

Traversing Distortion-Perception Tradeoff using a Single Score-Based Generative Model

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Abstract

The distortion-perception (DP) tradeoff reveals a fundamental conflict between distortion metrics (e.g., MSE and PSNR) and perceptual quality. Recent research has increasingly concentrated on evaluating denoising algorithms within the DP framework. However, existing algorithms either prioritize perceptual quality by sacrificing acceptable distortion, or focus on minimizing MSE for faithful restoration. When the goal shifts or noisy measurements vary, adapting to different points on the DP plane needs retraining or even re-designing the model. Inspired by recent advances in solving inverse problems using score-based generative models, we explore the potential of flexibly and optimally traversing DP tradeoffs using a single pre-trained score-based model. Specifically, we introduce a variance-scaled reverse diffusion process and theoretically characterize the marginal distribution. We then prove that the proposed sample process is an optimal solution to the DP tradeoff for conditional Gaussian distribution. Experimental results on two-dimensional and image datasets illustrate that a single score network can effectively and flexibly traverse the DP tradeoff for general denoising problems.

1 Introduction

In recent years, we have witnessed rapid progress in image restoration algorithms, especially deep learning-based implementations that have shown remarkable achievements in solving general inverse problems. How to evaluate the performance of emerging algorithms is a crucial yet complicated problem. Traditional full reference *distortion* metrics, such as Mean Square Error (MSE), focus on the pixel-level accuracy between the original image X and its reconstruction $\hat{X}$. On the other hand, *perceptual quality*, referring to the degree to which an image looks natural rather than algorithmically generated [14], is also an important mea-

sure. It has been demonstrated that perceptual quality could be associated with the distance between the distributions of natural images p_X and generated images $p_{\hat{X}}$ [2, 3, 12].

Improvements in distortion measures do not necessarily lead to enhancements in perceptual quality. In fact, [2] has demonstrated that there exists a tradeoff between distortion and perception, termed as distortion-perception (DP) tradeoff. Mathematically, it can be modeled as

$$D(P) = \min_{p_{\hat{X}|Y}} \mathbb{E}[\Delta(X, \hat{X})],$$

$$\text{s.t. } d(p_X, p_{\hat{X}}) \leq P,$$

where Y is the degraded observation, $\Delta : \mathcal{X} \times \hat{\mathcal{X}} \rightarrow \mathbb{R}^+$ is a full reference distortion function (e.g., such as square-error), and $d(\cdot, \cdot)$ is some divergence between probability distribution, such as Wasserstein distance [16] or Kulback-Leibler (KL) divergence.

Since the introduction of the DP tradeoff, more and more algorithm designs have focused on performance evaluation on the DP plane for specific denoising tasks, e.g., image deblurring [15, 25] and super-resolution [11, 13, 23]. However, most algorithms tend to seek better perceptual quality at the cost of distortion, or focus on optimizing MSE, a distortion measure, to ensure restoration fidelity. When the restoration objective shifts between distortion and perception, we need to retrain or even re-design the network, which can be computationally intensive. In many practical scenarios, it is significant to flexibly and effectively fulfill diverse task objectives or user demands with a *single* model at inference time.

There have been explorations into theories and algorithms to traverse the DP tradeoff in inference time [7, 15]. Specifically, in [15], the authors developed a framework based on conditional generative adversarial networks (CGAN) with an additional penalty on posterior expectation. By averaging different numbers of samples or adjusting the noise level injected into the generator, this CGAN-based framework can access various points along the tradeoff. However, using multiple image samples and averaging

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them may not be efficient during inference, and the method is not provably optimal. Furthermore, the training process relies on pairs of clean and noisy data. Thus, retraining is required for different noise levels and measurements. In [7], the DP tradeoff is studied in Wasserstein space. Theoretical results show that estimators on the DP curve can be constructed by linearly combining two estimators at the extremes: an optimal MSE minimizer and a perfect perception sampler with minimum distortion. This idea is further applied in burst restoration [27]. In practice, a method that achieves a relatively low MSE, alongside another method that achieves good perceptual quality, can serve as approximations of these two extreme estimators for a specific task. Nonetheless, different models must be deployed to address various measurements, rendering this method less flexible to general inverse problems.

Recent research has demonstrated the generative capability of score-based diffusion models [4, 8, 10, 18, 20, 21] in tackling general inverse problems using a single score network [5, 6, 19, 24, 26, 29]. Score-based diffusion models learn the prior distribution of the data $\mathbf{x}$ by training a score network to match the gradient of the logarithm density $\nabla_{\mathbf{x}} \log p(\mathbf{x})$, referred to as the *score*. After observing a noisy measurement $\mathbf{y}$, sampling from posteriors involves approximating the conditional score $\nabla_{\mathbf{x}} \log p(\mathbf{x}|\mathbf{y})$. Theoretical insights provided in [28] also reveal the potential for recovering Minimum Mean Square Error (MMSE) estimation by propagating the mean of the reverse diffusion process. Inspired by these advancements, we explore the potential of traversing DP tradeoffs for different tasks using a single score network. Our main contributions are summarized below:

- First, we propose a variance-scaled reverse diffusion process and theoretically characterize the marginal distributions produced by this novel reverse process. It is demonstrated that the mean of our proposed sampling converges towards the MMSE point while the marginal covariance is scaled by a scaling factor. By tuning a parameter that dictates the variance of reverse sampling, we can flexibly navigate from the MMSE point to the posterior distribution, where the two extremes are achieved by setting the parameter to zero and one, respectively.
- Subsequently, we show that the reconstruction obtained from the variance-scaled reverse sampling represents the optimal solution to the conditional DP tradeoff for multivariate Gaussian distributions, assessed through MSE and Wasserstein-2 distance metrics.
- Finally, we validate our methodology through experiments conducted on various two-dimensional datasets and a real-world image dataset. With a single pre-trained score network, we can traverse the DP tradeoff across different inverse problems with varying noise levels. The results indicate that the variance-scaled reverse diffusion

process achieves a more complete empirical DP tradeoff curve and better MSE than the GAN-based method and other inverse problem solvers, demonstrating the effectiveness and flexibility of the proposed framework.

