the proposed method yields the same ranking results and optimal solutions as most established methods, which proves that our approach is correct and effective. The results show that this method is reliable in dealing with problems in the q-ROF environment. Therefore, decision-makers can choose our method to make decisions on complex real-world problems in the future. The ranking orders of these methods are shown in Fig. 9.

Given the discrepancies in the ranking outcomes between the proposed q-ROFSs information measures-based WASPAS method and alternative methods, it is essential to compute the correlation coefficients among these methods. This research computed the Spearman rank correlation coefficient, as illustrated in Fig. 10. The proposed method exhibits correlation

Table 17: comparative results.

Method	Ranking order
Proposed method	$A_2 > A_1 > A_3 > A_4$
q-ROFSs TODIM [33]	$A_1 > A_3 > A_2 > A_4$
q-ROFSs TOPSIS [54]	$A_2 > A_1 > A_3 > A_4$
crisp number EDAS [55]	$A_2 > A_1 > A_4 > A_3$
SVNS-MULTIMOORA [51]	$A_2 > A_1 > A_3 > A_4$

coefficients of 0.4, 1, 0.8, and 1 when compared to the four alternative methods, respectively. These coefficients, which are positive and in close proximity to 1. The observation indicates that despite the presence of differences between the proposed method and other current methods, a robust correlation persists. Consequently, it can be concluded that the results yielded by this method are strongly aligned with those of other methods. This alignment further confirms the effectiveness and stability of the proposed method.

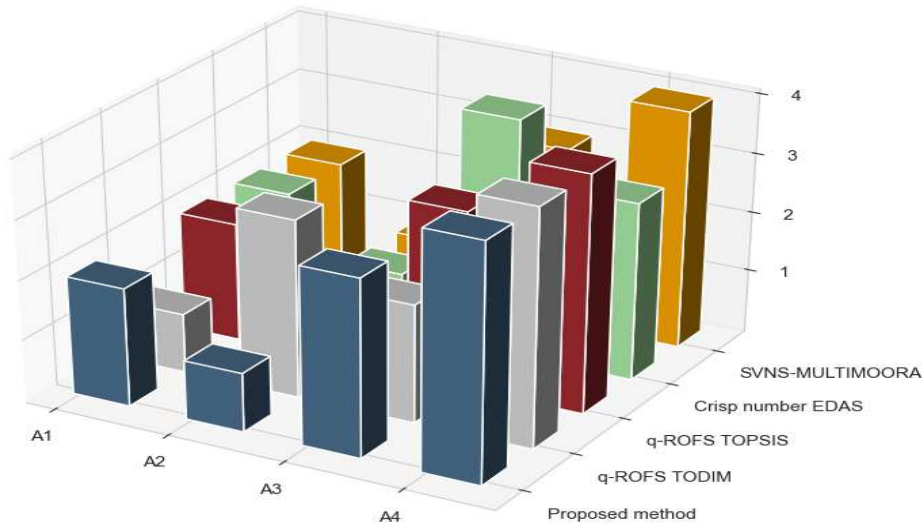


Fig. 9. Ranking results of different methods.

6.5 Discussion

Next, we consider the model from the perspective of application and method and further expand the comparison with existing methods. Table 18 summarizes the specific features of each model, as detailed below. This indicates that, compared with existing models, the proposed model has significant theoretical superiority. By comparison, the benefits of the approach introduced in this study can be summarized as follows:

(1) The proposed q-ROFSs information measures-based WASPAS method, q-ROFSs TODIM and q-ROFSs TOPSIS methods are all developed based on q-ROFSs, while the crisp number EDAS and single-valued neutrosophic MULTIMOORA methods are respectively developed based on crisp number and single-valued neutrosophic. Compared with crisp number and single-valued neutrosophic, q-ROFSs can more flexibly reflect uncertain information, thereby

improving the reliability of the method.

