

RateCount: Learning-Free Device Counting by Wi-Fi Probe Listening

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Abstract—A Wi-Fi-enabled device, or simply Wi-Fi device, sporadically broadcasts probe request frames (PRFs) to discover nearby access points (APs), whether connected to an AP or not. To protect user privacy, unconnected devices often randomize their MAC addresses in the PRFs, known as MAC address randomization. While prior works have achieved accurate device counting under MAC address randomization, they typically rely on machine learning, resulting in inefficient deployment due to the time-consuming processes of data cleaning, model training, and hyperparameter tuning. To enhance deployment efficiency, we propose RateCount, an accurate, lightweight, and learning-free counting approach based on the rate at which APs receive PRFs within a window. RateCount employs a provably unbiased closed-form expression to estimate the device count time-averaged over the window and an error model to compute the lower bound of the estimation variance. We also demonstrate how to extend RateCount to people counting by incorporating a device-to-person calibration scheme. Through extensive real-world experiments conducted at multiple sites spanning a wide range of counts, we show that RateCount, without any deployment costs for machine learning, achieves comparable counting accuracy with the state-of-the-art learning-based device counting and improves previous people counting schemes by a large margin.

Index Terms—Wi-Fi probing, Device counting, MAC address randomization, Unbiased estimator, People counting

I. INTRODUCTION

A Wi-Fi-enabled device, or simply *Wi-Fi device* in this paper, actively manages its wireless connection by sporadically broadcasting probe request frames (PRFs) to the access points (APs) in its proximity. The frames are broadcast no matter whether the device has been connected to an AP, with an interval generally ranging from seconds to minutes. Although each Wi-Fi device has a unique physical MAC address, to protect user privacy, unconnected devices often fabricate random (or virtual) MAC addresses in their PRFs, known as *MAC address randomization*. Consequently, an AP may receive multiple distinct virtual MAC addresses from an unconnected device while the device remains in the coverage area.

As reported in much of the literature, the number of Wi-Fi devices in a counting area (i.e., the joint coverage of one or more APs) may be estimated through listening to their PRFs broadcast within a window of a few to tens of minutes [1]–[3]. Such *device counting* is fundamental to many Internet of Things (IoT) applications, such as crowd estimation, Wi-Fi popularity assessment, people flow management, and public transportation scheduling [4]–[7]. To prevent overcounting devices caused by MAC address randomization, previous

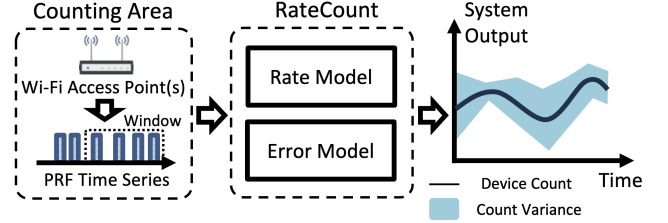


Fig. 1. Overview of the counting process of RateCount. Given a sliding window and a time series of PRFs as listened by the APs, RateCount uses a rate model to estimate the device count time-averaged over the window and an error model to compute the estimation variance.

approaches often employ machine learning to associate the PRFs with their broadcast devices [8]–[11]. However, these approaches require expert knowledge and professional efforts involving training data curation and cleaning, model training, and hyperparameter tuning [12], [13]. Their deployment hence could be inconvenient, costly, and time-consuming, which is unsuitable for scenarios of fast deployment or with restricted computing power.

To achieve efficient system deployment, we propose RateCount, an accurate, lightweight, and learning-free Wi-Fi device counting approach under MAC address randomization. Observing that more Wi-Fi devices lead to a higher probing rate, RateCount counts devices by a simple and effective *closed-form expression* that measures the rate at which APs receive PRFs within a window. Fig. 1 overviews the counting process of RateCount. Given a sliding window and a time series of PRFs as listened by the APs, RateCount employs a rate model and an error model to estimate device count and its variance, respectively. RateCount makes the following contributions:

- *A closed-form rate model to count Wi-Fi devices and its unbiased estimator proof:* We propose a rate model, a simple and closed-form expression, which estimates the device count by a ratio of the probing rate of all devices within a window to the mean probing rate of a single device. We further prove that our rate model is an unbiased estimator of the device count time-averaged over the window.
- *A closed-form error model to estimate the count variance:* We propose a simple closed-form error model for the lower bound of the count variance, in terms of the window size, the number of PRFs in the window, and the interval variance of device probing.
- *Extension to people counting via a multimodal calibration scheme:* As a showcase of its major application, we

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TABLE I
SUMMARY OF NOTATIONS (UNLESS SPECIFIED, ALL CONCEPTS ARE
RELATIVE TO A CERTAIN TIME WINDOW IN A CERTAIN COUNTING AREA).

Notation	Definition
U	Set of devices $\{u\}$
V	Set of counting areas $\{v\}$
w	Window size
M	Total number of people appearing in the window
$M(t)$	Number of people at time instant t
$\bar{M}$	Average number of people ($\bar{M} = (1/w) \int_0^w M(t)dt$)
N	Total number of devices appearing in the window
$N(t)$	Number of devices at time instant t
$\bar{N}$	Average number of devices ($\bar{N} = (1/w) \int_0^w N(t)dt$)
$\hat{N}$	Average number of devices estimated by RateCount
p_m	Dwell time of m th person
d_n	Dwell time of n th device
D	Total dwell time of all devices ($D = \sum_{n=1}^N d_n$)
b_n	Number of probing bursts from n th device
B	Total number of probing bursts ($B = \sum_{n=1}^N b_n$)
$\bar{\tau}$	Mean of probing intervals ($\bar{\tau} = \int_0^\infty \tau Pr(\tau v) d\tau$)
σ_τ	Standard deviation of probing intervals
$\bar{r}$	Mean probing rate of all devices ($\bar{r} = 1/\bar{\tau}$)
R	Aggregated probing rate ($R = B/w$)
α	Device-to-person ratio ($\alpha = N/M$)

demonstrate how to extend RateCount to people counting by means of a device-to-person ratio estimated from a multimodal calibration scheme.

We have implemented RateCount and conducted extensive experiments to count devices and people in diverse environments: a lab room, a building entrance, a campus atrium, and a bus station. Compared with previous approaches, our learning-free RateCount demonstrates comparable accuracy with the state-of-the-art learning-based device counting and significantly reduces the counting error of previous people counting by 66%. Furthermore, we show that our error model provides sound estimation of the count variance.

The remainder of the paper is organized as follows. We first introduce the preliminaries in Section II. Then, we discuss the design of RateCount in Section III, followed by its extension to people counting in Section IV. We present illustrative experimental results in Section V. We finally review related work in Section VI, and conclude in Section VII. We prove that RateCount gives an unbiased estimator in Appendix A.

