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Aggregating bipolar opinions through bipolar assumption-based argumentation

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Abstract
We introduce a novel method to aggregate bipolar argumentation frameworks expressing opinions of different parties in debates. We use Bipolar Assumption-based Argumentation (ABA) as an all-encompassing formalism for bipolar argumentation under different semantics. By leveraging on recent results on judgement aggregation in social choice theory, we prove several preservation results for relevant properties of bipolar ABA using quota and oligarchic rules. Specifically, we prove (positive and negative) results about the preservation of conflict-free, closed, admissible, preferred, complete, set-stable, well-founded and ideal extensions in bipolar ABA, as well as the preservation of acceptability, acyclicity and coherence for individual assumptions. Finally, we illustrate our methodology and results in the context of a case study on opinion aggregation for the treatment of long COVID patients.

Keywords Opinion aggregation · Computational argumentation · Social choice theory · Knowledge representation and reasoning

1 Introduction

There is a long and well-established tradition in knowledge representation and reasoning to formally describe debates as exchanges of opinions by the parties involved through attacks [11] and supports [39] between arguments, understood abstractly as in *Abstract Argumentation (AA)* [23] or as in *Bipolar Argumentation* (BA) [17, 18].

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When these debates emerge in multi-agent systems [27, 39], a key question concerns opinion aggregation, namely how we can obtain a collective consensus from several opinions expressed as argumentation frameworks, in such a way that the agents' opinions are well-portrayed in the collective outcome [9]. Recently, Chen and Endriss [11–13] applied aggregation procedures from Social Choice Theory [33] to AA Frameworks (AAFs), by leveraging specifically on judgement aggregation [26], and proving the preservation of interesting properties of AAFs, such as conflict-freeness, acyclicity, and extensions according to several semantics. Some efforts have been made to provide procedures for the aggregation of *Quantitative Argumentation Debate Frameworks* [8, 39, 40] (a form of *BA Frameworks (BAFs)* incorporating both attacks and supports, but equipped with gradual, rather than extension-based, semantics). However, the aggregation of BAFs is an open problem, and a challenging one mainly because there are different semantic interpretations of support in BAFs (e.g., as deductive or necessary [4, 18, 37]), which might generate "inconsistencies" in the aggregated framework. Chen [15, 16] has explored BAF aggregation using Social Choice Theory, focusing specifically on aggregating the support relation. However, their approach assumes that agents share a common interpretation of support.

Moving from the considerations above, we advance the state of the art on the application of Social Choice Theory to opinion aggregation in computational argumentation by focusing on BA. To address the problem that different parties may adopt different interpretations of support in their opinions (i.e. BAFs), we use *Bipolar Assumption-based Argumentation (ABA) frameworks* [19, 29, 30] for representing opinions. Bipolar ABA is a restricted form of ABA that provides a unified formalism to accommodate different interpretations of support [19]. Thus, by adopting Bipolar ABA, we let parties choose their interpretation of support before aggregation takes place.

Contributions of the paper. Our contribution is threefold. First, we define aggregation mechanisms for Bipolar ABA frameworks based on Social Choice Theory. Our investigations mainly focus on quota and oligarchic rules [26]. Second, we conduct a study on the preservation of several properties using the defined aggregation procedures. In some cases, restrictions need to be placed, for example, towards specific aggregation rules. Specifically, we prove (positive and negative) results about the preservation of conflict-free, closed, admissible, preferred, complete, well-founded [3], ideal (adapted to Bipolar ABA from [22]) and set-stable [19] extensions in Bipolar ABA, as well as the preservation of acceptability, acyclicity and coherence for individual assumptions. Lastly, we illustrate the potential usefulness of our framework in a realistic setting: decision-making for medical treatment, when several experts have different opinions, alongside the patient needing treatment.

Structure of the paper. The remainder of the paper is structured as follows. First, in Sect. 2, we discuss related work, outlining the context of the contributions of the paper. In Sect. 3 we provide the necessary background on Bipolar ABA. Section 4 sets the ground, by formulating the aggregation problem, aggregation rules, and preservation properties in our Bipolar ABA setting. Section 5 contains the main theoretical contribution of the paper providing preservation results for various properties of Bipolar ABA frameworks. Section 6 illustrates possible uses of our framework for deciding on the treatment of patients. Finally, Sect. 7 concludes and elaborates on several promising directions for future work.

This paper is a revised and extended version of [31], in particular including revisions of some results (as detailed in Sect. 5.10) and all proofs, as well as a novel illustration in Sect. 6.

2 Related work

Judgement aggregation in Social Choice Theory is widely studied (e.g. see [26, 38]). It uses mathematical formulations in the form of aggregation rules to aggregate opinions by several agents into a single collective opinion. These rules have been popular research topics in many areas, and judgement aggregation shows promise in several applications (including in economics, law and political science). The opinions to be aggregated may be expressed in several forms, notably they may be expressed in logic (e.g. [24]), as preferences (e.g. [41]), or as argumentation frameworks (e.g. [11–13]). We follow the latter approach, focusing specifically on BAFs and, at the same time, a simple form of structured argumentation frameworks: Bipolar ABA. Other approaches have instead focused on aggregating argumentation frameworks in the other forms, specifically AAFs ([1, 9, 11–13, 21]) and the extensions thereof [12], value-based argumentation [32], and abstract dialectical frameworks [6]. We adopt the view that aggregation takes place by means of rules drawn from Social Choice Theory, e.g. as in [1, 9, 11, 13, 21]. Others instead achieve aggregation via dialogues (e.g. [10, 39]).

This paper is closely related in terms of subject and methodology to some recent studies, which we will now discuss in further detail.

Graph aggregation. In [25], Endriss and Grandi analyse the effect of graph aggregation procedures on the preservation of graph-theoretical properties. Here, graph aggregation refers to the process of computing an output graph that constitutes a good compromise between several input graphs. The primary focus of the paper is the presentation of several preservation results for different aggregation procedure. Notably, the paper takes an axiomatic approach, characterising aggregation procedure according to *axioms* they fulfil and also characterising properties by the *meta-properties* they satisfy. Thus, Endriss and Grandi are able to state broad results in terms of these general characterisations, allowing results to be applied to many different combinations of aggregation rules and properties. This paper is highly relevant as the aggregation of argumentation frameworks can be viewed as an instance of graph aggregation and indeed this is mentioned as a particular domain of application for this axiomatic method of study. However, no technical results are presented in [25] that are related to argumentation.

Preservation of semantic properties during the aggregation of AAFs. In [13], Chen and Endriss provide results for the preservation of properties during the aggregation of AAFs. Here, the axioms and meta-properties defined in [25] are introduced in the context of AA and the properties under study are those related to AA semantics. It was found that most properties under investigation required restrictions, such as limiting the choice of aggregation rule or input AAFs, to preserve them. Although [13] provides a study of preservation of AAFs, it does not make any commentary on preservation of other types of argumentation frameworks. However, the structure of study in this paper is particularly relevant, and outlines a similar approach, both in the properties under investigation and the method of study itself that is taken in this paper. In particular, a similar axiomatic approach to the study was taken in [13] as in [25].

Collective argumentation. Chen [15, 16] investigates the aggregation of opinions in multi-agent systems using BAFs. These works focus on a specific aggregation model where agents share a common set of arguments and attacks, but propose different supports. Here positive and negative results are developed by employing the axiomatic method from graph aggregation to obtain results in terms of classes of aggregation rules that satisfy particular axioms from social choice theory. Additionally, Chen examines various properties individually, but also proves results using meta-properties to characterise results in terms of classes of properties. Their approach considers both deductive and necessary supports, although these

different notions of support are not handled simultaneously; agents need to agree on the type of support before aggregation takes place.

While our work shares similarities with Chen's in that both papers aim to aggregate BAFs and leverage meta-properties to prove results, there are differences. First, our aggregation model allows agents to propose Bipolar ABA rules, which ultimately correspond to attacks and supports in BAFs. This contrasts with Chen's model, where agents assume the same set of attacks and arguments, but can propose different supports. Second, our approach first transforms BAFs into an intermediate representation of Bipolar ABA, providing a different perspective on the aggregation process. Notably, this allows us to handle scenarios where different agents may have different interpretations of supports.

3 Background: bipolar ABA

Bipolar Assumption-based Argumentation [19], or Bipolar ABA for short, is a form of structured argumentation, where arguments and attacks are derived from assumptions, rules, and a contrary map from assumptions. Note that contrary should not be confused with negation, which may or may not occur in the underlying language [42].

Definition 1 [19] A *Bipolar ABA framework* is a quadruple $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$, where

- $\mathcal{A} \subseteq \mathcal{L}$ is a non-empty set of *assumptions*.
- $^- : \mathcal{A} \to \mathcal{L}$ is a total map; for $\alpha \in \mathcal{A}$, $\bar{\alpha}$ is the *contrary* of α.
- $(\mathcal{L}, \mathcal{R})$ is a *deductive system* with $\mathcal{L}$ a *language* (i.e., a set of sentences) and $\mathcal{R}$ a set of *rules* of the form $\phi \leftarrow \alpha$ where $\alpha \in \mathcal{A}$ and either $\phi \in \mathcal{A}$ or $\phi = \bar{\beta}$ for some $\beta \in \mathcal{A}$. We say that ϕ is the *head* and α is the *body* of rule $\phi \leftarrow \alpha$.

Note that in Bipolar ABA frameworks rules are of a restricted kind in comparison with generic ABA frameworks [3]: their bodies amount to a single assumption, and thus, in particular, there are no rules with an empty body; also, their heads are either assumptions or contraries thereof.