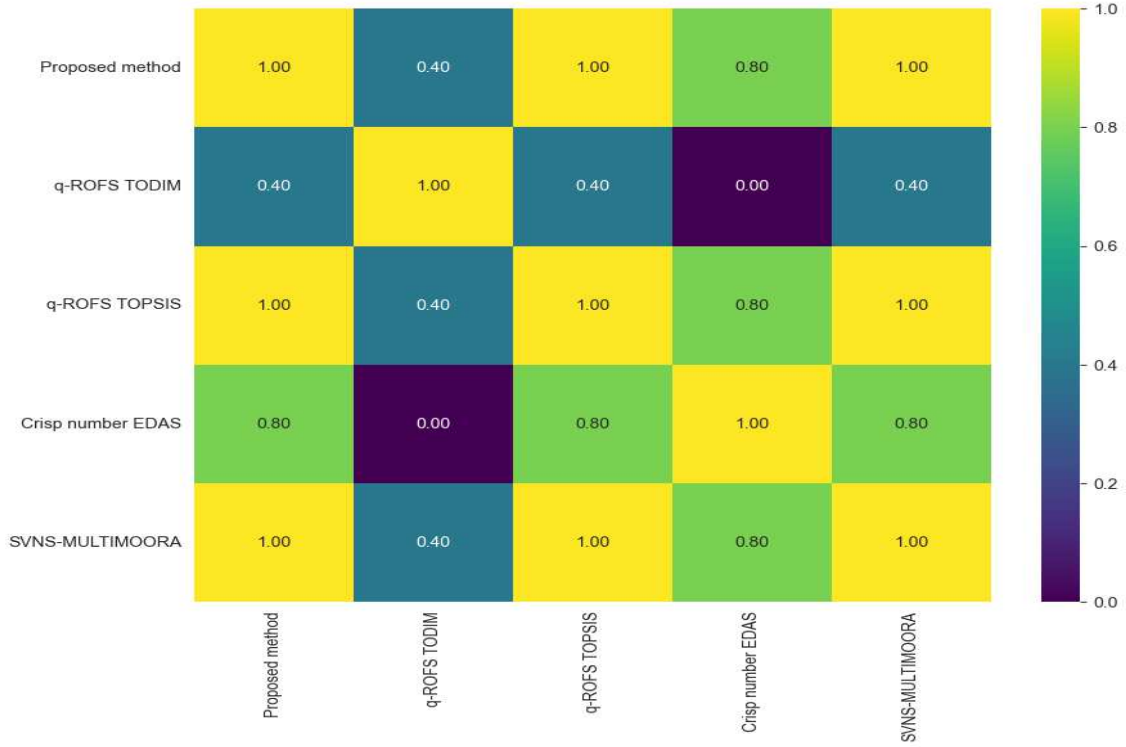


Fig. 10. Spearman's rank correlation coefficients of different methods.

(2) The proposed method employs the CRTIC model to determine the weights of experts and, in the process, uses the proposed score function to calculate the score matrix for each expert, thereby overcoming the limitations of other score functions. Compared with other methods for calculating expert weights, this method takes into account the interaction among experts, making the results more scientific and reasonable.

(3) This method determines attribute weights by combining subjective and objective informations. By introducing the proposed uncertainty measure, the certainty of attributes is calculated based on the evaluation matrix provided by the DMs, and the objective weight information of attributes is fully considered. At the same time, an optimization model is constructed to incorporate the subjective weight information provided by experts. Among the above four methods, some only consider objective attribute weights, while others only consider subjective attribute weights. These methods all have partiality and irrationality. In contrast, the weight calculation method that combines subjective weight information and objective weight information is more comprehensive and reliable.

(4) The proposed q-ROFSs information measures-based WASPAS method integrates the q-ROFAAWA operator and the q-ROFAAWG operator, and can select different parameters according to the problem requirements, assigning different levels of importance to the two weighted operators. Compared with methods that only use one weighted operator, it is more scientific and targeted.

Table 18: The comparative results between the previous models and proposed model.