II. PRELIMINARIES

In this section, we introduce the basics of device counting using Wi-Fi probing. We review Wi-Fi probing in Section II-A and window-averaged device counting in Section II-B. The major symbols are listed in Table I. Unless otherwise specified, all discussions are relative to a certain window in a certain counting area.

A. Wi-Fi Probing

Probing Burst: Wi-Fi devices sporadically broadcast probe request frames (PRFs) to probe the nearby access points (APs), known as Wi-Fi probing. To enhance network discovery, Wi-Fi devices generally broadcast PRFs in *bursts* [14]. That is, a device repeatedly broadcasts PRFs with the same message

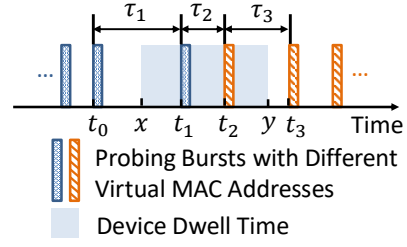


Fig. 2. Probing pattern of a single device. The start time of a probing burst is defined as the probing instant, shown as t , and the interval between two consecutive probing instants is defined as the probing interval, denoted as τ . The device dwells in a counting area from time x to y .

within a short period (typically less than 4 seconds), and the interval between two such bursts is much longer, ranging from 30 seconds to a few minutes. As shown in Fig. 2, each bar represents the period of a *probing burst* (or PRF burst). We define the start time of a probing burst as the *probing instant*, shown as t_i ($i = 0, 1, \dots$) and the interval between two consecutive probing instants as the *probing interval*, denoted as τ_j ($j = 0, 1, \dots$).

MAC Address Randomization: Each PRF, serving at the data-link layer, carries the MAC address of the Wi-Fi device. Although each Wi-Fi device has a unique physical MAC address, to protect user privacy, most devices irregularly fabricate virtual MAC addresses in their PRFs, known as MAC address randomization.¹ As illustrated in Fig. 2, the probing bursts at t_1 and t_2 carry different MAC addresses (indicated by color and hatch) due to MAC address randomization. Consequently, an AP can recognize two bursts from the same device if they share the same MAC address, but two bursts of the same device may not necessarily carry the same MAC address.²

Distribution of Probing Interval: Probing interval refers to the duration between two consecutive probing bursts from the same device. Typically, the probing interval of a device depends on the internal setting of the device (e.g., hardware specifications), its external environment (e.g., signal reception quality), and randomness [14]. In this regard, we consider the probing interval as an *independent and identically distributed* (IID) random variable sampled from a statistical distribution. Formally, for a single device $u \in U$ in a counting area $v \in V$, the probability that a probing interval is τ can be expressed as

$$Pr(\tau | u, v), \quad (1)$$

where we refer to U as the set of devices $\{u\}$ and V as the set of counting areas $\{v\}$, respectively.

In terms of device counting, we are more interested in the distribution of probing interval of a counting area, which is the marginal distribution based on (1). Formally, for a counting area $v \in V$, the probability that a probing interval of τ appears in the area is expressed as

$$Pr(\tau | v) = \sum_{u \in U} Pr(\tau | u, v) Pr(u | v). \quad (2)$$

¹MAC address randomization is usually considered as unpredictable [11].

²The probability of devices sharing virtual MAC addresses is minimal.

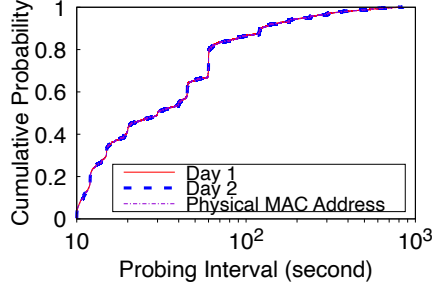


Fig. 3. Distributions of probing interval under different conditions in a counting area.

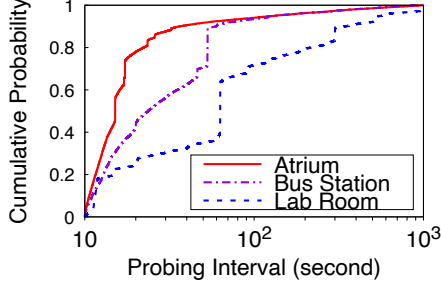


Fig. 4. Distributions of probing interval in different counting areas.

In our observation, the probing interval (τ) in (2) can be regarded as an IID variable with the number of sampling being sufficiently large. To validate this, we conduct an independence test shown in Fig. 3. In the test, we collected the probing intervals of a counting area on different days. Each day has approximately 266,000 PRFs, and we calculated the probing intervals from the bursts that share the same MAC address. We made a hypothesis that τ is IID given a counting area $v \in V$. Through *Ljung-Box* test to validate the independence and *Kolmogorov-Smirnov* test to validate the identical distribution, both tests *failed to reject* the hypothesis with $p\text{-values} > 0.9$. In addition, probing interval shows similar independence with MAC address randomization.³ Finally, we show in Fig. 4 how the distribution varies by counting areas, which demonstrates the nontrivial environmental impacts on the probing intervals.

B. Window-averaged Device Counting

Given the widespread use of Wi-Fi devices, the number of devices in an area could be used to estimate the population of the area, which is crucial for many IoT applications. For example, knowing the variation of device counts near bus stations is beneficial for scheduling public transportation. This knowledge is also useful for reflecting the queuing situation in various public venues such as canteens, banks, hospitals, toilets, which facilitates crowd management. To enable these applications, we study device counting, specifically, the counting of the number of devices present at a specific time instant in an area.

Device counting based on Wi-Fi probing has two challenges. First, since a probing burst is transient, it is difficult to know

³We check the 7th bit of the information element in PRF [15].