A *deduction* for $\phi \in \mathcal{L}$ supported by $A \subseteq \mathcal{A}$ and $R \subseteq \mathcal{R}$, denoted $A \vdash^R \phi$, is a finite tree [42] with the root labelled by ϕ; leaves labelled by assumptions, with A the set of all such assumptions; and each non-leaf node ψ has a single child labelled by the body of some ψ-headed rule in $\mathcal{R}$, with R the set of all such rules.[1]

This notion of deduction can be used to define two important notions in Bipolar ABA: *closedness* of sets of assumptions and *attacks* between sets of assumptions. These notions are the core ingredients for the definition of *semantics* of Bipolar ABA framework, in terms of sets of assumptions (also called *extensions*) satisfying given requirements.

For $A \subseteq \mathcal{A}$, let the *closure of* $A \subseteq \mathcal{A}$ be $Cl(A) = \{\alpha \in \mathcal{A} :$ for some $A' \subseteq A$, $R \subseteq \mathcal{R}$, we have $A' \vdash^R \alpha\}$. Then, A is *closed* iff $A = Cl(A)$. A (Bipolar or generic) ABA framework is *flat* iff every $A \subseteq \mathcal{A}$ is guaranteed to be closed [3], namely, intuitively, when assumptions are not "deducible" from other assumptions. Note that, in general, Bipolar ABA frameworks may not be flat, and thus may lack some of the properties that flat ABA frameworks exhibit [14].

[1] Note that Bipolar ABA is a restricted form of ABA, in that rule bodies consist of a single assumption [19]. As a result, the deduction $A \vdash^R \phi$ can be more simply seen as a sequence of sentences $\alpha_1, \ldots, \alpha_n$, where: $\alpha_1 \in A$; $\alpha_n = \phi$; and for each pair of adjacent elements α_i and α_{i+1} (where $1 \leq i < n$), there exists a rule $\alpha_{i+1} \leftarrow \alpha_i \in R$.

Table 1 Bipolar ABA semantics (for extension $A \subseteq \mathcal{A}$)

Semantics	Conditions
Admissible	A is closed, conflict-free and for every $B \subseteq \mathcal{A}$, if B is closed and attacks A, then A attacks B
Preferred	A is $\subseteq$-maximally admissible
Complete	A is admissible and $A = \{\alpha \in \mathcal{A} : A \text{ defends } \alpha\}$, where A *defends* $\alpha \in \mathcal{A}$ iff for all closed $B \subseteq \mathcal{A}$, if B attacks α then A attacks B
Set-stable	A is closed, conflict-free, and attacks $Cl(\beta)$ for each $\beta \in \mathcal{A} \setminus A$
Well-founded	A is the intersection of all complete extensions
Ideal	A is $\subseteq$-maximal such that it is admissible and $A \subseteq B$ for all preferred extensions $B \subseteq \mathcal{A}$

For $A \subseteq \mathcal{A}$ and $\beta \in \mathcal{A}$, A *attacks* β iff for some $A' \subseteq A$ and $R \subseteq \mathcal{R}$, it holds that $A' \vdash^R \bar{\beta}$. Then, $\alpha \in \mathcal{A}$ *attacks* $\beta \in \mathcal{A}$ iff $\{\alpha\}$ attacks β; and $A \subseteq \mathcal{A}$ *attacks* $B \subseteq \mathcal{A}$ iff for some $\beta \in B$, we have that A attacks β. A set of assumptions $A \subseteq \mathcal{A}$ is *conflict-free* iff A does not attacks A.

Table 1 gives definitions of the semantics for Bipolar ABA frameworks we will analyse in this paper, in addition to properties of closedness, conflict-freeness, acyclicity and coherence (the latter two defined later in the paper).[2]

Example 1 As a simple illustration of these semantics, consider a Bipolar ABA framework with $\mathcal{L} = \{\alpha, \beta, \gamma, \bar{\alpha}, \bar{\beta}, \bar{\gamma}\}$, $\mathcal{A} = \{\alpha, \beta, \gamma\}$ and $\mathcal{R} = \{\bar{\beta} \leftarrow \gamma, \gamma \leftarrow \alpha\}$.[3] This framework is non-flat. In particular, $\{\alpha\}$ and $\{\alpha, \beta\}$ are not closed (and thus not admissible). Further, $\{\beta\}$ is closed and conflict-free but not admissible, and $\{\alpha, \gamma\}$ is closed, conflict-free, admissible, preferred, complete, set-stable, well-founded and ideal.

Note that, in Bipolar ABA (as in flat ABA, but not in general ABA), admissible, preferred and ideal extensions are guaranteed to exist: in particular, since rules cannot have an empty body, the empty set of assumptions is closed, and thus admissible. Instead, complete, well-founded and set-stable extensions may not exist [19].

Bipolar ABA provides an all-encompassing framework for capturing different interpretations of support (under admissible, preferred and set-stable semantics) [19], as illustrated in the following example, adapted from [39].[4]

Example 2 The UK public may hold a range of views on Brexit:

α: The UK should leave the EU.

[2] Note that, in line with [19], we opt to focus on the set-stable semantics rather than the stable semantics originally given for (flat or non-flat) ABA [3]. (Indeed, the set-stable, rather than stable, semantics corresponds to various notions of stable extensions for BAFs.) Also, we adapt the notion of ideal semantics from [22] to (potentially non-flat) Bipolar ABA, even though the original notion focused on flat ABA frameworks. (The adaptation amounts to using it with the notion of admissible and preferred extensions for general, potentially non-flat ABA.) Finally, note that the well-founded semantics is equivalent to the grounded semantics [23] in the case of flat ABA frameworks, but not in the context of (non-flat) Bipolar ABA [19].

[3] With an abuse of notation, we use $\bar{x}$ to denote the $\mathcal{L}$-sentence amounting to the contrary of $x \in \mathcal{A}$ and omit to specify the contrary map explicitly.

[4] The formal definitions of how different interpretations of support are captured in Bipolar ABA and the formal proofs of the equivalence between the original representations and the Bipolar ABA formulations appear in [19] and are outside the scope of this paper.

β: The UK staying in the EU is good for its economy.
γ: The EU's immigration policies are bad for the UK's economy.
δ: EU membership fees are too high.
ε: The UK staying in the EU is good for world peace.

Here α may be deemed to be attacked by β and ε, but supported by γ and δ. A Bipolar ABA representation for the *deductive interpretation* of support [18] is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$, where

- $\mathcal{L} = \{\alpha, \beta, \gamma, \delta, \varepsilon, \bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}, \bar{\varepsilon}\}$;
- $\mathcal{R} = \{\bar{\alpha} \leftarrow \beta, \quad \bar{\alpha} \leftarrow \varepsilon, \quad \alpha \leftarrow \gamma, \quad \alpha \leftarrow \delta\}$;
- $\mathcal{A} = \{\alpha, \beta, \gamma, \delta, \varepsilon\}$.

Instead, a Bipolar ABA representation for the *necessary interpretation* of support [37] is $\langle \mathcal{L}, \mathcal{R}', \mathcal{A}, ^- \rangle$ with $\mathcal{R}' = \{\bar{\alpha} \leftarrow \beta, \quad \bar{\alpha} \leftarrow \varepsilon, \quad \gamma \leftarrow \alpha, \quad \delta \leftarrow \alpha\}$. As evident from this illustration, the two representations in Bipolar ABA differ in how they interpret the support of α by γ and δ, in line with the different semantics as defined by [18, 37]: whereas under the deductive interpretation α belongs to the closure of either $\{\gamma\}$ and $\{\delta\}$ (and thus is "deducible" from either), under the necessary interpretation γ and δ both belong to the closure of $\{\alpha\}$ (and thus α is necessary for "deducing" both γ and δ). Other interpretations of support can also be represented in Bipolar ABA, as proven in [19], but are not illustrated here as they lie outside the scope of this paper, given our focus directly on Bipolar ABA rather than BAFs.

Note that Bipolar ABA can naturally capture *supported attacks* under the deductive interpretation of support [18], for example the supported attack from α to β in a (standard) BAF where α supports γ and γ attacks β is matched by α attacking β in a Bipolar ABA framework where $\{\bar{\beta} \leftarrow \gamma, \gamma \leftarrow \alpha\} \subseteq \mathcal{R}$.

The use of Bipolar ABA addresses the problem of agents adopting different interpretations of supports within their respective BAFs. Consider a situation where the frameworks of two agents are to be aggregated, where one agent proposes a BAF with the necessary interpretation of support, while the other proposes a BAF with the deductive interpretation of support. Suppose a simple aggregation procedure is used, where a support relation is included in the aggregated framework if it appeared in the frameworks of both agents. This procedure would be fundamentally inconsistent as it would not accurately account for the views of the agents. Here, a support relation that appears in both frameworks will be added to the aggregated framework, despite the fact that the relations have different meanings in each agent's framework. Since each agent differs in their interpretation of support, the meaning of any proposed support relations between two given arguments will differ between the agents, affecting extensions admitted under the semantics and thus the views expressed by the agents' frameworks in different ways. By transforming BAFs into their corresponding Bipolar ABA frameworks prior to aggregation, we use a single formalism to represent the views of agents regardless of their individual interpretation of support. This permits us to define methods that aggregate rules, which will be consistently interpreted between frameworks, as opposed to attacks and supports.

Throughout the paper, we often depict Bipolar ABA frameworks as graphical BAFs. In such cases, we adopt the following representation.