Methods	Subjective attribute weight information	Objective attribute weight information	Interaction among experts	Expert's preference
Proposed method	✓	✓	✓	✓
q-ROFS TODIM [33]	✓	×	×	✓
q-ROFS TOPSIS [54]	✓	×	×	✓
crisp Number EDAS [55]	×	✓	×	×
SVNS-MULTIMOORA [51]	✓	×	×	✓

7 Conclusion, limitations and future works

In this paper, we mainly investigate two information measures of q-ROFSs and propose an extended MAGDM based on the two information measures. Firstly, to more accurately quantify the uncertain information of q-ROFSs, we propose an uncertainty measure with a novel axiomatic framework, which comprehensively considers the hesitation degree and fuzziness of q-ROFSs. In the aspect of comparison method, we define a novel score function for q-ROFSs, which considers the hesitation and address the limitations of current score functions. According to the proposed uncertainty measure and score function, we develop an extended MAGDM method by the Aczel-Alsina weighted operator and WASPAS approach under q-ROF environment. Finally, to illustrate the efficiency and adaptability of our MAGDM method, an example involving the selection of food waste treatment technology was utilized. The method introduced in this paper significantly enhances the current information measures techniques and MAGDM methods within the domain of q-ROFSs. Overall, this research has following contributions:

(1) An uncertainty measure for q-ROFSs and its axiomatic properties are defined. Compared to the existing entropy measures. The proposed uncertainty measure takes into account both fuzziness and hesitation simultaneously, and has obvious advantages over existing entropy measures in quantifying the uncertainty of information.

(2) A novel score function for q-ROFSs is proposed, and its properties are proven. Compared to previous score functions, the proposed one fully considers the hesitation and fuzziness of q-ROFNs. It also overcomes the shortcomings of previous score functions, such as counterintuitive behavior, inability to distinguish, and zero multiplication, making it more practical.

(3) Based on the proposed information measures, an extended MAGDM approach combines Aczel-Alsina weighted operator and WASPAS method under q-ROFSs is proposed. The uncertainty evaluations of experts are represented using q-ROFSs. The proposed uncertainty measure, combined with an optimization model, is used to determine attribute weights. The proposed score function, combined with the CRITIC model, is used to determine expert weights. The Aczel-Alsina WASPAS method is then applied to evaluate and rank alternatives. Finally, the stability and effectiveness of the method have been validated through a case study on the selection of food waste treatment technology.

7.1 Research limitations

There are some key limitations of the proposed q-ROFS information measures-based WASPAS method. Firstly, this method is highly dependent on precise data; if the data is missing, incomplete or incorrect, it will lead to incorrect ranking results, thereby hindering the decision-making process. Secondly, when the decision-making problem involves a large number of attributes and multiple experts, the computational load of this method is relatively high, which means that a large amount of calculation is required to ensure the rationality of the decision-making, and this may delay the decision-making process. Additionally, when using the WASPAS method for ranking, although the selection of parameter α does not affect the determination of the optimal and suboptimal solutions, it does have an impact on the overall ranking of the alternative options. Therefore, the reasonable selection of parameter α based on the actual problem is a key task. In the sensitivity analysis section, we conducted an analysis by selecting different parameter values; however, we did not analyze different expert weight sets and attribute weight sets. Finally, for the new information measurement method proposed in this paper, although it has certain theoretical advantages, its exploration in practical application scenarios is still insufficient.

7.2 Future works

After discussing the limitations of the research, the main directions for future research can be determined as follows:

(1) The proposed information measures and MAGDM approach can be widely applied to decision-making problems in various fields, such as evaluating commercial live-streaming platforms [56], medical waste treatment [1], supplier selection [2], the location selection of household waste recycling plants [57], thereby further enhancing their practical value and application scope.

(2) The proposed information measures and MAGDM approach can be extended to other types of fuzzy sets, such as interval-valued fermatean fuzzy sets [58], complex q-rung orthopair fuzzy sets [59] and so on.

(3) In decision-making problems, as they usually involve the opinions of multiple experts, future research will further explore how to evaluate the consistency of expert opinions and analyze its impact on the decision-making results.

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