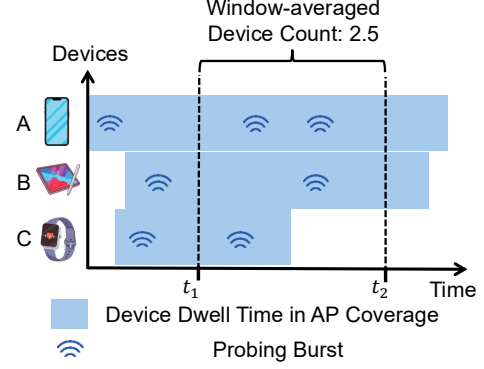


Fig. 5. The window-averaged device count is 2.5, with two devices (A and B) fully appearing in the window and one device (C) appearing for half of the window.

the device count at each time instant. Second, due to MAC address randomization, APs could receive more distinct MAC addresses than the actual number of devices, which further complicates the counting problem. To overcome the issues, previous approaches employ machine learning to associate the probing bursts with their broadcast devices and assume the probing intervals as the *dwell time* of the device in the counting area. To illustrate this, we present a counting example in Fig. 5, showing the dwell time of three devices and their probing bursts that have been associated with their broadcast devices. Based on the association results, the three devices (A , B and C) are correctly counted at time instant t_1 , because the probing intervals of the three devices overlap at t_1 . Although effective, as mentioned earlier, the system deployment of this approach could be costly and inefficient due to the PRF association based on machine learning. Also, it may under-count the devices, as the probing interval is expected to be shorter than the actual device dwell time (see the counting example at t_2).

In this paper, we propose a lightweight counting approach, which approximates the device count using the number of devices time-averaged over a window, namely, *window-averaged device count*. As illustrated in Fig. 5, the window-averaged device count from t_1 to t_2 is 2.5 because two devices (A and B) appear fully in the window and one device (C) appears for half of the window. With a proper window size (say, a few minutes), the window-averaged device count can effectively reflect the instantaneous device count in various real-world applications. Formally, letting $N(t)$ and w respectively denote the instantaneous device count at time t and the window size, the window-averaged device count is computed as

$$\bar{N} = \frac{1}{w} \int_0^w N(t) dt, \quad (3)$$

where the window extends from time 0 to w .

In the following, we discuss how to unbiasedly estimate the window-averaged device count ($\bar{N}$) under MAC address randomization without using machine learning and unveil how window size (w) affects the estimation variance.

III. RATECOUNT: LEARNING-FREE DEVICE COUNTING

This section presents the design of RateCount. First, we present the rate model for device counting in Section III-A, explain its approximation of ground truth in Section III-B, and give a proof of unbiasedness in Appendix A. Then, we discuss the error model to compute the lower bound of the count variance in Section III-C.

A. Rate Model

As mentioned earlier, a counting area is the joint coverage of one or more APs. Given a window of size w during which B probing bursts are received by the APs, we design a rate model to estimate the device count within the window. Our observation is that APs typically receive the bursts at a *higher* rate when *more* devices present in the area.

First, we define the rate at which APs receive probing bursts in the window as the *aggregated probing rate*, which is calculated as

$$R = \frac{B}{w}, \quad (4)$$

and the total number of probing bursts (B) is the sum of the number of probing bursts from all the devices ($\{b_n\}$), i.e.,

$$B = \sum_{n=1}^N b_n. \quad (5)$$

Intuitively, the aggregated probing rate quantifies how frequently Wi-Fi probing occurs in the window.

Second, we define the average rate that a single device probes in a counting area as the *mean probing rate*, which is calculated as

$$\bar{r} = \frac{1}{\bar{\tau}}, \quad (6)$$

where the *mean probing interval*, denoted as $\bar{\tau}$, is derived from the distribution of the probing interval (τ) given this counting area ($v \in V$) from (2), expressed as

$$\bar{\tau} = \int_0^\infty \tau \Pr(\tau | v) d\tau. \quad (7)$$

Intuitively, the mean probing rate quantifies how frequently each device probes in a counting area on average.

Given the above, the rate model counts devices by the *ratio* of the aggregated probing rate to the mean probing rate, which is given as

$$\hat{N} = \frac{R}{\bar{r}} = \frac{B}{w\bar{r}} = \frac{B\bar{\tau}}{w}. \quad (8)$$

We argue that $\hat{N}$ is an unbiased estimator of the ground truth $\bar{N}$ when the device count ($\bar{N}$) and window size (w) are sufficiently large.

Next, we explain that $\hat{N}$ approximates the ground truth $\bar{N}$ in Section III-B. The rigorous proof of unbiasedness based on the *Elementary Renewal Theorem* [16] is provided in Appendix A.

B. Approximation of Ground Truth

We explain that the rate model approximates the window-averaged device count ($\bar{N}$), i.e.,

$$\hat{N} = \frac{R}{\bar{r}} \approx \bar{N} \quad (9)$$

when $\bar{N}$ and w are sufficiently large.

We start with an intuitive explanation by definition. According to (4) and (6), the aggregated probing rate (R) describes how frequently all devices probe, and the mean probing rate ($\bar{r}$) describes how frequently a single device probes on average. If there is only one device, R is expected to equal $\bar{r}$. In this case, $\mathbb{E}(R/\bar{r}) = 1$ gives (9). Similarly, if we have n such devices, R is expected to be n times as frequent as it is in the single-device scenario, i.e., $\mathbb{E}(R) = n\bar{r}$. In this case, $\mathbb{E}(R/\bar{r}) = n$ still gives (9).

Next, we explicitly explain the approximation by introducing the dwell time of devices. Let N represent the total number of devices that appear in a certain counting area over a window, and d_n represent the dwell time of the n th device during which it probes b_n bursts. The total dwell time of all devices are

$$D = \sum_{n=1}^N d_n. \quad (10)$$

Recall that $\bar{\tau}$ is the mean probing interval, i.e., the expectation of the probing intervals in a counting area. In other words, one device is expected to probe once if it dwells in the area for a duration of $\bar{\tau}$.⁴ Generally, we expect one device to dwell *longer* when it broadcasts *more* probing bursts, and the expectation of dwell time can be estimated by $\mathbb{E}(d_n) = b_n\bar{\tau}$. Thus, we obtain the following approximation

$$d_n \approx b_n\bar{\tau}. \quad (11)$$

We elaborate on this using the single-device example shown in Fig. 2. In the figure, the APs received $b_n = 2$ bursts separately at t_1 and t_2 , and the device enters the area at time x and leaves at y . Since the device enters and leaves the area largely independent of the probing, x could be any value between t_0 and t_1 , and y could be any value between t_2 and t_3 .⁵ Therefore, the expected dwell time of this device is calculated as

$$\begin{aligned} \mathbb{E}(d_n) &= \mathbb{E}(y - x) \\ &= \frac{1}{t_3 - t_2} \int_{t_3}^{t_2} \left(\frac{1}{t_1 - t_0} \int_{t_1}^{t_0} (y - x) dx \right) dy \\ &= \frac{t_3 + t_2}{2} - \frac{t_1 + t_0}{2} = \frac{\tau_1 + 2\tau_2 + \tau_3}{2}. \end{aligned} \quad (12)$$

As mentioned in (1), τ is an IID random variable sampled from a conditional distribution. Therefore, $\bar{\tau}$ is the expectation of the variables τ_1, τ_2 and τ_3 . Thus, we have $\mathbb{E}(d_n) = 2\bar{\tau} = b_n\bar{\tau}$, where $b_n = 2$ in this example. Extending to the case of N

⁴This device may broadcast zero or multiple bursts, but the expected value is the mean across all possibilities.