Notation 1 (*Bipolar ABA frameworks as graphical BAFs*) Let $\rightarrow$ and $\Rightarrow$ denote two sets of directed edges (representing *attacks* and *supports*, respectively). Then, we can represent a given Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ as a graph with nodes $Nodes$ and edges $\rightarrow \cup \Rightarrow$ by the following procedure:

(i) for each $\alpha \in \mathcal{A}$, create exactly one node in $Nodes$ uniquely labelled with α;

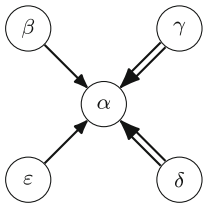


Fig. 1 Example bipolar ABA framework as a BA framework

(ii) for each $r \in \mathcal{R}$:

(a) if $r = \overline{\alpha} \leftarrow \beta$, for $\alpha, \beta \in \mathcal{A}$, add (β, α) to $\rightarrow$;
(b) if $r = \alpha \leftarrow \beta$, for $\alpha, \beta \in \mathcal{A}$, add (β, α) to $\Rightarrow$.

As an example, take the framework presented in Example 2. Then, the corresponding representation as a result of this procedure is shown in Fig. 1.

4 Setting the scene: social choice for bipolar ABA

Social choice theory mainly focuses on how to aggregate (people's or agents') opinions into a single collective decision. Broadly speaking, there are two main types of aggregation[5]: preference aggregation [7, 33] and judgement aggregation [33]. Here we focus on the latter, and adapt notions given in [11] to accommodate opinions drawn from Bipolar ABA frameworks.

Hereafter, we assume a set $N = \{1, \ldots, n\}$ of agents (for $n > 1$), with agents' opinions represented as Bipolar ABA frameworks:

$$\langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, \bar{} \rangle, \ldots, \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, \bar{} \rangle$$

We assume that the agents' frameworks are built on the same language, assumptions and contraries. Thus, aggregation combines the rules of the frameworks. This choice is consistent with intuition, as aggregating BAFs means aggregating the attacks and supports in the original frameworks, which correspond to the rules in BAFs. For convenience, we use $\mathscr{F}$ to refer to the set of Bipolar ABA frameworks defined on the same language $\mathcal{L}$, set of assumptions $\mathcal{A}$ and contrary mapping $\bar{}$.

When aggregating agents' opinions, it is reasonable to assume that the agents behave *rationally*. When the opinions are expressed as Bipolar ABA frameworks, there are various options for enforcing agents' *rationality*. The most basic rationality requirement is that for every $\alpha \in \mathcal{A}$ it holds that $\{\alpha\}$ is conflict-free in each agent's framework. In particular, this implies that rules $\overline{\alpha} \leftarrow \alpha$ (for $\alpha \in \mathcal{A}$) will never belong to any $\mathcal{R}_i$. Stronger requirements are also possible and may be desirable, e.g. as follows:

Requirement 1 An agent is *rational* if for every $\alpha \in \mathcal{A}$ it holds that $Cl(\{\alpha\})$ is conflict-free in the agent's framework.

Requirement 1 is stronger than the most basic requirement, as can be seen in Fig. 2. See that $Cl(\{\alpha\}) = \{\alpha, \beta, \gamma\}$, and so $Cl(\{\alpha\})$ is not conflict-free as α attacks γ. Thus this framework is not rational under Requirement 1. However, $\{\alpha\}$ is conflict-free, as α does not attack itself (as there exists no deduction for $\overline{\alpha}$). Thus this same framework is deemed rational under the basic requirement.

[5] Other aggregation types exist but are omitted here.

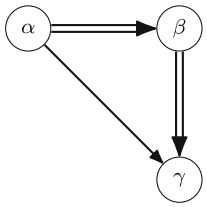


Fig. 2 A framework that satisfies the basic rationality requirement but fails to meet Requirement 1

In the remainder of the paper, we will impose this rationality requirement on all agents. This implies, for example, that rules $\bar{\alpha} \leftarrow \beta, \beta \leftarrow \alpha$ (for $\alpha, \beta \in \mathcal{A}$) will never belong to any $\mathcal{R}_i$. Note that additional requirements may also be enforced on "deducibility" of assumptions, e.g., by requiring that no assumption is self-supporting, i.e., there exists no $\alpha \in \mathcal{A}$ such that $\{\alpha\} \vdash^R \alpha$ with R a non-empty set of rules. This implies, for example, that $\alpha \leftarrow \alpha$ will never belong to any $\mathcal{R}_i$. This requirement can be further strengthened to enforce that no assumption is "deducible" from its own closure. This implies, for example, that $\alpha \leftarrow \beta, \beta \leftarrow \alpha$ will never belong to any $\mathcal{R}_i$. Further elaboration on agents' rationality, when they are defined argumentatively, can be found in [39], but for our purposes Requirement 1 will suffice.

To collectively combine the opinions of all agents, i.e., the agents' Bipolar ABA frameworks, we need aggregation rules.

Definition 2 (*Aggregation Rule*) A *Bipolar ABA aggregation rule* is a mapping $F : \mathscr{F}^n \rightarrow \mathscr{F}$ from n Bipolar ABA frameworks into a single Bipolar ABA framework. Given n Bipolar ABA frameworks:

$$\boldsymbol{\mathcal{F}} = \big(\langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, \bar{} \rangle, \ldots, \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, \bar{} \rangle\big) \in \mathscr{F}^n$$

(as opinions of agents in $N = \{1, \ldots, n\}$), F returns a single *aggregated Bipolar ABA framework* $F(\boldsymbol{\mathcal{F}}) = \langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \bar{} \rangle$ with $\mathcal{R}_{agg} \subseteq \boldsymbol{\mathcal{R}}$ where $\boldsymbol{\mathcal{R}} = \bigcup_{i \in N} \mathcal{R}_i$.

As attacks and supports correspond to rules when BAFs are captured as Bipolar ABA frameworks [19], our aggregation model can be seen as allowing agents to each propose different attacks and supports in their respective BAFs before aggregation, while also requiring that the agents agree on the set of arguments (which corresponds with fixing the assumptions in our Bipolar ABA setting).

The requirement for agents to agree on a common set of assumptions might initially appear restrictive, implying that agents need to know about all existing assumptions (arguments) in a debate, including those they have no opinion on. In practice, however, this requirement can be addressed through a simple preliminary step, where the union of all assumptions is incorporated into each individual agent's framework before aggregation takes place. In this way, agents can individually propose their own frameworks without requiring knowledge or opinions about all assumptions in the debate, whilst still ensuring that the agents share the same set of assumptions.

There are several potential ways to aggregate argumentation frameworks, ranging from combining fundamental structural elements (such as rules in our case) to more high-level methods like directly aggregating extensions admitted under different semantics across input frameworks.

Our model, which focuses on aggregating structural elements, is motivated by two key factors. Firstly, it aligns with approaches in neighboring literature (e.g., [13, 15, 16]) where attacks and/or support relations are typically aggregated. This similarity-as aggregating rules

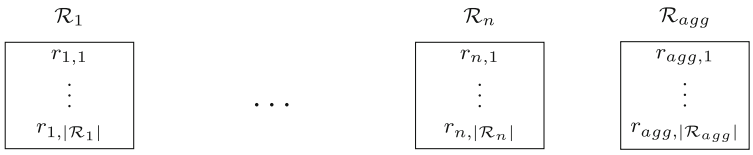


Fig. 3 The depiction of an aggregation scenario, as defined in Notation 2

corresponds to aggregating attacks and supports in BAFs-allows our study to remain comparable with existing research.

Secondly, this approach offers a straightforward method for combining frameworks, making it a logical starting point for aggregation. While alternative methods may prove viable and warrant future investigation, our model and its results serve as a reference point. It establishes a foundation for evaluating and comparing different aggregation techniques in future research, balancing simplicity with the potential for more complex approaches.

Sometimes, the effect of an aggregation rule on a particular set of frameworks may not be immediately obvious. In these situations, we illustrate such *aggregation scenarios* using the following graphical convention.

Notation 2 (*Aggregation Scenario*) Suppose n input frameworks $\boldsymbol{\mathcal{F}} = \langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, \bar{} \rangle, \ldots, \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, \bar{} \rangle$ are aggregated by an aggregation rule F such that $F(\boldsymbol{\mathcal{F}}) = \langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \bar{} \rangle$. For each input framework $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \bar{} \rangle$, we denote, as shown in Fig. 3, the rules in $\mathcal{R}_i$, that agent i adopts, as $r_{i,1}, \ldots, r_{i,|\mathcal{R}_i|}$ in a box labelled by $\mathcal{R}_i$, and the rules in $\mathcal{R}_{agg}$, resulting from the aggregation, as $r_{agg,1}, \ldots, r_{agg,|\mathcal{R}_{agg}|}$ in a box labelled by $\mathcal{R}_{agg}$.

Inspired by graph aggregation [25], we restrict attention in this paper to aggregation rules in the form of either quota rules or oligarchic rules, defined below in our Bipolar ABA setting.
Quota Rules. These set a quota $q \in N$ as a threshold to accept some set of rules $R \subseteq \boldsymbol{\mathcal{R}}$, i.e., there should be at least q agents that accept the rules in R.

Definition 3 (*Quota Rule*) The *quota rule* F_q, for $q \in N$, is a Bipolar ABA aggregation rule such that, given $\boldsymbol{\mathcal{F}} = \langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, \bar{} \rangle, \ldots, \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, \bar{} \rangle$ and $\boldsymbol{\mathcal{R}} = \bigcup_{i \in N} \mathcal{R}_i$, it holds that $F_q(\boldsymbol{\mathcal{F}}) = \langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \bar{} \rangle$ with $\mathcal{R}_{agg} = \{r \in \boldsymbol{\mathcal{R}} : r \in \bigcap_{j \in N'} \mathcal{R}_j$ for some $N' \subseteq N, |N'| \geq q\}$

The quota q can be any natural number up to, and including, n, but there are several special quotas that are commonly used: *weak majority* has a quota $q = \lfloor \frac{n}{2} \rfloor$; *strict majority* has a quota $q = \lceil \frac{n}{2} \rceil$; *nomination* returns all rules adopted by at least 1 agent, i.e., $q = 1$; and *unanimity* requires all agents to adopt the rules resulting from aggregation, i.e., $q = n$.