Notations: For a random variable X denoted by a capital letter, we use small letter $\mathbf{x}$ to denote its realizations, and use $p_X(\mathbf{x})$ to denote the distribution over its alphabet $\mathcal{X}$. When there is no ambiguity, the distribution can be abbreviated as $p(\mathbf{x})$. For a discrete sequence of random variables $X_0, X_1, \dots, X_T$, we abbreviate them as $\{X_k\}_{k=0}^T$ and use $\mathbf{x}_{0:T}$ to denote their realizations. The expectation and conditional expectation of X given $Y = \mathbf{y}$ is $\mathbb{E}_{p(\mathbf{x})}[X]$ and $\mathbb{E}_{p(\mathbf{x}|\mathbf{y})}[X]$. We use $\text{Cov}_{p(\mathbf{x})}[X]$ and $\text{Cov}_{p(\mathbf{x}|\mathbf{y})}[X]$ to represent the covariance and conditional covariance of X given $Y = \mathbf{y}$. Matrices are denoted by uppercase boldface letters (e.g., Σ). $\text{Tr}(\Sigma)$ and Σ^{-1} represent the trace and inverse.

2 Backgrounds

2.1 Score-based generative model

Score-based generative models or diffusion models define the generative process as the reverse of a diffusion process. Specifically, the diffusion process (i.e., forward process) is a Markov chain $\{X_k\}_{k=0}^T$, with joint distribution

$$p(\mathbf{x}_{0:T}) := p(\mathbf{x}_0) \prod_{k=0}^{T-1} p(\mathbf{x}_{k+1}|\mathbf{x}_k), \quad (1)$$

which gradually diffuses the data $X_0 \in \mathbb{R}^d$ with distribution $p(\mathbf{x}_0) \triangleq p_{\text{data}}$. Variance preserving (VP) diffusion [8] adopts $p(\mathbf{x}_{k+1}|\mathbf{x}_k) \triangleq \mathcal{N}(\mathbf{x}_{k+1}; \sqrt{1 - \beta_{k+1}}\mathbf{x}_k, \beta_{k+1}\mathbf{I})$ with monotonically increasing variance schedule $\beta_T \geq \beta_{T-1} \geq \dots \geq \beta_0 = 0$. When $T \rightarrow \infty$, $\mathbf{x}_T$ converges to an isotropic Gaussian distribution $\mathcal{N}(\mathbf{0}, \mathbf{I})$.

To generate a sample following the data distribution $p(\mathbf{x}_0)$, we can start from the standard Gaussian $p_{\theta}(\mathbf{x}_T) := \mathcal{N}(\mathbf{0}, \mathbf{I})$ and follow the reverse process

$$p_{\theta}(\mathbf{x}_{0:T}) := p_{\theta}(\mathbf{x}_T) \prod_{k=0}^{T-1} p_{\theta}(\mathbf{x}_k|\mathbf{x}_{k+1}), \quad (2)$$

with transitions $p_{\theta}(\mathbf{x}_k|\mathbf{x}_{k+1}) \sim \mathcal{N}(\mu_{\theta}(\mathbf{x}_k, k), \Sigma_{\theta}(\mathbf{x}_k, k))$ parameterized by θ . It will be shown later that the reverse sampling is closely related to the *score function* for each step, i.e., $\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k)$, for $k = 0, 1, \dots, T$.

In continuous-time, we can describe the diffusion process $(X_t)_{t \in [0, T_c]}$, $X_t \in \mathbb{R}^d$ with a stochastic differential equation (SDE) [21]

$$dX_t = -\frac{1}{2}\beta(t)X_t dt + \sqrt{\beta(t)}dW_t, \quad X_0 \sim p_{\text{data}}, \quad (3)$$

where $(W_t)_{t \in [0, T_c]}$ is a standard Brownian motion, and $\beta(t)$ is the noise schedule. Let $(p_t)_{t \in [0, T_c]}$ be the path distribution associated with (3). To perform data sampling following the distribution p_{data} , we can reverse the SDE (3) and

apply discretization. According to [1] and [21], the reverse SDE corresponding to (3) is

$$d\tilde{X}_t = \left[\frac{1}{2}\beta(T_c-t)\tilde{X}_t + \beta(T_c-t)\nabla \log p_{T_c-t}(\tilde{X}_t) \right] dt + \sqrt{\beta(T_c-t)}d\tilde{W}_t, \quad \tilde{X}_0 \sim p_{T_c}, \quad (4)$$

where $(\tilde{W}_t)_{t \in [0, T_c]}$ is another standard Brownian motion. The reverse SDE produces a time-reverse process $(\tilde{X}_t)_{t \in [0, T_c]}$ where $\tilde{X}_t$ has the same distribution with X_{T_c-t} . Note that the drift function in (4) depends on the score function $\nabla_{\mathbf{x}_t} \log p_t(\mathbf{x}_t)$, which can be approximated by a time-aware neural network trained with denoising score matching [22].

2.2 Score-based model for posterior sampling

In many application scenarios, we may observe a noisy version of the measurement of the data X_0 , given by

$$Y = \mathcal{A}(X_0) + N,$$

where $\mathcal{A}(\cdot) : \mathbb{R}^d \rightarrow \mathbb{R}^n$ is a measurement operator and $N \in \mathbb{R}^n$ is the measurement noise with $N \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I})$.

Score-based generative models have shown powerful capability in solving general inverse problems [6, 10, 19, 26, 28, 29]. Leveraging the diffusion model as the prior, sampling from the reverse posterior distribution

$$p(\mathbf{x}_{0:T}|\mathbf{y}) := p(\mathbf{x}_T|\mathbf{y}) \prod_{k=0}^{T-1} p(\mathbf{x}_k|\mathbf{x}_{k+1}, \mathbf{y}) \quad (5)$$

requires the knowledge of conditional scores $\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k|\mathbf{y})$, which can be expressed as $\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k|\mathbf{y}) = \nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k) + \nabla_{\mathbf{x}_k} \log p(\mathbf{y}|\mathbf{x}_k)$. The first term can be approximated by a score network $s_\theta(\mathbf{x}_k, k)$ [8, 21], and different methods have been proposed to estimate the second term $\nabla_{\mathbf{x}_k} \log p(\mathbf{y}|\mathbf{x}_k)$ [6, 19]. In each step the conditional sampling process in VP diffusion is [6, 19]

$$\begin{aligned} \mathbf{x}_{k-1} &\leftarrow \frac{1}{\sqrt{\alpha_k}}(\mathbf{x}_k + (1-\alpha_k)\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k|\mathbf{y})) + \tilde{\sigma}_k \mathbf{z}, \\ &\approx \frac{1}{\sqrt{\alpha_k}}(\mathbf{x}_k + (1-\alpha_k)(s_\theta(\mathbf{x}_k, k) + \nabla_{\mathbf{x}_k} \log p(\mathbf{y}|\mathbf{x}_k))) + \tilde{\sigma}_k \mathbf{z}, \end{aligned} \quad (6)$$

where $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ and $\tilde{\sigma}_k^2$ is the approximated variance of the reverse posterior distribution.