⁵It should be noted that x cannot be too close to t_0 since a burst of PRFs could last for a few seconds, but this is negligible because $\bar{\tau}$ is much larger.

devices, the total dwell time of N devices can be approximated by⁶

$$D = \sum_{n=1}^N d_n \approx \sum_{n=1}^N (b_n \bar{\tau}) = B\bar{\tau}. \quad (13)$$

By the *Law of Large Numbers*, with sufficiently large N and w , the approximation error would be negligible.

On the other hand, if we consider device dwell time by the definition of window-averaged device count in (3), we get

$$D = \int_0^w N(t) dt = \bar{N}w, \quad (14)$$

recall that $N(t)$ is the number of devices at time instant t . Combining (13) and (14), we have

$$D = \bar{N}w \approx B\bar{\tau}. \quad (15)$$

Recall that $\hat{N}$ is the counting result given by RateCount, and $\bar{N}$ is the ground truth of the window-averaged device count. With (4) and (6), we rewrite (15) as

$$\bar{N} \approx \frac{B/w}{1/\bar{\tau}} = \frac{R}{\bar{\tau}} = \hat{N}, \quad (16)$$

which brings us (9).

C. Error Model

We have demonstrated that RateCount approximates the ground truth. In the following, we deduce an error model to reveal how the lower bound of its counting error varies with the window size, the number of bursts in the window, and the interval variance of a single device probing.

The mean squared error (MSE) of the rate model consists of the count variance, denoted as $\text{Var}(\hat{N})$, and the counting bias, denoted as $\text{Bias}(\hat{N}, \bar{N})$. The error of the rate model is computed as

$$\begin{aligned} \text{MSE}(\hat{N}) &= \mathbb{E} \left[\left(\hat{N} - \bar{N} \right)^2 \right] \\ &= \mathbb{E} \left[\left(\hat{N} - \mathbb{E}(\hat{N}) \right)^2 \right] + \mathbb{E} \left[\left(\mathbb{E}(\hat{N}) - \bar{N} \right)^2 \right] \\ &= \text{Var}(\hat{N}) + \text{Bias}^2(\hat{N}, \bar{N}). \end{aligned}$$

Since the rate model is unbiased with sufficiently large $\bar{N}$ and w , we neglect the bias term and estimate MSE using the count variance, i.e.,

$$\text{MSE}(\hat{N}) \approx \text{Var}(\hat{N}), \quad (17)$$

where $\text{MSE}(\hat{N}) \geq \text{Var}(\hat{N})$.

Given that b and w are constants, the count variance depends on the variance of the mean probing interval, given as

$$\text{Var}(\hat{N}) = \text{Var}\left(\frac{B\bar{\tau}}{w}\right) = \frac{B^2}{w^2} \text{Var}(\bar{\tau}), \quad (18)$$

where $\text{Var}(\bar{\tau})$ represents the variance of the mean over a group of samples with a sample size of B , corresponding to the number of bursts.

Since τ is an IID random variable, its variance contributes to the random error. Therefore, the *lower bound* of the count variance depends on the variance of its sample mean, i.e.,

$$\text{Var}(\bar{\tau}) \geq \frac{\sigma_\tau^2}{B}, \quad (19)$$

where σ_τ is the standard deviation of τ , i.e., the standard deviation of the distribution in (2). By combining (18) and (19), the error model is given as

$$\text{MSE}(\hat{N}) \approx \text{Var}(\hat{N}) \geq \frac{B\sigma_\tau^2}{w^2}. \quad (20)$$

The error model conveys three key messages regarding the counting system. First, it shows a tradeoff in selecting the window size, where a large window could reduce the errors but may delay the counting results, and vice versa. Second, the counting area has an impact on the counting error as the variance of probing intervals contributes to the count variance. Third, the error model indicates that the count variance increases with the number of bursts (B). In fact, B does not have a direct impact on the count variance. Rather, the ground-truth number of devices could explain their correlation. That is, more devices lead to both more bursts and larger counting variance.

To clarify the third point, given that $\bar{N} \approx B\bar{\tau}/w$ from (8) and (9), we present the MSE relative to the ground-truth device count ($\bar{N}$) in normalized root-mean-square error (NRMSE), expressed as

$$\text{NRMSE}(\hat{N}) = \frac{\sqrt{\text{MSE}(\hat{N})}}{\bar{N}} \approx \frac{\sigma_\tau}{\sqrt{\bar{N}w\bar{\tau}}} = \frac{\sigma_\tau}{\bar{\tau}\sqrt{B}}. \quad (21)$$

As indicated by NRMSE, the counting is more accurate relative to the ground-truth device count ($\bar{N}$). In other words, RateCount tends to be more accurate in the more crowded scenarios in terms of relative counting accuracy.

IV. APPLICATION IN PEOPLE COUNTING

This section presents how to extend RateCount to people counting. We discuss a multimodal calibration scheme in Section IV-A, the derivation of the device-to-person ratio from the scheme in Section IV-B, and its error analysis in Section IV-C.

A. Multimodal Calibration Scheme

Given the popularity of Wi-Fi devices today, the device count is often used to estimate crowd density. However, in some scenarios, it is necessary to determine the exact number of people, regardless of whether they have Wi-Fi devices. To address this need, we demonstrate a calibration scheme that extends RateCount to people counting.

The gap between the number of devices and people lies in how many Wi-Fi devices a person typically carries, referred to as the *device-to-person* ratio, and the ratio often shows significant correlation with the counting area [17]–[19]. For example, the ratio is usually less than one in an elderly home because many elders do not use Wi-Fi devices, whereas it is often greater than one on a university campus where

⁶The τ here is sampled from the marginal distribution given in (2).

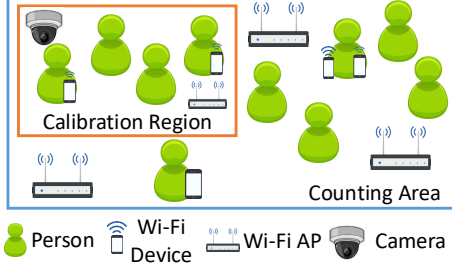


Fig. 6. Illustration of Wi-Fi-based people counting using a multimodal calibration scheme. The device-to-person ratio, computed from the calibration region, is used to calibrate the device count to people count in any other subareas of the counting area.

many students carry multiple devices, such as smartphones, wristbands, laptops, etc.