Example 3 Assume that $n = 3$ and agents adopt sets of rules $\mathcal{R}_1 = \{\bar{\alpha} \leftarrow \beta\}, \mathcal{R}_2 = \{\alpha \leftarrow \gamma\}$, and $\mathcal{R}_3 = \{\bar{\alpha} \leftarrow \beta,\ \alpha \leftarrow \delta\}$. Using weak majority, the aggregated Bipolar ABA framework has set of rules $\mathcal{R}_{agg} = \{\bar{\alpha} \leftarrow \beta,\ \alpha \leftarrow \gamma,\ \alpha \leftarrow \delta\}$; with strict majority, $\mathcal{R}_{agg} = \{\bar{\alpha} \leftarrow \beta\}$; nomination gives $\mathcal{R}_{agg} = \{\bar{\alpha} \leftarrow \beta,\ \alpha \leftarrow \gamma,\ \alpha \leftarrow \delta\}$, while unanimity returns $\mathcal{R}_{agg} = \{\}$.

Oligarchic Rules. These give relevant agents the power to veto rules. Clearly, oligarchic rules are not fair in that the opinions of agents without veto power are disregarded. However, in some cases they are necessary to avoid conflicts among agents.

Definition 4 (*Oligarchic Rule*) Take some $N_v \subseteq N$, which we refer to as *agents with veto power*. Then the *oligarchic rule* F_o is a Bipolar ABA aggregation rule such that given $\boldsymbol{\mathcal{F}} = \langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, \bar{} \rangle, \ldots, \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, \bar{} \rangle$ and $\boldsymbol{\mathcal{R}} = \bigcup_{i \in N} \mathcal{R}_i$, it holds that $F_o(\boldsymbol{\mathcal{F}}) = \langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \bar{} \rangle$ with $\mathcal{R}_{agg} = \{r \in \boldsymbol{\mathcal{R}} : r \in \bigcap_{j \in N_v} \mathcal{R}_j\}$. If $|N_v| = 1$ then the oligarchic rule is called *dictatorship*.

If all agents have veto powers, then the oligarchic rule coincides with unanimity. Note that if an agent $i \in N$ has *veto powers* then $\mathcal{R}_{agg} \subseteq \mathcal{R}_i$ for $F_o(\boldsymbol{\mathcal{F}}) = \langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \bar{} \rangle$.

Example 4 Assume, as in Example 3, that $n = 3$ and the agents adopt sets of rules $\mathcal{R}_1 = \{\bar{\alpha} \leftarrow \beta\}$, $\mathcal{R}_2 = \{\alpha \leftarrow \gamma\}$, and $\mathcal{R}_3 = \{\bar{\alpha} \leftarrow \beta, \quad \alpha \leftarrow \delta\}$. If agents 1 and 3 are given veto powers, then the aggregated Bipolar ABA framework has set of rules $\mathcal{R}_{agg} = \{\bar{\alpha} \leftarrow \beta\}$. On the other hand, if all agents have veto powers, then $\mathcal{R}_{agg} = \{\}$, as with unanimity.

This paper focuses on quota and oligarchic rules due to their extensive study in the existing literature. In general, these rules are of specific interest because they reflect common practices in real-world decision-making bodies, where majority requirements or veto powers are often employed. These rules are already well-studied in the social choice literature in terms of desirable properties (called axioms) [2, 34]. For instance, oligarchic rules are not *anonymous*, in that they give preferential treatment to certain agents, whereas the quota rules are. Both the quota and oligarchic rules are: *unanimous* (universal acceptance of a rule guarantees that it appears in the aggregated framework); *grounded* (all rules in the aggregated framework originate from the input frameworks); *neutral* (all rules are treated the same by the aggregation method); *independent* (deciding whether to include one rule in the aggregate is not affected by the existence of other rules); and *monotonic* (additional acceptance of a rule in an input framework does not change the decision to include it in the aggregate). In studying these rules in the context of Bipolar ABA aggregation, we aim to provide a further evaluation of their capabilities.

Hereafter we study whether the properties of the agents' Bipolar ABA frameworks, including conflict-freeness, closedness, semantics, and "graph" properties, are preserved in the aggregated Bipolar ABA framework.

Definition 5 (*Preservation*) Let P be a property of Bipolar ABA frameworks. If $\Delta \subseteq \mathcal{A}$ satisfies P in each agents' Bipolar ABA framework $\mathcal{F}_i = \langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, \bar{} \rangle$ (for $i \in N$), then P is *preserved for* Δ in the aggregated Bipolar ABA framework $\mathcal{F}_{agg} = \langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, \bar{} \rangle$ iff Δ satisfies P in $\mathcal{F}_{agg}$.

Example 5 Continuing Example 3, $\{\alpha\}$ is conflict-free in all agents' Bipolar ABA frameworks and in the aggregated Bipolar ABA framework obtained with any of weak majority, strict majority and unanimity. Thus, conflict-freeness is preserved for $\{\alpha\}$ in the aggregated Bipolar ABA framework by any of these aggregation rules here.

We will say that *an aggregation rule* resulting in $\mathcal{F}_{agg}$ *preserves a property* P (or that P *is preserved by the aggregation rule*) to mean that P is preserved in $\mathcal{F}_{agg}$ for every Δ satisfying P, as specified in Definition 5.

5 Preservation results

In this section, we present preservation results for several properties P as summarised in Table 2. Throughout the section, unless specified otherwise, we assume that $\mathcal{F}_{agg} =$

Table 2 Overview of Sect. 5

Properties P	Results
Closedness	Theorem 1
Conflict-freeness	Theorems 2, 3, 4
Admissibility	Theorems 5, 6
Set-stable	Theorem 7
Assumption acceptability	Theorems 11, 12, 13, 14
Preferred, complete, well-founded and ideal extension	Theorems 8, 9, 10
Non-emptiness of the well-founded extension	Theorem 15
Acyclicity	Theorem 16
Coherence	Theorems 17, 18, 19

The "assumption acceptability" properties and the "graph" properties of "acyclicity" and "coherence" are defined later in the section. The exact conditions under which the properties P are preserved are presented in the specific result statement

$\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle$ is the aggregated Bipolar ABA framework resulting from deploying the Bipolar ABA aggregation rules as considered with agents' Bipolar ABA frameworks $\mathcal{F}_1 = \langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, ^- \rangle, \ldots, \mathcal{F}_n = \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, ^- \rangle$.

5.1 Closedness

Closedness is an important property in Bipolar ABA frameworks, made so by the presence of "support" between assumptions (in the form of rules whose head and body are assumptions). Instead, it is not meaningful in AA and is indeed not studied in [11].

Theorem 1 *Every quota and oligarchic rule preserves closedness.*

Proof Assume that $\Delta \subseteq \mathcal{A}$ is closed in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$. By contradiction, assume that Δ is not closed in $\mathcal{F}_{agg}$. Then, there is some $\beta \notin \Delta$ such that $\beta \in Cl(\Delta)$, i.e., there exists $\Delta' \vdash^R \beta$ where $\Delta' \subseteq \Delta$, $R \subseteq \mathcal{R}$. Without loss of generality, $R = \{\beta \leftarrow \delta_1, \ldots, \delta_m \leftarrow \alpha\}$ where $\alpha \in \Delta'$, $m \geq 0$. By definition of quota and oligarchic rules, each $r \in R$ must be adopted by at least one agent. So $\delta_m \leftarrow \alpha$ must appear in *some* agent's $\mathcal{R}_i$. For Δ to be closed in each agent, we must have that $\delta_m \in \Delta$. But then as $\delta_{m-1} \leftarrow \delta_m$ appears in some (possibly different) $\mathcal{R}_j$, we must have $\delta_{m-1} \in \Delta$, etc. Eventually, we have that $\beta \leftarrow \delta_1$ appears in some (possibly different) $\mathcal{R}_k$, but then as $\beta \notin \Delta$, we have that Δ is not closed in $\langle \mathcal{L}, \mathcal{R}_k, \mathcal{A}, ^- \rangle$, which is a contradiction. □

5.2 Conflict-freeness

Conflict-freeness is not always preserved in our setting, as illustrated by the following counter-example.

Example 6 Consider the following scenario (as shown in Fig. 4). Assume there exists three agents with opinions expressed as $\mathcal{F}_i = \langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for $i \in \{1, 2, 3\}$, with $\mathcal{R}_1 = \{\bar{\beta} \leftarrow \delta_1, \delta_1 \leftarrow \delta_2\}$, $\mathcal{R}_2 = \{\delta_1 \leftarrow \delta_2, \delta_2 \leftarrow \alpha\}$ and $\mathcal{R}_3 = \{\delta_2 \leftarrow \alpha, \bar{\beta} \leftarrow \delta_1\}$. Then by the quota rule with $q = 2$ (strict majority), $\mathcal{R}_{agg} = \{\bar{\beta} \leftarrow \delta_1, \delta_1 \leftarrow \delta_2, \delta_2 \leftarrow \alpha\}$, thus the extension $\Delta = \{\alpha, \beta\}$ is conflict-free in each $\mathcal{F}_i$, but is not conflict-free in $\mathcal{F}_{agg}$.

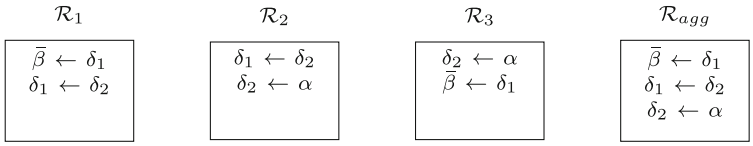


Fig. 4 Counter-example for the preservation of conflict-freeness (Example 6)

Although conflict-freeness is not preserved directly, if we place restrictions on the aggregation rule or impose closedness then this property can be preserved, as shown in the following results.