Meanwhile, [28] proposed to directly approximate the reverse conditional $p(\mathbf{x}_k|\mathbf{x}_{k+1}, \mathbf{y})$ and propagate the mean in each step. Specifically, when $T \rightarrow \infty$, the mean of $p(\mathbf{x}_k|\mathbf{x}_{k+1}, \mathbf{y})$ in VP-diffusion is given by [28]

$$\boldsymbol{\mu}_k(\mathbf{x}_{k+1}, \mathbf{y}) = \mathbf{U}_k \mathbf{x}_{k+1} + \mathbf{V}_k \mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[X_0], \quad (7)$$

where

$$\begin{aligned} \mathbf{U}_k &:= \sqrt{\alpha_{k+1}}((1-\bar{\alpha}_k)\mathbf{I} + \bar{\alpha}_k \text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[X_0]) \\ &\quad \cdot ((1-\bar{\alpha}_{k+1})\mathbf{I} + \bar{\alpha}_{k+1} \text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[X_0])^{-1}, \\ \mathbf{V}_k &:= \sqrt{\bar{\alpha}_k}((1-\alpha_{k+1})\mathbf{I})((1-\bar{\alpha}_{k+1})\mathbf{I} + \bar{\alpha}_{k+1} \text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[X_0])^{-1}. \end{aligned}$$

Here $\alpha_k = 1 - \beta_k$ and $\bar{\alpha}_k = \prod_{i=0}^k \alpha_i$ in VP-diffusion. The authors proved that by propagating the mean at each reverse step (i.e., $\boldsymbol{\mu}_T \rightarrow \boldsymbol{\mu}_{T-1}(\mathbf{x}_T = \boldsymbol{\mu}_T, \mathbf{y}) \rightarrow \cdots \rightarrow \boldsymbol{\mu}_0(\mathbf{x}_1 = \boldsymbol{\mu}_1, \mathbf{y})$), the end point $\boldsymbol{\mu}_0(\mathbf{x}_1 = \boldsymbol{\mu}_1, \mathbf{y})$ represents the MMSE estimator $\mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[X_0]$ when $\bar{\alpha}_T \rightarrow 0$. The proof of reverse mean propagation converging to the MMSE point [28] and its connection to conditional score are included in Appendix A.

2.3 Connection between reverse mean and conditional score

The mean derived in (7) can be viewed as an approximation of the mean in (6). Specifically, when $T \rightarrow \infty$, $p(\mathbf{x}_k|\mathbf{y})$ can be approximately expressed in the form of a Gaussian distribution. We can show that (see details in Appendix A)

$$\frac{1}{\sqrt{\alpha_k}}(\mathbf{x}_k + (1-\alpha_k)\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k|\mathbf{y})) = \mathbf{U}_k \mathbf{x}_{k+1} + \mathbf{V}_k \mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[X_0].$$

Theoretically, when $\tilde{\sigma}_k = 0$ in each step (6), the endpoint of the reverse process converges to the MMSE estimator, which achieves the best possible performance on distortion. When $\tilde{\sigma}_k$ is the true posterior variance, the reverse process samples from the posterior distribution $p(\mathbf{x}_{0:T}|\mathbf{y})$, which results in a reconstruction with perfect perception measured in conditional distribution. The above observations motivate us to **bridge the two extreme points with the score-based generative model** by tuning the scale of σ_k in the reverse process. It remains to be answered whether the bridging can provide us with an optimal and flexible estimator to traverse along the distortion-perception plane.

3 Traversing DP Tradeoff with Scaled Reverse Diffusion

In this section, we first propose a novel reverse sampling method based on a variance-scaled version of the joint inference distribution (5) and theoretically derive the mean and variance of the *marginal* distribution. Then, we prove that the resulting end distribution of the variance-scaled reverse sampling provides an optimal DP tradeoff with respect to MSE and Wasserstein-2 distance.

3.1 Reverse mean and variance

Consider the joint inference distribution with true mean but scaled variance, i.e.,

$$p_\lambda(\mathbf{x}_k|\mathbf{x}_{k+1}, \mathbf{y}) := \mathcal{N}(\boldsymbol{\mu}_k(\mathbf{x}_{k+1}, \mathbf{y}), \lambda \mathbf{C}_k), \quad (8)$$

where the expectation $\boldsymbol{\mu}_k(\mathbf{x}_{k+1}, \mathbf{y})$ is given in (7), and the true covariance $\mathbf{C}_k$ is given by [28]

$$\begin{aligned} \mathbf{C}_k &:= \frac{\beta_{k+1}}{1-\beta_{k+1}}((1-\bar{\alpha}_k)\mathbf{I} + \bar{\alpha}_k \text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[X_0]) \\ &\quad \cdot \left(\left(\frac{\beta_{k+1}}{1-\beta_{k+1}} + 1 - \bar{\alpha}_k \right) \mathbf{I} + \bar{\alpha}_k \text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[X_0] \right)^{-1}. \end{aligned} \quad (9)$$

We have the following theorem to characterize the marginal distribution given by the variance-scaled version of the joint inference distribution.

Theorem 1. For VP-diffusion and $0 \leq \lambda \leq 1$, consider the joint inference distribution given by

$$p_\lambda(\mathbf{x}_{0:T}|\mathbf{y}) = p_\lambda(\mathbf{x}_T|\mathbf{y}) \prod_{k=0}^{T-1} p_\lambda(\mathbf{x}_k|\mathbf{x}_{k+1}, \mathbf{y}) \quad (10)$$

where $p_\lambda(\mathbf{x}_T|\mathbf{y}) = \mathcal{N}(0, \mathbf{I})$ and $p_\lambda(\mathbf{x}_k|\mathbf{x}_{k+1}, \mathbf{y})$ is given by (8). Then, the corresponding margin has the distribution $p_\lambda(\mathbf{x}_k|\mathbf{y}) = \mathcal{N}(\boldsymbol{\mu}_k^\lambda, \boldsymbol{\Sigma}_k^\lambda)$, where