To obtain the device-to-person ratio, we set up a multimodal calibration region in the counting area, which is illustrated in Fig. 6. Specifically, we calculate the device-to-person ratio from the region, using the device count estimated by RateCount and the people count estimated by a camera-based approach [20].⁷ Then, this ratio can be applied for people counting in other subareas without a camera. For example, we can set up a calibration region in an atrium to estimate the device-to-person ratio of the university campus, and the ratio can be used to count people elsewhere on the campus. In our implementation, the sensing areas of Wi-Fi and camera are aligned by Wi-Fi geo-fencing [21].

Note that this calibration framework has been studied in the literature [17], [19]. In the following, we contribute to the field by discussing in detail how to incorporate RateCount with the calibration scheme.

B. Deriving Device-to-Person Ratio

We define the device-to-person ratio rigorously as the mean number of devices a person carries in a counting area, calculated as

$$\alpha = \frac{N}{M}, \quad (22)$$

where N and M are the numbers of devices and people in the counting area, respectively.

Given this ratio, we can estimate the window-averaged people count in any subarea of the counting region without a camera, given as

$$\begin{aligned} \bar{M} &= \frac{1}{w} \int_0^w M(t) dt \\ &\approx \frac{1}{w} \int_0^w \frac{N(t)}{\alpha} dt = \frac{\bar{N}}{\alpha}, \end{aligned} \quad (23)$$

where $M(t)$ is the number of people at time instant t , and $\bar{N}$ is estimated by RateCount.

We estimate the device-to-person ratio from the calibration region by

$$\alpha \approx \frac{\bar{N}}{\bar{M}}, \quad (24)$$

⁷We empirically adopt camera-based people counting, while other people-sensing technologies should also be applicable [17].

where $\bar{N}$ is estimated by RateCount, and $\bar{M}$ is estimated by the camera-based people counting.

Next, we explain (24) by considering the dwell time of devices and people. Recalling from (10), the total dwell time of all devices is the sum of the dwell time of each individual device. On the other hand, the dwell time of devices equals their users' dwell time (devices that do not have an associated user can be excluded by using the method in [22]). Therefore, the dwell time of all devices is given as

$$D = \sum_{n=1}^N d_n = \sum_{m=1}^M p_m \alpha_m, \quad (25)$$

where p_m is the dwell time of the m th person who has α_m as the number of devices in total.⁸

Here, we consider a general case where a person's dwell time is independent of how many devices it carries. Therefore, with a sufficiently large number of people, we have

$$\sum_{m=1}^M p_m \alpha_m \approx \frac{N}{M} \sum_{m=1}^M p_m, \quad (26)$$

recalling that N/M is the mean number of devices carried by a person. By combining (25) and (26), we get

$$\frac{\bar{N}}{\bar{M}} = \frac{\frac{1}{w} \sum_{n=1}^N d_n}{\frac{1}{w} \sum_{m=1}^M p_m} \approx \frac{\frac{N}{M} \sum_{m=1}^M p_m}{\sum_{m=1}^M p_m} = \alpha. \quad (27)$$

This indicates that we can estimate α using (24).

From the above analysis, the ratio is preferably calculated from a counting area with large crowds, such as a campus atrium or the entrance of a shopping mall, and a large window size is also preferred.

C. Error Analysis

In this section, we analyze the counting error of the people counting scheme. In particular, we consider the error of camera-based people counting based on the error model discussed in Section III-C.

Let $\hat{M}_c$ and $\hat{N}_c$ denote the people count and device count estimations in the calibration region, respectively, and $\hat{N}$ denote the device count estimation in the counting area. The people count estimation in the counting area, denoted by $\hat{M}$, is given as

$$\hat{M} = \frac{\hat{M}_c}{\hat{N}_c} \hat{N}. \quad (28)$$

According to error propagation [23], the NRMSE of the people count estimation, denoted by $\text{NRMSE}(\hat{M})$, is given as

$$\sqrt{\text{NRMSE}^2(\hat{M}_c) + \text{NRMSE}^2(\hat{N}_c) + \text{NRMSE}^2(\hat{N})}.$$

In our implementation, we empirically obtain the error of camera-based people counting ($\text{NRMSE}(\hat{M}_c)$).

⁸Rigorously, α_m represents the device-to-person ratio. Since it is specific to one person (the m th person), its value can be interpreted as the number of devices that the person carries.

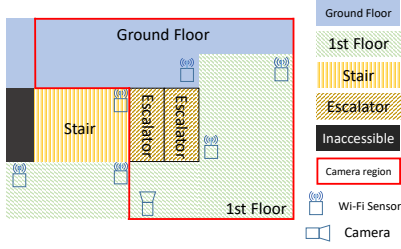


Fig. 7. Illustration of the experimental setup at a campus atrium. Several infrastructural Wi-Fi APs listen to the PRFs for device counting, and a camera is installed to provide the near-ground truth for people counting experiments.

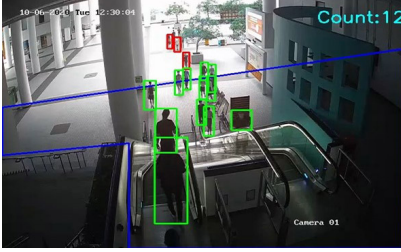


Fig. 8. Example of system operation in calibration region. In our experiment, we use YOLO to count people and Wi-Fi geo-fencing technology to align the sensing areas of Wi-Fi and camera [20], [21].

V. ILLUSTRATIVE EXPERIMENTAL RESULTS

This section presents experiments to validate RateCount. We first discuss the experimental setting in Section V-A. Then, we present the illustrative experimental results of device counting in Section V-B and people counting in Section V-C.

A. Environmental Setup and Performance Metrics

To validate RateCount, we have conducted extensive experiments of device and people counting in various sites. At each site, we employ several off-the-shelf Wi-Fi APs (GL-AR150 with OpenWrt 18.06) to capture PRFs, which were transmitted to a personal computer (with an Intel Core i7-4770 3.6 GHz CPU and 32 GB RAM) where RateCount operates, and deploy a camera to provide the near-ground truth of people counting. An example of the experimental setup is illustrated in Fig. 7.

We conducted the device counting experiment in a laboratory room for five days, which captures PRFs from six APs. To obtain the ground truth for the number of devices, we manually track the entering and leaving time of each device, leveraging the enclosed environment. To further validate RateCount in various scenarios and counting levels, we conduct people counting experiments in a campus atrium, a building entrance, and a bus station, each using six APs to capture PRFs. We use YOLOv7 to provide the near-ground truth [20], and the device-to-person ratios were computed and used interchangeably among those sites (given that they are on the same university campus), illustrated in Fig. 8.