Theorem 2 *Conflict-freeness is preserved by every oligarchic rule.*

Proof Assume $\Delta \subseteq \mathcal{A}$ is conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$. By contradiction, assume that Δ is not conflict-free in $\mathcal{F}_{agg}$, i.e., for some $\alpha, \beta \in \Delta$, β *attacks* α. Then, by definition of *attacks*, there is some $R \subseteq \mathcal{R}_{agg}$ such that $\beta \vdash^R \bar{\alpha}$. By definition of oligarchic rules, R must appear in the framework of each oligarch. However, this would imply that the same deduction $\beta \vdash^R \bar{\alpha}$ holds, and therefore β *attacks* α holds in the each oligarch's framework. However, this would imply that Δ is conflicting in some $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$, which is a contradiction. □

Theorem 3 *If $\Delta \subseteq \mathcal{A}$ is closed in each agent, then conflict-freeness is preserved for Δ by every quota rule.*

Proof Assume $\Delta \subseteq \mathcal{A}$ is conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$. By contradiction, assume that Δ is not conflict-free in $\mathcal{F}_{agg}$, i.e., for some $\alpha, \beta \in \Delta$, β *attacks* α. Then, by definition of *attacks*, there is some $R = \{\bar{\alpha} \leftarrow \gamma_1, \ldots, \gamma_m \leftarrow \beta\}$, $m \geq 0$. As closedness is preserved, we have that Δ is closed in $\mathcal{F}_{agg}$. Therefore, as $\beta \in \Delta$, we must have that $\gamma_1, \ldots, \gamma_m \subseteq \Delta$. If we take $\bar{\alpha} \leftarrow \gamma_1$, we note that it must appear in some $\mathcal{R}_i$ by definition of the quota rules. However, as $\gamma_1 \in \Delta$ and Δ is closed, $\alpha \in \Delta$ and it is not conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$, thus contradicting our assumption. □

Finally, we can guarantee preservation of conflict-freeness by quota rules, under a bound for the quota, as follows.

Theorem 4 *Let $\mathcal{E} \neq 0$ be the size of the largest (wrt cardinality) minimally (wrt to set inclusion) non-empty conflicting subset(s) of $\bigcup_{i \in N} \mathcal{R}_i$. Then every quota rule with $q > n\frac{\mathcal{E}-1}{\mathcal{E}}$ preserves conflict-freeness.*

Proof Take an arbitrary $\Delta \subseteq \mathcal{A}$ that is conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$. By contradiction, assume that Δ is not conflict-free in $\mathcal{F}_{agg}$, that is, there exists some $R = \{\bar{\beta} \leftarrow \gamma_1, \ldots, \gamma_m \leftarrow \alpha\}$, where $R \subseteq \mathcal{R}_{agg}$, $m \geq 0$ (*wlog*, R is assumed to be minimally conflicting). By definition of the quota rule, each $r \in R$ is adopted by at least q agents, and so it follows that there are at least $|R| \cdot q$ occurrences of rules in R across all the agents in total. By the pigeonhole principle, at least one agent must have accepted at least $\lceil \frac{|R| \cdot q}{n} \rceil$ of the total $|R| \cdot q$ rules. We note that:

i. since $q > n\frac{\mathcal{E}-1}{\mathcal{E}}$, it holds that $\lceil \frac{|R| \cdot q}{n} \rceil > |R|\frac{\mathcal{E}-1}{\mathcal{E}}$;
ii. since $|R| \leq \mathcal{E}$, it holds that $|R|\frac{\mathcal{E}-1}{\mathcal{E}} \geq |R| - 1$;

and, therefore, $\lceil \frac{|R| \cdot q}{n} \rceil > |R| - 1$. However, this means that $\lceil \frac{|R| \cdot q}{n} \rceil \geq |R|$, i.e., some agent must have individually adopted all of the rules in a conflicting subset, thus contradicting our assumption that Δ is conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$. □

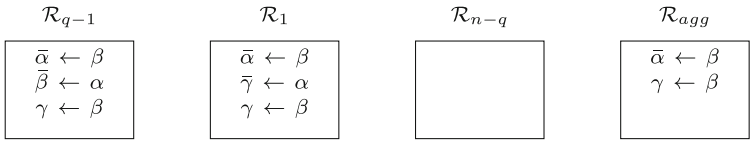


Fig. 5 Counter-example for Theorem 5

Note that in the case of $\mathcal{E} = 0$, there are no conflicts, and so conflict-freeness is trivially preserved.

Intuitively, the largest minimally conflicting subset of $\bigcup_{i \in N} \mathcal{R}_i$ is the largest chain that can be formed across $\bigcup_{i \in N} \mathcal{R}_i$ that leads to a conflict that does not include 'irrelevant' rules.

Example 7 Take $\mathcal{R}_1 = \{\bar{\alpha} \leftarrow \beta\}$, $\mathcal{R}_2 = \{\beta \leftarrow \gamma,\ \bar{\varepsilon} \leftarrow \theta\}$ and $\mathcal{R}_3 = \{\gamma \leftarrow \delta,\ \theta \leftarrow \lambda\}$. Then $\{\bar{\alpha} \leftarrow \beta,\ \beta \leftarrow \gamma,\ \gamma \leftarrow \delta,\ \theta \leftarrow \lambda\}$ is conflicting but not minimal. Both $\{\bar{\alpha} \leftarrow \beta,\ \beta \leftarrow \gamma,\ \gamma \leftarrow \delta\}$ and $\{\bar{\varepsilon} \leftarrow \theta,\ \theta \leftarrow \lambda\}$ are minimally conflicting subsets (the former being the largest).

5.3 Admissible extensions

Theorem 5 *For $|\mathcal{A}| \geq 3$, nomination is the only quota rule that preserves admissibility.*

Proof Assume that $\Delta \subseteq \mathcal{A}$ is admissible for each agent. By contradiction, assume that Δ is not admissible in $\mathcal{F}_{agg}$. As conflict-freeness and closedness are preserved by quota rules by Theorems 1 and 3, there must be some $B \subseteq \mathcal{A} \backslash \Delta$ such that B *attacks* Δ, and Δ does not attack B. As B *attacks* Δ, there exists $R = \{\bar{\alpha} \leftarrow \delta_1, \ldots, \delta_{j-1} \leftarrow \delta_j, \ldots, \delta_m \leftarrow \beta\} \subseteq \mathcal{R}_{agg}$, where $m \geq 1$, $\beta \in B$ and $\alpha \in \Delta$. By the nomination rule, there is at least one agent i such that $\bar{\alpha} \leftarrow \delta_1 \in \mathcal{R}_i$. As Δ is admissible in $\mathcal{F}_i = \langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$, we must have some $\gamma \in \Delta$ such that γ *attacks* δ_1, i.e., there exists some $R' = \{\bar{\delta_1} \leftarrow \gamma_1, \ldots, \gamma_m \leftarrow \gamma\} \subseteq \mathcal{R}_i$ for $m \geq 1$. Then, by the nomination rule, we must have that $R' \subseteq \mathcal{R}_{agg}$. As $\beta \in B$ and B is closed, we must have that $\delta_1, \ldots, \delta_m \subseteq B$. But then as γ *attacks* δ_1 and $\gamma \in \Delta$, $\delta_1 \in B$, we have that Δ *attacks* B, thus contradicting our assumption that Δ does not attack B.

To complete the proof, we need to show that for $|\mathcal{A}| \geq 3$, any other quota rules (besides nomination) do not preserve admissibility. Then, consider the following scenario (shown in Fig. 5), where admissibility is not preserved for an arbitrary quota rule with $q > 1$:

Assume there exists n agents with opinions expressed as $\mathcal{F}_i = \langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for $i \in N$, with $\mathcal{A} = \{\alpha, \beta, \gamma\}$. Suppose $q - 1$ agents have rules $\mathcal{R}_{q-1} = \{\bar{\alpha} \leftarrow \beta, \bar{\beta} \leftarrow \alpha, \gamma \leftarrow \beta\}$, 1 agent has rules $\mathcal{R}_1 = \{\bar{\alpha} \leftarrow \beta, \bar{\gamma} \leftarrow \alpha, \gamma \leftarrow \beta\}$, and the remaining $n - q$ agents have $\mathcal{R}_{N-q} = \emptyset$. Observe that $\Delta = \{\alpha\}$ is admissible in each $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$. However, by quota rule with quota q, we obtain that $\mathcal{R}_{agg} = \{\bar{\alpha} \leftarrow \beta, \gamma \leftarrow \beta\}$, and thus Δ is not admissible in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle$. □

Theorem 6 *For $|\mathcal{A}| \leq 2$, every quota and oligarchic rule preserves admissibility.*

Proof If $|\mathcal{A}| = 1$, the result holds vacuously.
If $|\mathcal{A}| = 2$, assume that $\mathcal{A} = \{\alpha, \beta\}$. Let us consider each case of $\Delta \subseteq \mathcal{A}$:

$\Delta = \{\alpha, \beta\}$. Assume Δ is admissible in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$. By contradiction, suppose Δ is not admissible in $\mathcal{R}_{agg}$. As $\Delta = \mathcal{A}$, clearly any $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle$ will be vacuously closed and defend against all attacks. Thus Δ must not be conflict-free in $\mathcal{R}_{agg}$, accepting some attack $\bar{\beta} \leftarrow \alpha$ and/or $\bar{\alpha} \leftarrow \beta$. However, by definition of quota and oligarchic