$$\begin{aligned} \boldsymbol{\mu}_k^\lambda &= (\mathbf{U}_k(\mathbf{U}_{k+1}(\cdots(\mathbf{U}_{T-1}\mathbf{V}_{T-1}+\mathbf{V}_{T-2})\cdots)+\mathbf{V}_{k+1})+\mathbf{V}_k) \\ &= \sqrt{\bar{\alpha}_k}(1-\alpha_{k+1}\cdots\alpha_T)((1-\bar{\alpha}_T)\mathbf{I}+\bar{\alpha}_T\text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[\mathbf{X}_0])^{-1}\mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[\mathbf{X}_0], \\ \boldsymbol{\Sigma}_k^\lambda &= \boldsymbol{\Sigma}_k(\lambda\mathbf{I}+(1-\lambda)\alpha_{k+1}\alpha_{k+2}\cdots\alpha_T\boldsymbol{\Sigma}_{T-1}^{-1}\boldsymbol{\Sigma}_k), \end{aligned}$$

where $\boldsymbol{\Sigma}_k$ is the covariance of $p(\mathbf{x}_k|\mathbf{y})$ in VP diffusion. In particular, when $\bar{\alpha}_T \rightarrow 0$, the variance of $p_\lambda(\mathbf{x}_0|\mathbf{y})$ is

$$\boldsymbol{\Sigma}_0^\lambda = \boldsymbol{\Sigma}_0(\lambda\mathbf{I}+(1-\lambda)\bar{\alpha}_T\boldsymbol{\Sigma}_{T-1}^{-1}\boldsymbol{\Sigma}_0) \rightarrow \lambda\text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[\mathbf{X}_0],$$

and the mean is

$$\begin{aligned} \boldsymbol{\mu}_0^\lambda &= \sqrt{\bar{\alpha}_0}((1-\bar{\alpha}_T)\mathbf{I})((1-\bar{\alpha}_T)\mathbf{I}+\bar{\alpha}_T\text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[\mathbf{X}_0])^{-1} \\ &\quad \cdot \mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[\mathbf{X}_0] \rightarrow \mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[\mathbf{X}_0]. \end{aligned}$$

Proof. See the details in Appendix B. $\square$

The above theorem shows that the proposed variance-scaled reverse diffusion process will lead to a scaled marginal distribution at each step, which finally results in the posterior mean and scaled posterior variance of the original data distribution at the end of the reverse process. This seemingly intuitive result actually provides the optimal solution to the conditional distortion-perception tradeoff for multivariate Gaussian distribution.

3.2 DP tradeoff for conditional Gaussian case

Consider the d -dimensional source $X \in \mathbb{R}^d$ with distribution $p(\mathbf{x})$. Let Y be the noisy version of the source data. Denote the conditional expectation and variance given an observation $\mathbf{y}$ as $\boldsymbol{\mu}_\mathbf{y} = \mathbb{E}_{p(\mathbf{x}|\mathbf{y})}[\mathbf{X}]$ and $\boldsymbol{\Sigma}_\mathbf{y} = \text{Cov}_{p(\mathbf{x}|\mathbf{y})}[\mathbf{X}]$, respectively. The goal of signal restoration is to find a good reconstruction $\hat{X}$ based on the observed Y .

From the optimization perspective, given a specific observation $Y = \tilde{\mathbf{y}}$, we can define the *conditional* distortion-perception function as

$$\begin{aligned} D(P) &= \min_{p_{\hat{X}|Y}(\hat{\mathbf{x}}|\tilde{\mathbf{y}})} \mathbb{E}_{p_{X,\hat{X}|Y}(\mathbf{x},\hat{\mathbf{x}}|\tilde{\mathbf{y}})}[\|\mathbf{X} - \hat{X}\|_2^2] \\ &\quad \text{s.t. } d(p_{X|Y}(\mathbf{x}|\tilde{\mathbf{y}}), p_{\hat{X}|Y}(\hat{\mathbf{x}}|\tilde{\mathbf{y}})) \leq P, \end{aligned}$$

where X, Y and $\hat{X}$ form a Markov chain.

Theorem 2. Consider the source X and an observation $\tilde{\mathbf{y}}$ that satisfy $p_{X|Y}(\mathbf{x}|\tilde{\mathbf{y}}) \sim \mathcal{N}(\boldsymbol{\mu}_{\tilde{\mathbf{y}}}, \boldsymbol{\Sigma}_{\tilde{\mathbf{y}}})$. The optimal distortion-perception tradeoff given $\tilde{\mathbf{y}}$ with MSE and Wasserstein-2 distance is

$$D(P) = \begin{cases} \text{Tr}(\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}}) + (\sqrt{\text{Tr}(\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}})} - P)^2, & \text{for } P \leq \sqrt{\text{Tr}(\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}})} \\ \text{Tr}(\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}}), & \text{for } P > \sqrt{\text{Tr}(\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}})}. \end{cases} \quad (11)$$

Meanwhile, the optimal tradeoff can be achieved by sampling from the joint inference distribution (10) in Theorem 1 and traversing different $0 \leq \lambda \leq 1$.

Proof. See details in Appendix C. $\square$

Theorem 2 explicitly characterizes the optimal DP tradeoff for conditional Gaussian distribution. When $P = 0$ (i.e., perfect perceptual quality is expected), the best achievable MSE is $2 \text{Tr}(\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}})$. As P increases, the optimal MSE gradually converges to the MMSE value $\text{Tr}(\boldsymbol{\Sigma}_{\tilde{\mathbf{y}}})$. The proof in Appendix C also shows that the variance-scaled reverse diffusion process can attain the entire optimal DP curve for conditional Gaussian distribution. Although the actual data distribution may not follow the conditional Gaussian distribution (e.g., mixture Gaussian, 2D distribution like S-curve, and image dataset), the forthcoming experiments will show that the proposed variance-scaled reverse sampling can effectively and flexibly traverse the DP tradeoff on general datasets using a single score network.

3.3 Variance-scaled reverse sampling process

In this subsection, we discuss how to perform variance-scaled reverse sampling by estimating the conditional score. With a *single pre-trained score network*, we can traverse different DP tradeoffs for general inverse problems.