We compare RateCount with the following approaches:

- *MAC Counting* [1] is a classic device-counting approach, which however fails to consider MAC address randomization. In the experiment, we implement it to count

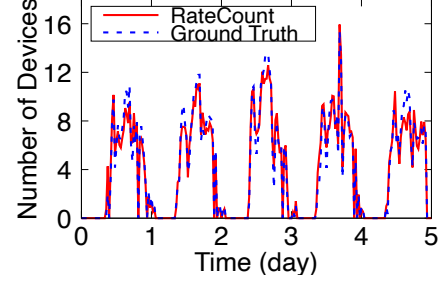


Fig. 9. Device counting in a lab room over five days.

the number of unique MAC addresses averaged over a window.

- *Learning-based Counting* [2] is a state-of-the-art (SOTA) device-counting approach that uses machine learning to associate the PRFs of the same devices. In the experiment, the counting is implemented based on the association results averaged over a window.
- *CountMeIn* [19] is a SOTA probing-based people-counting approach, which calibrates the number of MAC addresses in a window to people counts using a camera-based calibration region.

We evaluate the performance of RateCount by window-averaged counting as defined in (3) (for both device and people counting). Let $\hat{y}$ represent the counting result, $\bar{y}$ represent the (near) ground truth and J represent the number of observations in the experiment. We use the following metrics:

- *Root-Mean-Square Error* (RMSE):

$$\text{RMSE}(\hat{y}) = \sqrt{\frac{1}{J} \sum_{j=1}^J (\hat{y}_j - \bar{y}_j)^2}. \quad (29)$$

- *Mean Absolute Percentage Error* (MAPE):

$$\text{MAPE}(\hat{y}) = \frac{1}{J} \sum_{j=1}^J \frac{|\hat{y}_j - \bar{y}_j|}{\bar{y}_j}. \quad (30)$$

- *Normalized Root-Mean-Square Error* (NRMSE):

$$\text{NRMSE}(\hat{y}) = \frac{\text{RMSE}(\hat{y})}{\frac{1}{J} \sum_{j=1}^J \bar{y}_j}. \quad (31)$$

In other words, RMSE calculates the counting error in terms of the number of devices or people, and MAPE and NRMSE reflect the error relative to the ground truth. Also, RMSE and NRMSE calculate the error based on second-order norm, which is more sensitive to large errors than MAPE.

The whole system is implemented in Python, and the Wi-Fi sniffer for listening to PRFs is built on *libpcap*. Due to privacy regulations, the camera operated for five hours in the atrium and building entrance, and one hour at the bus station (during a peak hour). The distributions of probing intervals at each site were collected using a sliding window of one day. Unless specified otherwise, we use the window size of 15 minutes for counting in the lab room (since users there often stay for extended periods) and the window size of 3 minutes in other locations. The step size of the sliding window is 3 minutes.

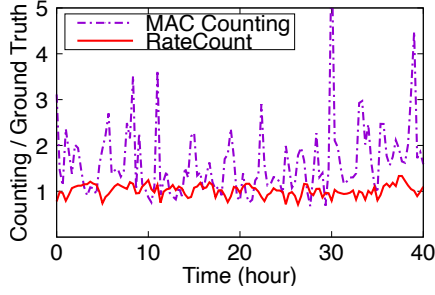


Fig. 10. Comparison of RateCount with MAC-counting approach in window-averaged counting.

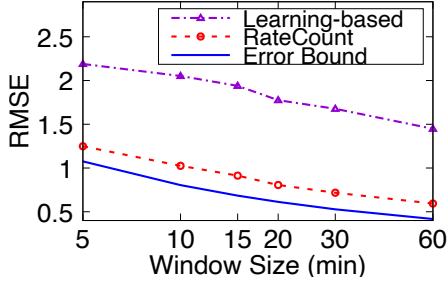


Fig. 11. Evidence that RateCount achieves comparable counting accuracy with the SOTA learning-based approach in device counting.

B. Device Counting

Fig. 9 demonstrates the device counting of RateCount in a lab room over five days. In the figure, RateCount accurately tracks the ground-truth number of devices. In particular, the counting results effectively reflect the usage pattern of the lab room, where most lab users arrive in the morning and leave after the evening, peaking in the number of devices at noon. This commendable result only uses the infrastructural Wi-Fi APs without any sensor installation or learning from datasets.

In Fig. 10, we compare RateCount with the classic MAC counting approach. The figure shows the ratio of counting results over the ground-truth device counts in the first 40 hours of data (the divided-by-zero scenario is filtered out). Though the MAC counting approach does not need machine learning, it *fails* under MAC address randomization. In particular, the MAC counting approach is vulnerable to the randomized MAC addresses that lead to the fluctuating overcounted results (note that the results have already been normalized by the window size). In comparison, RateCount can accurately count devices under MAC address randomization, while being as efficient as the classic MAC counting in computation and system deployment.

In Fig. 11, we compare RateCount with the SOTA learning-based device counting approach, showing their RMSE versus window size. From the results, RateCount achieves comparable, or even slightly better, results than the learning-based approach, without any deployment costs for machine learning. The slightly better accuracy owes to the counting unbiasedness of RateCount. Another reason is that RateCount directly counts devices using probing rate, which avoids the complexity of PRF association that may introduce errors.

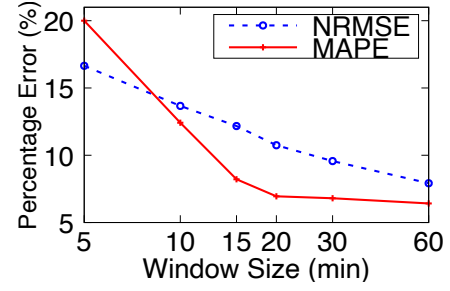


Fig. 12. Analysis on experimental errors in device counting.

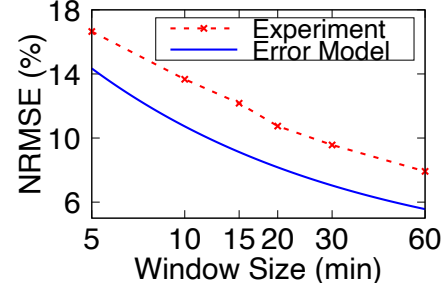


Fig. 13. Comparing the error model with experimental errors in device counting.

Fig. 12 shows the counting error of RateCount, where the error reduces with an increasing window size. This aligns with our analysis in Section III, where a large window reduces the count variance. Besides the decreasing error, the error distribution becomes more fat-tailed before the window size of 20 minutes and conversely thereafter, given that the NRMSE is more sensitive to large errors than MAPE.