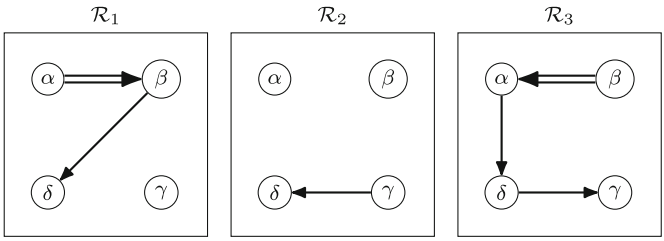


Fig. 6 Counter example for Theorem 7

rules, such attack(s) must be present in at least one agent's framework, which would contradict that assumption that Δ is conflict-free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$.
$\Delta = \{\alpha\}$ (equivalently $\Delta = \{\beta\}$). Assume Δ is admissible in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$. Then, in each $\mathcal{R}_i$, either $r = \bar{\alpha} \leftarrow \beta \notin \mathcal{R}_i$ or $R = \{\bar{\alpha} \leftarrow \beta, \bar{\beta} \leftarrow \alpha\} \subseteq \mathcal{R}_i$. Suppose the latter is the case for p agents (and for the remaining $n - p$ agents we have the former case). If $p < q$, then clearly $r \notin \mathcal{R}_{agg}$. Otherwise, if $p \geq q$ then $R \subseteq \mathcal{R}_{agg}$, and so Δ is attacked but defended. Therefore, every quota and oligarchic rule yields $\{\alpha\}$ as an admissible extension in $\mathcal{F}_{agg}$.
$\Delta = \{\}$. The empty extension is always admissible, thus it will be preserved in all cases. □

5.4 Set-stable extensions

The set-stable semantics for Bipolar ABA frameworks generalises the stable semantic for AAFs to accommodate supports (see [19]). This generalisation though does not affect preservation. Thus, Theorem 7 extends Proposition 5 in [11].

Theorem 7 *Nomination is the only quota rule that preserves set-stability.*

Proof Assume that $\Delta \subseteq \mathcal{A}$ is set-stable in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for all $i \in N$. By Theorems 3 and 1, nomination preserves closedness and conflict-freeness. Therefore, Δ is both closed and conflict-free in $\mathcal{F}_{agg}$. To be set-stable, Δ has to attack the closure of every assumption $\beta \notin \Delta$, i.e. for some $R \subseteq \mathcal{R}_{agg}$ and $\gamma \in Cl(\{\beta\})$, $\{\alpha\} \vdash^R \bar{\gamma}$. This is trivially the case if $\Delta = \mathcal{A}$. For all other Δ, as Δ is set-stable in all agents' frameworks, then $R_i \subseteq \mathcal{R}_i$ must exist, for all $i \in N$, such that $\{\alpha\} \vdash^{R_i} \bar{\gamma}$ for some $\gamma \in Cl(\{\beta\})$ in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$. Thus, by using nomination, $Cl(\{\beta\})$, for $\beta \in \mathcal{A} \setminus \Delta$, is attacked also in $\mathcal{F}_{agg}$.

Other quota rules do not preserve set-stable extensions because, while preserving conflict-freeness and closedness, they do not guarantee that the closure of every assumption not in the extension is attacked. A counter-example can be given as follow: assume three Bipolar ABA frameworks with rules $\mathcal{R}_1 = \{\bar{\delta} \leftarrow \beta, \quad \beta \leftarrow \alpha\}$, $\mathcal{R}_2 = \{\bar{\delta} \leftarrow \gamma\}$, and $\mathcal{R}_3 = \{\bar{\delta} \leftarrow \alpha, \quad \bar{\gamma} \leftarrow \delta, \quad \alpha \leftarrow \beta\}$, as illustrated in Fig. 6. In each framework, the set of assumptions $\{\alpha, \beta, \gamma\}$ is set-stable. However, using any other quota rules with $q > 1$, $\mathcal{R}_{agg} = \{\}$, and $\{\alpha, \beta, \gamma\}$ is not set-stable anymore as the assumption δ is not included in it and it is not attacked either ($Cl(\{\delta\}) = \{\delta\}$ in this example). □

5.5 Preferred, complete, well-founded, and ideal extensions

In this section, we present results for preserving entire extensions under the preferred, complete, well-founded and ideal semantics.

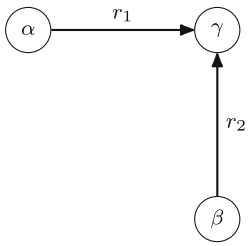


Fig. 7 Disjunctive case for Theorem 9

Theorem 8 *For $|\mathcal{A}| \geq 3$, no oligarchic rule except dictatorship preserves the preferred, complete, well-founded or ideal semantics.*

Proof Let P be any property amongst a given extension being preferred, set-stable, complete, well-founded or ideal. We prove that any such P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$, for $|\mathcal{A}| \geq 3$. Then, the theorem holds by Lemma 2. Consider the example in Fig. 7, where $\mathcal{R} = \{\}$, $r_1 = \bar{\beta} \leftarrow \alpha$, $r_2 = \bar{\beta} \leftarrow \gamma$. Then, P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ as $\Delta = \{\alpha, \gamma\}$ is preferred, set-stable, complete, well-founded and ideal in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$ for all $\mathcal{S} \subseteq \{r_1, r_2\}$ except when $\mathcal{S} = \emptyset$ (where, in this case, $\Delta' = \{\alpha, \beta, \gamma\}$ is the only preferred, set-stable, complete, well-founded and ideal extension). Observe that this example can be extended for all $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ with $|\mathcal{A}| \geq 3$ by adding redundant assumptions. □

Theorem 9 *For $|\mathcal{A}| \geq 3$, the quota rules do not preserve preferred, complete, well-founded or ideal semantics.*

Proof To prove this result we provide a counter-example, showing that for each property the quota rules, as a whole, do not preserve that property for $|\mathcal{A}| \geq 3$.

Let $\mathcal{A} = \{\alpha, \beta, \gamma\}$ and suppose there are 4 agents. Suppose two agents have frameworks with rules $\mathcal{R}_{1,2} = \{\bar{\gamma} \leftarrow \alpha\}$ and the other two agents have $\mathcal{R}_{3,4} = \{\bar{\gamma} \leftarrow \beta\}$. In both sets of frameworks, note that $\{\alpha, \beta\}$ is preferred, complete, well-founded and ideal as γ is defeated by some attack from α or β. Then, if we take any quota rule with $q > 2$, the resulting aggregated framework has rules $\mathcal{R}_{agg} = \{\}$, yielding $\{\alpha, \beta, \gamma\}$ as the only preferred, complete and ideal extension. Thus the property is not preserved. Note that this counter-example can be extended for $|\mathcal{A}| \geq 3$ by adding redundant assumptions. □

Theorem 10 *For $|\mathcal{A}| \leq 2$, every quota and oligarchic rule preserves preferred, complete, well-founded and ideal semantics.*

Proof For $|\mathcal{A}| = 1$, let $\mathcal{A} = \{\alpha\}$. Then, the result holds vacuously as if the agents are all individually rational, then the only extension is $\Delta = \{\alpha\}$.

For $|\mathcal{A}| = 2$, let $\mathcal{A} = \{\alpha, \beta\}$. Then, if $\Delta = \{\alpha\}$ is preferred, complete, well-founded or ideal in all the agents' frameworks, then, for each $i \in N$, if the agents are all individually rational (so that individual assumptions, notably β, are conflict-free), then there must be a deduction $\{\alpha\} \vdash^{R_i} \bar{\beta}$. Thus, any quota and oligarchic rules preserve P. The other cases ($\Delta = \{\beta\}$ and $\Delta = \{\alpha, \beta\}$) can be proven similarly. □

5.6 Assumption acceptability

Assumption acceptability concerns the preferred, complete, set-stable, well-founded and ideal semantics, but at the level of single assumptions rather than full extensions. If an

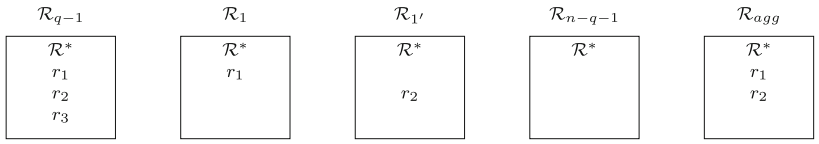


Fig. 8 Distribution of rules for implicative impossibility result

assumption is acceptable in one of those semantics (by belonging to a set of assumptions accepted by the semantics) in each of the agents' Bipolar ABA framework, then preservation amounts to that assumption being still acceptable under the same semantics in the aggregated Bipolar ABA framework.

Definition 6 (*Assumption Acceptability*) An assumption $\alpha \in \mathcal{A}$ is *acceptable* (in a Bipolar ABA framework) under preferred, complete, set-stable, well-founded, or ideal semantics iff there is $\Delta \subseteq \mathcal{A}$ with $\alpha \in \Delta$ such that Δ is (respectively) a preferred, complete, set-stable, well-founded or ideal extension (in the Bipolar ABA framework).

The proofs of preservation results regarding acceptability uses adaptations of results from [25] on *implicative* and *disjunctive* meta-properties to represent impossibility results with dictatorship. We cast these meta-properties for Bipolar ABA frameworks in the following definitions. Note that a rule or set of rules is said to be *based on* $\mathcal{L}$ when the rule or each rule in the set is of the form $\beta \leftarrow \alpha$ or $\bar{\beta} \leftarrow \alpha$ where $\alpha, \beta, \bar{\beta} \subseteq \mathcal{L}$. Further, we henceforth use (in this section and the subsequent) the notation $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ to refer to the set of all frameworks with the same language $\mathcal{L}$, set of assumptions $\mathcal{A}$ and contrary-map $^-$, but different sets of rules $\mathcal{R}$ based on $\mathcal{L}$. Intuitively, the underscore symbol _ serves as a wildcard that can match any set of rules that are based on $\mathcal{L}$.