Recall that the proposed variance-scaled reverse sampling process is given by $p_\lambda(\mathbf{x}_{0:T}|\mathbf{y}) = p_\lambda(\mathbf{x}_T|\mathbf{y}) \prod_{k=0}^{T-1} p_\lambda(\mathbf{x}_k|\mathbf{x}_{k+1}, \mathbf{y})$, where for VP diffusion, we have the posteriors $p_\lambda(\mathbf{x}_{k-1}|\mathbf{x}_k, \mathbf{y}) := \mathcal{N}(\boldsymbol{\mu}_{k-1}^\lambda(\mathbf{x}_k, \mathbf{y}), \lambda\mathbf{C}_{k-1})$. The sampling process in each step can be approximately expressed as

$$\mathbf{x}_{k-1} = \frac{1}{\sqrt{\alpha_k}}(\mathbf{x}_k + (1-\alpha_k)\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k|\mathbf{y})) + \lambda\mathbf{C}_k\mathbf{z}, \quad (12)$$

where $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$. To draw a sample $\mathbf{x}_{k-1}$ according to (12), we need to compute the conditional score $\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k|\mathbf{y})$. Specifically, the conditional scores can be decomposed as $\nabla_{\mathbf{x}_k} p(\mathbf{x}_k|\mathbf{y}) = \nabla_{\mathbf{x}_k} p(\mathbf{x}_k) + \nabla_{\mathbf{x}_k} p(\mathbf{y}|\mathbf{x}_k)$ by Bayes' rule, where the first term can be approximated by a score network $s_\theta(\mathbf{x}_k, k)$ [8, 21]. To deal with the second term, Denoising Posterior Sampling (DPS) [6] proposes to approximate $p(\mathbf{y}|\mathbf{x}_k) = \mathbb{E}_{p(\mathbf{x}_0|\mathbf{x}_k)}[p(\mathbf{y}|\mathbf{x}_0)]$ with $p(\mathbf{y}|\hat{\mathbf{x}}_0(\mathbf{x}_k))$, where

$$\hat{\mathbf{x}}_0(\mathbf{x}_k) = \mathbb{E}_{p(\mathbf{x}_0|\mathbf{x}_k)}[\mathbf{X}_0] = \frac{1}{\alpha_k}(\mathbf{x}_k + (1-\bar{\alpha}_k)\nabla_{\mathbf{x}_k} p(\mathbf{x}_k)) \quad (13)$$

by Tweedie’s formula. The approximation error here is upper-bounded by the Jensen gap [17]. Note that $p(\mathbf{y}|\hat{\mathbf{x}}_0(\mathbf{x}_k))$ is analytically tractable when the measurement distribution is given. Specifically, when the measurement noise is $\mathcal{N}(0, \sigma_n^2 \mathbf{I})$, the second term in the conditional score takes the form of $\nabla_{\mathbf{x}_k} \log p(\mathbf{y}|\mathbf{x}_k) = \frac{1}{\sigma_n^2} \nabla_{\mathbf{x}_k} \|\mathbf{y} - \mathcal{A}(\hat{\mathbf{x}}_0(\mathbf{x}_k))\|_2^2$. Thus, the conditional scores can be approximated as $\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k|\mathbf{y}) = s_\theta(\mathbf{x}_k, k) + \frac{1}{\sigma_n^2} \nabla_{\mathbf{x}_k} \|\mathbf{y} - \mathcal{A}(\hat{\mathbf{x}}_0(\mathbf{x}_k))\|_2^2$.

In the following experiments, we adopt the DPS framework to approximate the conditional score and sample from (12), which is sketched as Algorithm 1.

Algorithm 1 Variance-Scaled Reverse Sampling with DPS

Require: $T, \mathbf{y}, \lambda, \{\tilde{\sigma}_k\}_{k=0}^T, \{\zeta_{k,\lambda}\}_{k=0}^T$
 $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
for $k = T - 1$ to 1 **do**
 $\hat{\mathbf{s}} \leftarrow s_\theta(\mathbf{x}_k, k)$
 $\hat{\mathbf{x}}_0 \leftarrow \frac{1}{\sqrt{\alpha_k}}(\mathbf{x}_k + (1 - \bar{\alpha}_k)\hat{\mathbf{s}})$
 $\hat{\mathbf{c}}(\hat{\mathbf{x}}_0) = -\frac{\sqrt{\alpha_k}}{1 - \alpha_k} \nabla_{\mathbf{x}_k} \|\mathbf{y} - \mathcal{A}(\hat{\mathbf{x}}_0(\mathbf{x}_k))\|_2^2$
 $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
 $\mathbf{x}_{k-1} \leftarrow \frac{1}{\sqrt{\alpha_k}}(\mathbf{x}_k + (1 - \alpha_k)(\hat{\mathbf{s}} + \zeta_{k,\lambda}\hat{\mathbf{c}}(\hat{\mathbf{x}}_0))) + \lambda\tilde{\sigma}_k\mathbf{z}$
end for
return $\mathbf{x}_0$

In Algorithm 1, $\tilde{\sigma}_k$ is set to β_k [8, 28], and $\zeta_{k,\lambda}$ is a hyperparameter to control the weight of the conditional score, and may differ for different λ . The heuristic choices of $\zeta_{k,\lambda}$ and the rationale for fine-tuning have been included in Appendix E.

Remark 1. In this paper, we adopt the conditional score approximation proposed in [6] for illustration. To implement the variance-scaled reverse sampling process (12), one can employ any approach to directly estimate the conditional score [19] or estimate the posterior mean as a whole [28] for general inverse problems. For example, Pseudoinverse-guided Diffusion Models (PIGDM) [19] approximated the unconditional posteriors $p(\mathbf{x}_0|\mathbf{x}_k)$ with Gaussian, i.e., $p(\mathbf{x}_0|\mathbf{x}_k) \approx \mathcal{N}(\hat{\mathbf{x}}_0, r_k^2 \mathbf{I})$, where $\hat{\mathbf{x}}_0$ is given by (13), and r_k^2 is the hyperparameter. Thus, the score term $\nabla_{\mathbf{x}_k} p(\mathbf{y}|\mathbf{x}_k)$ can be analytically expressed by the pseudoinverse of the measurement model. In [28], the posterior mean is estimated as a whole. Specifically, we have $\mu_{k-1}(\mathbf{x}_k, \mathbf{y}) = \mathbf{U}_{k-1}\mathbf{x}_k + \mathbf{V}_{k-1}\mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[X_0]$. Since the expression involves $\mathbb{E}_{p(\mathbf{x}_0|\mathbf{y})}[X_0]$ and $\text{Cov}_{p(\mathbf{x}_0|\mathbf{y})}[X_0]$, it can not be computed directly. Instead, the authors approximate the joint conditional posterior with variational Gaussian distributions and apply natural gradient descent to perform the sampling. Note that the gradient calculation also involves the approximation of the conditional score $\nabla_{\mathbf{x}_k} \log p(\mathbf{y}|\mathbf{x}_k)$ as in DPS [6].