In Fig. 13, we compare the experimental error of RateCount with its error model given in (21). From the figure, the error model not only outlines the lower bound but also accurately estimates the counting error. In particular, the experimental errors are slightly larger than the estimates from the error model by merely 2% on average over different window sizes. This validates that the error model can effectively estimate the counting error in practice.

C. People Counting

To validate RateCount in more scenarios and count levels, we conducted people counting experiments in an atrium, a building entrance, and a bus station on a university campus.

Fig. 14 shows the counting result of RateCount in the atrium. The device-to-person ratio is computed at the building entrance (around 1.14). The experimental results show that RateCount, as a privacy-preserving Wi-Fi-based approach, can accurately track the results of the camera-based people counting. Note that the camera-based counting could only provide the near-ground truth, whose NRMSE is around 0.08 from our observation.

In Fig. 15, we compare RateCount with a SOTA probing-based people counting scheme (i.e., CountMeIn). The device-to-person ratio is estimated from the campus atrium. In the experiment, CountMeIn calibrates the number of unique MAC

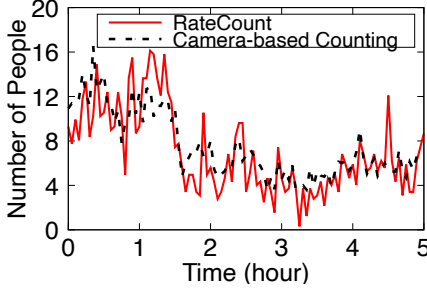


Fig. 14. People counting at a campus atrium.

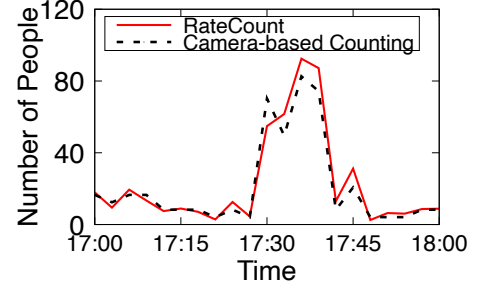


Fig. 16. People counting in a bus station during rush hour.

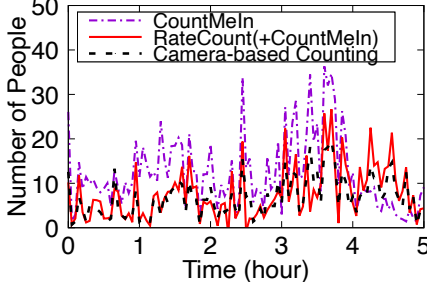


Fig. 15. Evidence that RateCount improves the SOTA Wi-Fi-based people counting scheme, demonstrated at a building entrance.

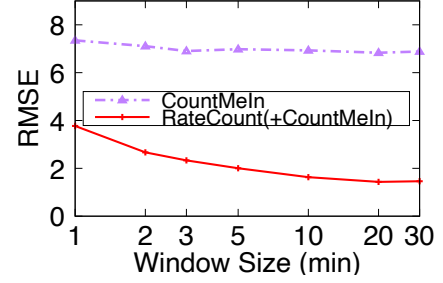


Fig. 17. Analysis of people-counting errors at the building entrance.

addresses to the people counts, while our scheme calibrates RateCount to the people count. From the figure, the counting of CountMeIn fluctuates due to the noisy and unpredictable MAC address randomization. In comparison, the calibration scheme, incorporated with RateCount, significantly reduces the counting variance owing to its unbiased device counting, showing similar counting results to the camera-based counting. The result validates that RateCount can effectively improve the performance of probing-based people counting.

Fig. 16 shows the performance of RateCount at the bus station during a peak hour. Compared with the previous sites, the bus station is crowded with more people. Despite this, RateCount still shows similar counting results to the camera-based counting, effectively capturing the usage pattern of the bus station (students went to bus stations after the last class concluded at 5:30 PM). This validates RateCount in more scenarios and its scalability to different count levels.

Fig. 17 shows the RMSE of CountMeIn and RateCount at the building entrance, using camera-based people counting as the near-ground truth. From the figure, the RMSE reduces with the window size as expected. However, the RMSE of CountMeIn reduces much slower than RateCount, since it is affected by the noisy and unpredictable MAC address randomization. Overall, by incorporating RateCount, the people-counting scheme has reduced around 4.57 RMSE (that is, 0.66 NRMSE) at the window of 3 minutes.

Finally, we validate the error model in people counting by comparing the experimental and estimated errors on the three sites in Fig. 18. The error is estimated from (IV-C), and the NRMSE of the camera-based counting is around 0.1 from our observation. From the figure, the gap between the experimental and estimated errors is roughly the same as that of the device

counting. This validates that our error model can provide a sound estimate of the error in people counting.

VI. RELATED WORK

In this section, we review the related works in Wi-Fi-based device counting and the existing sensing technologies for people counting.

To count Wi-Fi devices, the simplest method is to count the number of devices connected to Wi-Fi access points (APs), i.e., the connection-based approach [24]–[26]. While straightforward, it fails to consider the unconnected devices, making it suitable only for areas with a high connection ratio [27]–[30]. To include the unconnected ones, subsequent works have explored Wi-Fi probing, estimating the count by the number of unique MAC addresses in the probe request frames (PRFs) over a window, i.e., the MAC-counting approach [1], [31], [32]. However, this approach is vulnerable to MAC address randomization, where one device may have multiple virtual MAC addresses.

Since it is very challenging to address the MAC address randomization [33]–[35], state-of-the-art approaches leverage machine learning to associate the PRFs with their broadcasting devices [2], [3], [8], [9]. Specifically, efforts have been made by using machine learning models, such as Bayesian networks, decision trees, Gaussian mixture models, and deep learning, to associate PRFs based on their messages, such as information elements, sequence numbers, received signal strength, arrival times [2], [3], [10], [22], [36], [37]. Although accurate device counting has been achieved based on the association results, their system deployment is inefficient and time-consuming due to the professional efforts for machine learning and heavy computing requirements [12], [13], which is unsuitable for scenarios of fast deployment or with restricted computing

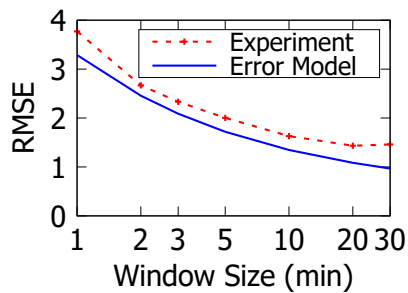


Fig. 18. Comparing the error model with experimental errors in the three people counting experiments.

power. In addition, these works often lack a theoretical understanding of counting errors. In summary, RateCount is the first learning-free approach to count Wi-Fi devices under MAC address randomization, with theoretically derived counting unbiasedness and an error model.