Definition 7 (*Implicative Properties*) P is *implicative* in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ iff there exists $\mathcal{R}$ and three rules $r_1, r_2, r_3 \notin \mathcal{R}$ based on $\mathcal{L}$ such that for all $\mathcal{S} \subseteq \{r_1, r_2, r_3\}$, $P(\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle)$ iff $\mathcal{S} \neq \{r_1, r_2\}$.

Definition 8 (*Disjunctive Properties*)) P is *disjunctive* in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ iff there exists $\mathcal{R}$ and two rules $r_1, r_2 \notin \mathcal{R}$ based on $\mathcal{L}$ such that for all $\mathcal{S} \subseteq \{r_1, r_2\}$, $P(\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle)$ iff $\mathcal{S} \neq \{\}$.

Lemma 1 *Suppose P is implicative in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then P is not preserved by any quota rule with $1 \leq q < n$.*

Proof As P is implicative, we know that there is a particular $\mathcal{R}^*$ and r_1, r_2, r_3 such that for all $\mathcal{S} \subseteq \{r_1, r_2, r_3\}$ iff $P(\langle \mathcal{L}, \mathcal{R}^* \cup \mathcal{S}, \mathcal{A}, ^- \rangle) \leftrightarrow \mathcal{S} \neq \{r_1, r_2\}$. Consider the following distribution of rules (as outlined in Fig. 8): $q-1$ agents have rules $\mathcal{R}_{q-1} = \mathcal{R}^* \cup \{r_1, r_2, r_3\}$, 1 agent has rules $\mathcal{R}_1 = \mathcal{R}^* \cup \{r_1\}$, 1 agent has rules $\mathcal{R}_{1'} = \mathcal{R}^* \cup \{r_2\}$, and $n-q-1$ agents have rules $\mathcal{R}_{n-q-1} = \mathcal{R}^*$. Here, the agents are distinct and accept precisely the stated rules. Then, after aggregation by a quota rule with quota q, we have that $\mathcal{R}_{agg} = \mathcal{R}^* \cup \{r_1, r_2\}$. However, by implicativeness, we conclude that $\neg P(\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle)$ and so P is not preserved. □

Lemma 2 *Suppose P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then P is not preserved by any oligarchic rule except dictatorship.*

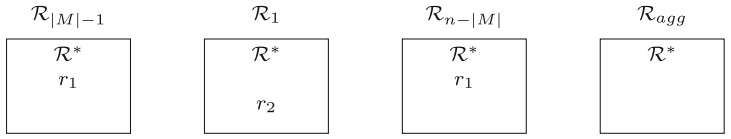
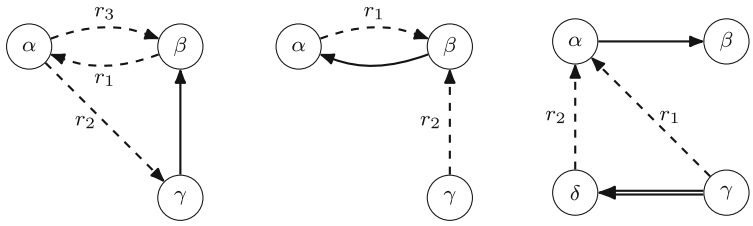


Fig. 9 Distribution of rules for disjunctive impossibility result



(a) Theorem 11 (b) Theorem 12 (c) Theorem 13

Fig. 10 Implicative and disjunctive cases for acceptability theorems

Proof Let $M \subseteq N$ be the oligarchs (thus including unaniminity when $M = N$). As P is disjunctive, we know there is $\mathcal{R}^*$ and r_1, r_2 such that for all $\mathcal{S} \subseteq \{r_1, r_2\}$, $P(\langle \mathcal{L}, \mathcal{R}^* \cup \mathcal{S}, \mathcal{A}, ^- \rangle)$ iff $\mathcal{S} \neq \{\}$. Consider the following distribution of rules (as outlined in Fig. 9): $|M|-1$ oligarchs have rules $\mathcal{R}_{|M|-1} = \mathcal{R}^* \cup \{r_1\}$, 1 oligarch has rules $\mathcal{R}_1 = \mathcal{R}^* \cup \{r_2\}$, and $n - |M|$ agents have rules $\mathcal{R}_{n-|M|} = \mathcal{R}^* \cup \{r_1\}$. Here, the oligarchs/agents are distinct and adopt precisely the stated rules. After aggregation, we have $\mathcal{R}_{agg} = \mathcal{R}^*$. However, by disjunctiveness, we conclude that $\neg P(\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle)$ and so P is not preserved. □

Lemma 3 *Suppose P is implicative and disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then P is preserved by an aggregation rule F iff F is a dictatorship.*

Proof We prove both directions.

($\Leftarrow$): Let $i^* \in N$ be the dictator. Then, assume $P(\langle \mathcal{L}, \mathcal{R}_{i^*}, \mathcal{A}, ^- \rangle)$. By definition of dictatorship, $F(\langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, ^- \rangle, \ldots, \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, ^- \rangle) = \langle \mathcal{L}, \mathcal{R}_{i^*}, \mathcal{A}, ^- \rangle$. Thus, $P(F(\langle \mathcal{L}, \mathcal{R}_1, \mathcal{A}, ^- \rangle, \ldots, \langle \mathcal{L}, \mathcal{R}_n, \mathcal{A}, ^- \rangle))$ as required.

($\Rightarrow$): We prove the contrapositive. Suppose F is not a dictatorship. It is therefore a quota or oligarchic rule. We show that P is not preserved by F in both cases. As P is implicative, we can conclude that P is not preserved by any quota rule with $1 \leq q < n$ by Lemma 1. As P is disjunctive, we can conclude that P is not preserved by any oligarchic rule (except dictatorship) by Lemma 2. □

Theorem 11 *For $|\mathcal{A}| \geq 3$ no quota rule with $1 \leq q < n$ preserves assumption acceptability under preferred, complete, set-stable, well-founded and ideal semantics.*

Proof Let P be acceptability of an assumption α under preferred, complete, set-stable, well-founded, or ideal semantics. We need to prove that for $|\mathcal{A}| \geq 3$, P is implicative in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then, by Lemma 1, the theorem holds.

To show that P is implicative for $|\mathcal{A}| \geq 3$. Let $\mathcal{A} = \{\alpha, \beta, \gamma\}$, $\mathcal{R} = \{\bar{\beta} \leftarrow \gamma\}$, $r_1 = \{\bar{\alpha} \leftarrow \beta\}$, $r_2 = \{\bar{\gamma} \leftarrow \alpha\}$ and $r_3 = \{\bar{\beta} \leftarrow \alpha\}$ (outlined in Fig. 10a). Then consider that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$ *iff* $\mathcal{S} \neq \{r_1, r_2\}$ for all $\mathcal{S} \subseteq \{r_1, r_2, r_3\}$. When $\mathcal{S} \neq \{r_1, r_2\}$, we have that α

is acceptable in all semantics. However, when $\mathcal{S} = \{r_1, r_2\}$, then $\{\alpha, \beta, \gamma\}$ form cyclic attacks so that the assumptions α, β and γ are not acceptable under preferred, complete, set-stable, well-founded and ideal semantics. However, when $\mathcal{S} = \{r_1, r_2, r_3\}$, the addition of r_3 breaks the cyclic attacks, rendering α to be acceptable again. Thus P is implicative in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Observe that this scenario can be extended for any $|\mathcal{A}| \geq 3$ by adding redundant assumptions (that is, assumptions and contraries thereof that do not appear in the head or body of rules in the given framework). Therefore, P is implicative in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ for $|\mathcal{A}| \geq 3$. □

Theorem 12 *For $|\mathcal{A}| \geq 3$ no oligarchic rule except dictatorship preserves assumption acceptability under preferred, complete and set-stable semantics.*

Proof Let P be acceptability of an assumption α under preferred, complete or set-stable semantics. We need to prove that for $|\mathcal{A}| \geq 3$, P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then, by Lemma 2, the theorem holds.

To show that P is disjunctive for $|\mathcal{A}| \geq 3$. Let $\mathcal{A} = \{\alpha, \beta, \gamma\}$, $\mathcal{R} = \{\bar{\alpha} \leftarrow \beta\}$, $r_1 = \{\bar{\beta} \leftarrow \alpha\}$ and $r_2 = \{\bar{\beta} \leftarrow \gamma\}$ (outlined in Fig. 10b). Then consider whether P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$ iff $\mathcal{S} \neq \{\}$, for all $\mathcal{S} \subseteq \{r_1, r_2\}$. If $\mathcal{S} = \{r_1\}$, $\{r_2\}$ or $\{r1,\ r2\}$, then the assumption α is defended by the attack from β, and thus α is acceptable under the five semantics. However, if $\mathcal{S} = \{\}$, then the assumption α is attacked by β and is not defended, thus α is unacceptable under preferred, complete and set-stable semantics. Thus P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Observe that this scenario can be extended for any $|\mathcal{A}| \geq 3$ by adding redundant assumptions. Therefore, P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ for $|\mathcal{A}| \geq 3$. □

Theorem 13 *For $|\mathcal{A}| \geq 4$, no oligarchic rule except dictatorship preserves assumption acceptability under well-founded and ideal semantics.*

Proof Let P be acceptability of an assumption β under well-founded or ideal semantics. We need to prove that for $|\mathcal{A}| \geq 4$, P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then, by Lemma 2, the theorem holds.