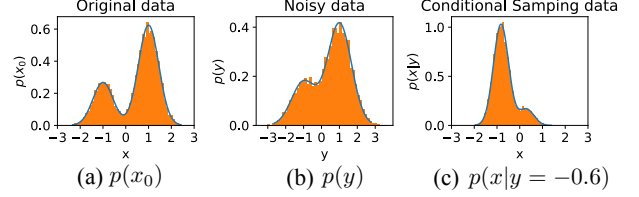


Figure 1. An example of mixture Gaussian, the noisy observation, and the conditional distribution given an observation $y = -0.6$

3.4 An illustrative example

Consider the mixture Gaussian distribution $X_0 \sim p(x_0)$ with two components, where $p(x_0) = w_1\mathcal{N}(\mu_1, \sigma_1^2) + w_2\mathcal{N}(\mu_2, \sigma_2^2)$. The noisy observation is obtained by $Y = aX_0 + \sigma_0\epsilon$, where $\epsilon \sim \mathcal{N}(0, 1)$, i.e., $p(y|x_0) = \mathcal{N}(ax_0, \sigma_0^2)$. The MMSE estimator and posterior distribution can be theoretically derived (see Appendix D for details). Fig. 1 shows an example of mixture Gaussian distribution and the conditional distribution given an observation $y = -0.6$.

Given an observation y and a chosen λ , we iteratively perform the variance scaled reverse process (12) for $k = T, \dots, 1$, where the variance is approximated by $\mathbf{C}_k = \beta_k$ as the conventional VP-diffusion. Starting from a random sample $x_T \sim \mathcal{N}(0, 1)$, Fig. 2 shows the trajectories $x_T \rightarrow \dots x_k \rightarrow \dots x_0$ of multiple reconstructions for each λ . We can observe that for $\lambda = 0$, given an initial x_T , the trajectories are deterministic and converge to the MMSE point. This phenomenon collaborates with results in [28]. When λ increases, the generated trajectories follow the form of the posterior distribution and show more stochasticity. When $\lambda = 1$, the reconstruction distribution matches the ground truth posterior.

The resulting DP tradeoffs are shown in Fig. 3. It can be observed that as λ increases, the divergence between the true posterior distribution and reconstruction distribution continuously decreases, with an increase of MSE value. The overall curve is convex, and both Wasserstein-2 and KL-divergence come to zero. Meanwhile, the MSE achieved at $\lambda = 1$ is approximately two times the MSE at $\lambda = 0$ for both Wasserstein-2 and KL-divergence cases, coinciding with the result in Theorem 2.

4 Experimental Results

In this section, we conduct a series of experiments on two-dimensional datasets and the FFHQ dataset [9]. It is demonstrated that the variance-scaled reverse sampling shows great flexibility and effectiveness in traversing DP tradeoff compared to the GAN-based approach and other denoising methods.

4.1 Two-dimensional points example

First, we evaluate the validity and effectiveness of our approach on two-dimensional datasets, including pinwheel, S-

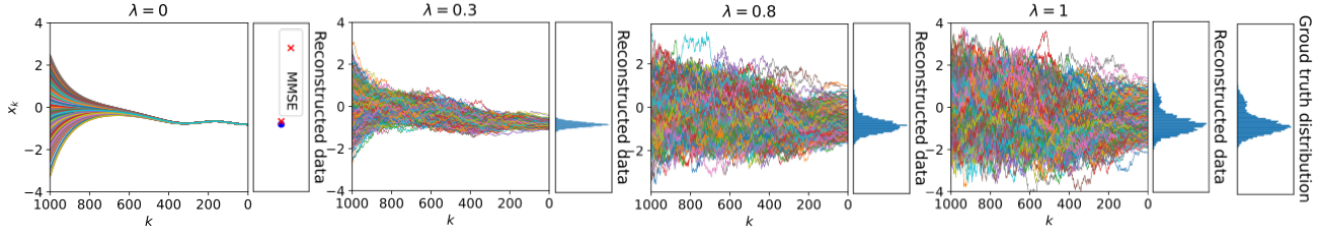


Figure 2. Trajectories $x_T \rightarrow \dots x_k \rightarrow \dots \rightarrow x_0$ of different reconstructions for $\lambda = 0, 0.3, 0.8$ and 1 . The initial x_T is $\mathcal{N}(0, 1)$.

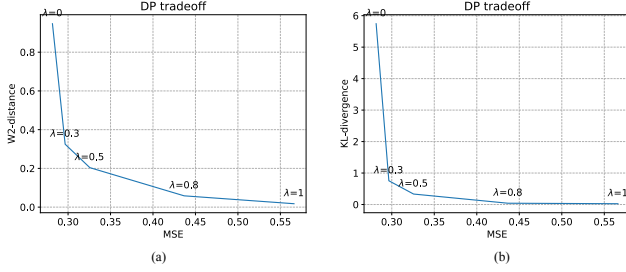


Figure 3. Distortion-perception tradeoff traversed by variance-scaled reverse sampling given different λ 's. (a) tradeoff between Wasserstein-2 distance and MSE; (b) tradeoff between KL-divergence and MSE.

curve, and moon-shape data distributions. Taking the pinwheel dataset as an example, the first row of Fig. 4 shows the distribution of pinwheel data points X_0 and the noisy observation Y , where $Y = aX + N$ for $N \sim \mathcal{N}(0, \sigma_n^2 \mathbf{I})$.

For each dataset, we train a conventional score network to approximate $p(x_0)$ on the original distribution. Then, we perform the variance-scaled reverse diffusion process (12) on the noisy observation. As a comparison, we adopt a GAN-based DP-traversing approach PSCGAN [15]. This scheme introduces a penalty of the posterior expectation to the training of the conditional GAN. The authors proposed two strategies to navigate the DP tradeoff during the inference time: (1) PSCGAN- N : This method involves sampling N instances from the generator and then averaging them. As N increases, the averaged image is closer to the conditional expectation, leading to a smaller distortion but larger perception loss. (2) PSCGAN- z : It varies the standard deviation σ_z of the noise injected into the generator to control the stochasticity. For $0 \leq \sigma_z \leq 1$, a larger σ_z results in a better perceptual quality and higher MSE. Note that the training of PSCGAN relies on noisy data points Y . Thus, the model needs to be retrained for different measurement scenarios and different levels of σ_n^2 .

Fig. 4 shows the reconstruction of our variance-scaled reverse diffusion process for different λ 's, as well as the reconstruction of PSCGAN when setting $N = 16, \sigma_z = 1$. It can be observed that when $\lambda = 0$, our sampling process leads to a more concentrated reconstruction, and gradually approaches the true distribution as λ increases.