Several sensing technologies have been considered for people counting, such as camera, LiDAR, and mmWave radar [6], [38], [39]. However, these sensors often have limited sensing range, incur extra cost due to infrastructure setup, and might pose privacy concerns. To address these issues, Wi-Fi-based people counting has been explored, which can be categorized into active and passive approaches. The active approach counts people by analyzing the physical Wi-Fi signal propagating over the counting area (i.e., channel state information), which however requires specialized devices [40]–[43]. On the other hand, the passive approach listens to the PRFs and studies how to calibrate the number of PRFs to estimate the people count [17], [44]–[47]. In this paper, we demonstrate that RateCount, incorporating the previous calibration scheme, can significantly improve the accuracy of people counting.

VII. CONCLUSION

For network discovery, Wi-Fi devices sporadically broadcast probe request frames (PRFs) with randomized MAC addresses to the nearby access points (APs). While prior works have achieved accurate device counting under MAC address randomization, the counting relies on machine learning to associate the PRFs, leading to inefficient and time-consuming system deployment involving training data curation and cleaning, model training, and hyperparameter tuning. To enable deployment efficiency, we study a lightweight and learning-free approach to count Wi-Fi devices under MAC address randomization. Observing that more Wi-Fi devices lead to a higher PRF rate, we propose a counting approach named RateCount, which counts devices by a simple and effective closed-form expression that measures the rate of APs receiving PRFs within a window. We further prove that the rate model gives an unbiased estimator of the device count time-averaged over the window and deduce the lower bound of the counting error in terms of the window size, number of PRFs in the window, and the variance of the probing interval. In addition, we demonstrate how to extend RateCount to people counting via a multimodal calibration scheme. We have implemented RateCount and conducted extensive experiments of device and

people counting in various environments, specifically, in a lab room, a building entrance, a campus atrium, and a bus station. The results show that our learning-free RateCount achieves comparable counting accuracy with the SOTA learning-based device counting and significantly improves the previous people counting by cutting 66% errors. Furthermore, our error model has demonstrated sound estimations of the counting error.

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APPENDIX A

PROOF OF THE UNBIASEDNESS OF RATECOUNT

We provide a proof of $\mathbb{E}(\hat{N}) = \bar{N}$ below by presenting that the whole system is equivalent to a *renewal process*. In a renewal process, we consider the summation of a sequence of positive IID random variables X_k given as

$$S_K = \sum_{k=1}^K X_k, \quad (32)$$

where $\mathbb{E}(X_k) = \bar{X}$. We define a *counting random variable* $L(S)$ as the maximum number of IID random variables (X_k) such that their summation $S_{L(S)} \leq S$. Namely, we have $\sum_{k=1}^{L(S)} X_k \leq S$ and $\sum_{k=1}^{L(S)+1} X_k > S$.

An important property of the renewal process is the *Elementary Renewal Theorem*, which states that

$$\frac{\mathbb{E}(L(S))}{S} = \frac{1}{\bar{X}} \quad (33)$$

when S is sufficiently large.

We elaborate that our system is equivalent to a *renewal process* using the single-device example shown in Fig. 2. In this figure, the APs received $b_n = 2$ bursts separately at t_1 and t_2 , and the device enters the area at time x and leaves at y . The dwell time of this device is calculated as

$$d_n = y - x = \beta_1 \tau_1 + \tau_2 + \beta_3 \tau_3, \quad (34)$$

where β_1 and β_3 represent the fraction of τ_1 and τ_3 with $0 < \beta_1, \beta_3 < 1$. Since the device enters and leaves the area largely independent of the probing, with equal probabilities, x could be any value between t_0 and t_1 , and y could be any value between t_2 and t_3 . Therefore, β_1 and β_3 are IID *uniform random variables* with $\mathbb{E}(\beta_1) = \mathbb{E}(\beta_3) = 1/2$.

When computing the total dwell time (D), we use T_F to denote the set of *fractional intervals* $\{\beta_i \tau_i\}$ where only part of τ_i is summed up (e.g., $\beta_1 \tau_1$ and $\beta_3 \tau_3$), and T_I to denote the set of *integral intervals* $\{\tau_i\}$ where the whole of τ_i is summed up (e.g., τ_2). It is important to note that both T_F and T_I are sets of IID random variables where $\mathbb{E}(\beta_i \tau_i) = \bar{\tau}/2$ and $\mathbb{E}(\tau_i) = \bar{\tau}$.

Generally speaking, N devices with B probing bursts would have $2N$ fractional intervals and $B - N$ integral intervals. Therefore, the dwell time of all the devices can be written as

$$D = \sum_{\beta_i \tau_i \in T_F} \beta_i \tau_i + \sum_{\tau_i \in T_I} \tau_i. \quad (35)$$

We let $D_F = \sum \beta_i \tau_i$ for $\beta_i \tau_i \in T_F$ and $D_I = \sum \tau_i$ for $\tau_i \in T_I$. Both D_F and D_I are the sums of IID random variables similar to S_K in (32). Clearly, we have $D = D_F + D_I$. By applying the Elementary Renewal Theorem given in (33), we have

$$\frac{\mathbb{E}(B - N)}{D_I} = \frac{1}{\bar{\tau}} \quad (36)$$

and

$$\frac{\mathbb{E}(2\mathbf{N})}{D_F} = \frac{2}{\bar{\tau}} \quad (37)$$

when D_F and D_I are sufficiently large.

Given (8), we also have

$$\begin{aligned} \mathbb{E}(\hat{N}) &= \mathbb{E}\left(\frac{R}{\bar{r}}\right) = \mathbb{E}\left(\frac{B\bar{\tau}}{w}\right) = \frac{\bar{\tau}}{w}\mathbb{E}(B) \\ &= \frac{\bar{\tau}}{w}\left(\mathbb{E}(B - \mathbf{N}) + \frac{1}{2}\mathbb{E}(2\mathbf{N})\right). \end{aligned} \quad (38)$$

Based on (3), (36), (37), and (38), when $\bar{N}$ and w are sufficiently large so that D is also sufficiently large, we have

$$\mathbb{E}(\hat{N}) = \frac{\bar{\tau}}{w}\left(\frac{D_I}{\bar{\tau}} + \frac{1}{2}\frac{2D_F}{\bar{\tau}}\right) = \frac{D}{w} = \bar{N}. \quad (39)$$

This shows that the rate model is an unbiased estimator of the window-averaged device count, which ends the proof.