Fig. 5 compares the numerical DP tradeoffs achieved

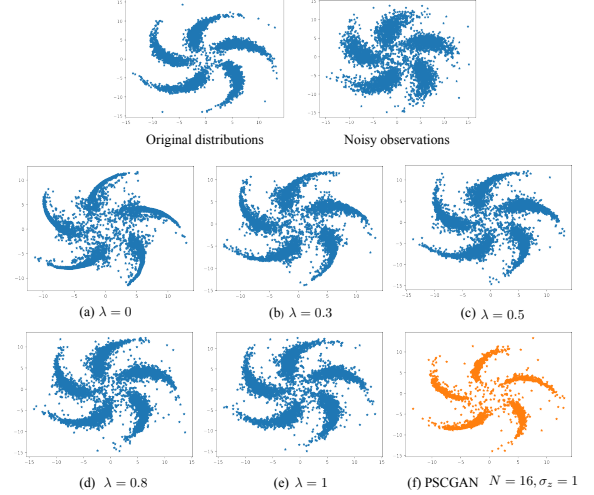


Figure 4. Experiments on a two-dimensional dataset. The first row illustrates the original distribution (left) of pinwheel and the noisy observation Y , where $Y = aX + N$ for $N \sim \mathcal{N}(0, \sigma_n^2 \mathbf{I})$ (right). The second and third row shows the reconstructions: (a)-(e) variance-scaled reverse diffusion process with different λ 's; (f) PSCGAN with $N = 16, \sigma_z = 1$.

by our method versus PSCGAN. Our score-based method achieves a much larger range of tradeoffs compared to the GAN-based approach, demonstrating superior performance. Moreover, unlike PSCGAN, our method requires only a single score network to handle varying measurements and noise levels, offering greater flexibility. Additional experimental results on other distributions are provided in Appendix E.

4.2 FFHQ dataset

In this subsection, we evaluate our method on the FFHQ 256×256 dataset [9]. For different measurement scenarios, including Gaussian blur and super-resolution, we use the same pre-trained score-based model from [6], which was trained from scratch using 49k training data for 1M steps. We consider measurement scenarios with noise levels more severe than those in [6, 28, 29]. Specifically, for Gaussian blur, the images are blurred by a Gaussian blur kernel with size 61×61 and a standard deviation of 3.0. For super-resolution, the images are downsampled by a factor of 8.

We compare our method against two benchmarks: (1)

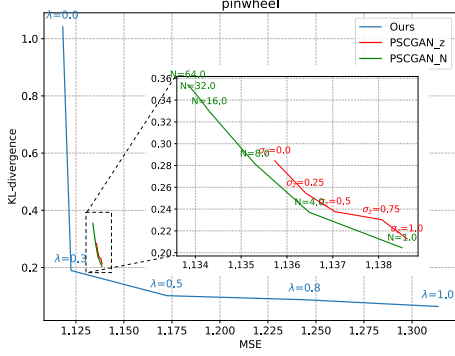


Figure 5. DP tradeoff on pinwheel dataset traversed by our variance-scaled reverse diffusion process and PSCGAN.

We compare the DP tradeoffs provided by our score-based method and PSCGAN [15] on the Gaussian deblur task. Note that PSCGAN is not designed for other measurement scenarios and requires retraining for different noise levels. (2) We evaluate the performance of DiffPIR [29] to see where it falls within the tradeoff spectrum for different inverse problems. DiffPIR is a plug-and-play image restoration method based on diffusion denoising implicit model (DDIM) [18]. It has shown competitive performance on many denoising problems with fewer diffusion steps.

4.2.1 Gaussian deblur task

Fig. 6 shows the DP tradeoff on the Gaussian deblur task. We test our variance-scaled reverse diffusion process, PSCGAN [15], and DiffPIR [29] on two different additive noise levels $\sigma_n = 0.3$ and $\sigma_n = 0.5$. Note that our sampling method and DiffPIR rely on a pre-trained score network on the FFHQ dataset. For PSCGAN, we use the model trained on images with additive Gaussian noise of $\sigma_n = 0.3$.

For noise level $\sigma_n = 0.3$, PSCGAN shows suboptimal and more limited tradeoffs compared to our method. To obtain a reconstruction with similar MSE achieved in our $\lambda = 0.9$ case, PSCGAN requires averaging 64 sampled images. Its distortion degrades rapidly when fewer images are averaged. Our variance-scaled reverse diffusion sampling achieves both better fidelity and a broader range of DP tradeoffs. Moreover, when the models are tested on the higher noise level of $\sigma_n = 0.5$, the overall tradeoff shifts to the left. Our sampling method maintains effective tradeoff traversal. PSCGAN’s performance (trained at $\sigma_n = 0.3$) deteriorates significantly, showing much larger MSE and excessively high FID values.

Significant visual improvements can be observed in reconstructed samples in Fig. 7. When $\lambda = 0$, the restored images maintain high fidelity and faithfully reconstruct the original images, though appear relatively blurry. As λ increases, the images become progressively sharper, revealing more fine details such as hair strands, eye corners, and back-

ground elements. When $\lambda = 1$, while the overall reconstruction appears very natural, subtle details deviate from the original images. In comparison, both PSCGAN and DiffPIR have relatively good perceptual quality but compromise fidelity on crucial details. Notable deviations appear in features like eyes and mouth shapes, ultimately altering the facial expression and overall style of faces. We also report more metrics (PSNR and LPIPS) of Gaussian deblur task with additive noise level $\sigma_n = 0.3$ in Appendix E.

4.2.2 Super-resolution task

Similar phenomena are observed in the super-resolution (SR) task. By adjusting different λ ’s, we can traverse a wide range of points on the DP plane, allowing flexible selection of the most suitable point for specific application scenarios. Fig. 8 shows the numerical results and reconstruction samples given by our sampling process and DiffPIR. Note that DiffPIR performs close to our $\lambda = 1$ case, and achieves a better FID score but larger distortion than our method. While DiffPIR demonstrates strong generative capabilities in both Gaussian deblur and super-resolution tasks with fewer diffusion steps [29], we examine more challenging degradation scenarios ($\sigma_n = 0.3$ and 0.5 for deblurring, and $8\times$ downscaling for SR). Under these severe conditions, DiffPIR’s reduced diffusion steps lead to fidelity issues, with generated details less faithful compared to our $\lambda = 1$ case. Despite these differences, theoretical connections exist between DPS and DiffPIR [29]. We will consider the exploration of DDIM-style sampling for faster tradeoff traversal as future work.

In summary, using a single pre-trained score network, the proposed sampling method can effectively traverse a more complete DP tradeoff and achieve better MSE than the benchmarks. Experimental details and more results are included in Appendix E.

5 Conclusions

In this paper, we proposed a variance-scaled reverse diffusion process to traverse the DP tradeoff for general inverse problems using a single score-based model. By tuning a parameter that controls the scale of the reverse variance, we can navigate from the MMSE point to the whole posterior distribution, generating a complete DP curve. We proved that the proposed reverse sampling process serves as an optimal solution to the conditional DP tradeoff for multivariate Gaussian distribution. Meanwhile, we conducted experiments on the mixture Gaussian example, two-dimensional datasets, and the FFHQ dataset. Our results show that using a single pre-trained score network, we have achieved a more complete empirical DP tradeoff than the GAN-based method and other inverse problem solvers, demonstrating the effectiveness and flexibility of the proposed framework.

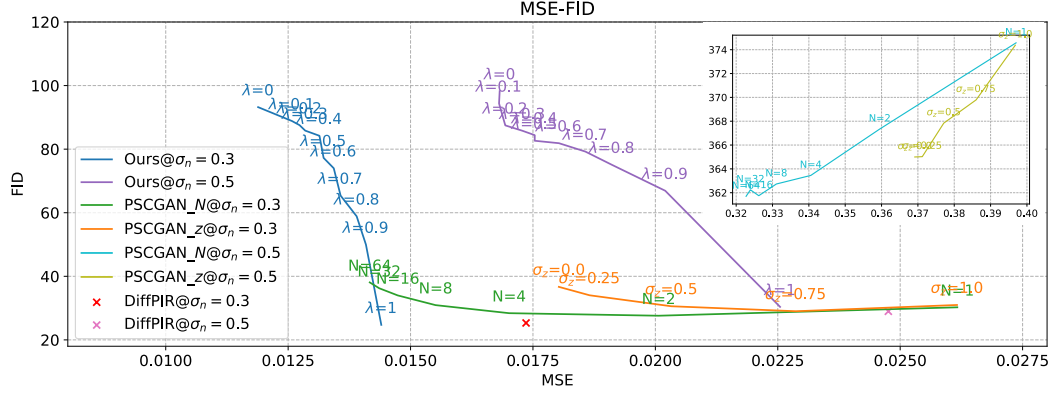


Figure 6. DP tradeoff on FFHQ dataset. Note that the PSCGAN model in this figure is trained on images contaminated with additive Gaussian noise of $\sigma_n = 0.3$. It can be observed that when the noise level in inference time does match the trained model (e.g., when the noise level becomes $\sigma_n = 0.5$ during the test time), PSCGAN fails to provide valid reconstructions. On the other hand, our variance-scaled reverse diffusion process and DiffPIR method rely on a pre-trained score network on the FFHQ dataset, which is not dedicated to any noisy measurement and is robust for different noise levels.

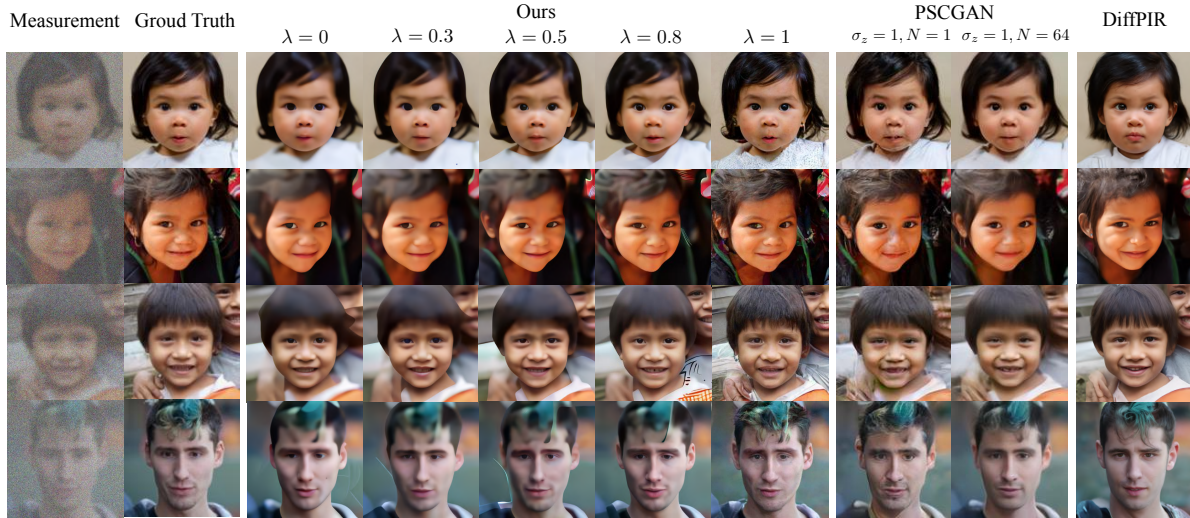


Figure 7. Samples from our sampling process, PSCGAN, and DiffPIR for Gaussian deblur task with noise level $\sigma_n = 0.3$.

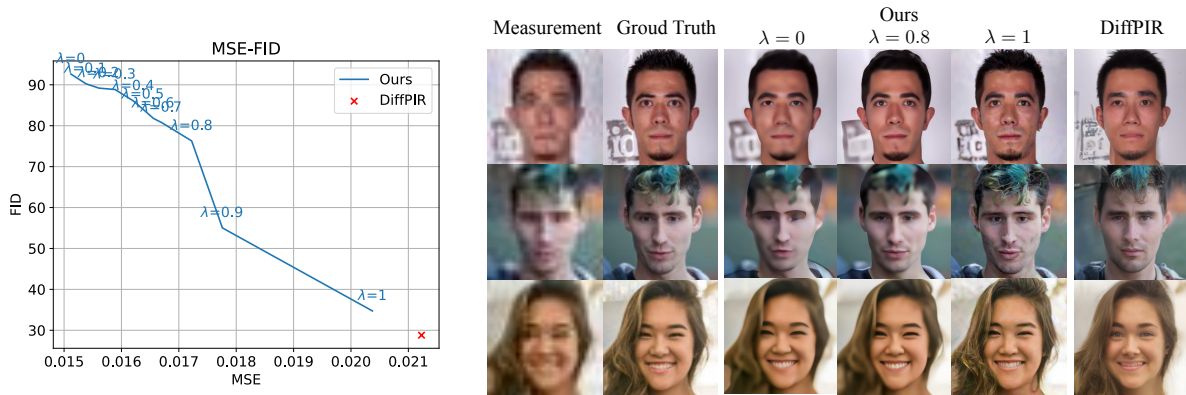


Figure 8. Samples from our sampling process, PSCGAN, and DiffPIR for super-resolution task with downsampling scale $\times 8$.

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