To show that P is disjunctive for $|\mathcal{A}| \geq 4$, let $\mathcal{A} = \{\alpha, \beta, \gamma, \delta\}$, $\mathcal{R} = \{\bar{\beta} \leftarrow \alpha,\ \delta \leftarrow \gamma\}$, $r_1 = \{\bar{\alpha} \leftarrow \gamma\}$ and $r_2 = \{\bar{\alpha} \leftarrow \delta\}$ (outlined in Fig. 10c). Then consider that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$ *iff* $\mathcal{S} \neq \{\}$ for all $\mathcal{S} \subseteq \{r_1, r_2\}$. If $\mathcal{S} = \{r_1\}$, $\{r_2\}$ or $\{r1,\ r2\}$, then the assumption β is defended, and thus β is acceptable under the five semantics. However, if $\mathcal{S} = \{\}$, then the assumption β is attacked by α and is not defended, thus β is unacceptable under preferred, complete, set-stable, well-founded and ideal semantics. Thus P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Observe that this scenario can be extended for any $|\mathcal{A}| \geq 4$ by adding redundant assumptions. Therefore, P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ for $|\mathcal{A}| \geq 4$. □

Theorem 14 *For $|\mathcal{A}| \leq 2$, every quota and oligarchic rule preserves assumption acceptability under preferred, complete, set-stable, well-founded and ideal semantics.*

Proof If $|\mathcal{A}| = 1$, the result holds vacuously for rational agents as $\alpha \in \mathcal{A}$ must be accepted under all five semantics for each agent's framework and thus for the aggregated framework for any quota and oligarchic rule.

If $|\mathcal{A}| = 2$, let $\mathcal{A} = \{\alpha, \beta\}$. Assume $\alpha \in \mathcal{A}$ is accepted under those semantics in all agents' frameworks. Then, in each framework, it is the case that either α is not attacked by β, or there exists an attack $r = \bar{\alpha} \leftarrow \beta$ and a corresponding defence $r' = \bar{\beta} \leftarrow \alpha$. Thus, after aggregation, either $r \notin \mathcal{R}_{agg}$ (and so α is acceptable under the given semantics) or $r \in \mathcal{R}_{agg}$. In the latter case, we must also have that $r' \in \mathcal{R}_{agg}$ as r' is accepted in at least in every framework where r is accepted. Therefore, it must also be accepted in the aggregate by the definition of quota and oligarchic rules. □

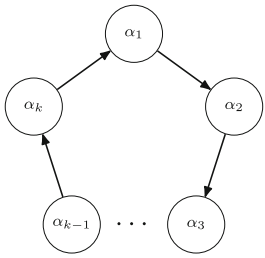


Fig. 11 Graphical illustration of the *k-exclusivity* property (for the proof of Theorem 15)

5.7 Non-emptiness of well-founded extensions

The well-founded extension is guaranteed to exist in a Bipolar ABA framework. However, to make sure that the well-founded extension is not empty, the framework must have at least one assumption whose closure is unattacked. This way, such assumptions are included in all complete extensions, and the intersection always has the assumptions in it. To derive preservation results, we directly adapt a meta-property developed in [11] in the context of Bipolar ABA, as follows.

Definition 9 (*k-exclusivity*) Let P be a property of Bipolar ABA frameworks and $k \geq 1$. P is *k-exclusive* in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ if there exists a set of rules $\mathcal{R}$ based on $\mathcal{L}$ and a set of rules $\mathcal{S} = \{r_1, \ldots, r_k\}$ also based on $\mathcal{L}$, such that if $\mathcal{R} \supseteq \mathcal{S}$ then P does not hold in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$, but if $\mathcal{R} \subset \mathcal{S}$ then P holds in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$.

Thus, there exists a specific framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ amongst $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$, where, for P to hold, the rules in a given $\mathcal{S}$ cannot be adopted together, but a strict subset of them can.

Lemma 4 *Let P be a property of a Bipolar ABA framework which is k-exclusive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ and let F be an aggregation rule. For $k \geq n$, P is preserved by F only if at least one of the n agents has veto power under F.*

Proof We prove the contrapositive. Assume that none of the agents have veto powers under F. Then, F must be a quota rule with $1 \leq q < n$ (consider that unanimity gives all agents veto powers). We show that P is not preserved.

As P is k-exclusive, there exists a distinguished set of rules $\mathcal{S} = \{r_1, \ldots, r_k\}$ with $k \geq n$. Consider the following distribution of rules. For each agent $i \in N$, we ensure that they adopt precisely $\mathcal{S} \setminus \{r_i\}$. As $k \geq n$, $S \nsubseteq \mathcal{R}_i$ for any $i \in N$. As a result, P holds in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$ for each $i \in N$ due to the k-exclusivity property. However, as each $r \in \mathcal{S}$ appears in at least $n-1$ agents, then any quota rule with $1 \leq q < n$ will result in $\mathcal{S} \subseteq \mathcal{R}_{agg}$. Then, by k-exclusivity, it holds that P does not hold in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle$. □

Theorem 15 *If $|\mathcal{A}| \geq n$, then any aggregation rule F that preserves non-emptiness of the well-founded extension must give at least one agent veto powers.*

Proof Let P be the Bipolar ABA framework property of non-emptiness of the well-founded extension. We show that, for $|\mathcal{A}| \geq n$, P is *k-exclusive* in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ with $k = |\mathcal{A}|$. Let $\mathcal{A} = \{\alpha_1, \ldots, \alpha_k\}$. Assume that $\mathcal{S}$ consists of all rules $\bar{\alpha}_{i+1} \leftarrow \alpha_i$ for $1 \leq i < |\mathcal{A}|$, as well as $\bar{\alpha}_1 \leftarrow \alpha_k$, as illustrated in Fig. 11. This $\mathcal{S}$ fits the definition of *k-exclusive*. Indeed, for

any Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$, if $\mathcal{S} \subseteq \mathcal{R}$, then the well-founded extension is empty due to the cyclic attacks. However, if only a strict subset $\mathcal{R} \subset \mathcal{S}$ is adopted, then the well-founded extension is not empty as the closure of at least one assumption is not attacked. Thus, by Lemma 4, P is preserved when at least one agent has veto power to prevent cyclic attacks. □

5.8 Acyclicity

We consider the *acyclicity* property of Bipolar ABA frameworks, where a Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ is acyclic if there does not exist any *cyclic attacks* among the assumptions in $\mathcal{A}$, amounting to a chained connection between some of the assumptions in $\mathcal{A}$, as follows:

Definition 10 (*Cyclic Attacks*) The rule set $\mathcal{R}$ in $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ contains *cyclic attacks* if there exists $\{\overline{\alpha_1} \leftarrow \alpha_2, \overline{\alpha_2} \leftarrow \alpha_3, \ldots, \overline{\alpha_k} \leftarrow \alpha_1\} \subseteq \mathcal{R}$ for $\alpha_i \in \mathcal{A}$ for all $i \in \{1, \ldots, k\}$ and $k \geq 2$.

We use *k-exclusivity* to also deal with cyclic attacks. The following Theorem 16 is extended from Theorem 8 in [11] in that supports are also considered.

Theorem 16 *For $|\mathcal{A}| \geq n$, at least one agent must have veto power to preserve acyclicity.*

Proof Let P be acyclicity. We show that P is *k-exclusive*.

Consider $\mathcal{S}$ as in the proof of Theorem 15. This $\mathcal{S}$ fits the definition of *k-exclusivity*. Indeed, if $\mathcal{S} \subseteq \mathcal{R}$, then the cyclic attacks remain in the Bipolar ABA framework of interest. However, if only a subset of $\mathcal{S}$ is adopted, then the cyclic attacks are broken because at least one rule that connects the cycle disappears. Therefore, by Lemma 4, P is preserved when at least one agent has veto power. □

Note that the addition of supports cannot transform an acyclic Bipolar ABA framework into a cyclic one.

Example 8 Let $k = |\mathcal{A}|$, $\mathcal{A} = \{\alpha_1, \ldots, \alpha_k\}$, and $\mathcal{S} = \{\overline{\alpha}_{i+1} \leftarrow \alpha_i \ : \ 1 \leq i < |\mathcal{A}|\}$. Then the rules in $\mathcal{S}$ are acyclic and, if a support $\alpha_1 \leftarrow \alpha_k$ or $\alpha_k \leftarrow \alpha_1$ is added, for example, then they will remain acyclic.

5.9 Coherence

Coherence is the general property that the extensions of two or more semantics coincide for a given Bipolar ABA framework. In this paper we focus on coherence with respect to the set-stable and preferred semantics:

Definition 11 (*Coherence*) A Bipolar ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ is *coherent* if the preferred and set-stable extensions coincide. That is, for each $\Delta \subseteq \mathcal{A}$, it holds that Δ is preferred if and only if Δ is set-stable.

We already know that, if a set of assumptions is set-stable, then it is preferred as well (see Proposition 3 in [19]): for coherence to hold, preferred extensions need to be set-stable too. Our next preservation result extends Theorem 9 in [11] and shows that, in order to preserve (our form of) coherence, the aggregation rule must be dictatorial. The proof for the result uses the concept of implicativeness and disjunctiveness introduced in Sect. 5.6.



(a) Implicative case (b) Disjunctive case

Fig. 12 Implicative and disjunctive cases for Theorem 17

Theorem 17 *For $|\mathcal{A}| \geq 3$, the only aggregation rule preserving coherence is dictatorship.*

Proof Let P be coherence. We need to prove that, for $|\mathcal{A}| \geq 3$, P is implicative and disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$.

To show that P is implicative, let $\mathcal{A} = \{\alpha, \beta, \gamma\}$, $\mathcal{R} = \{\bar{\alpha} \leftarrow \gamma\}$, $r_1 = \{\bar{\beta} \leftarrow \alpha\}$, $r_2 = \{\bar{\gamma} \leftarrow \beta\}$, and $r_3 = \{\bar{\alpha} \leftarrow \beta\}$, as illustrated in Fig. 12a. Now consider the effect of $\mathcal{S} \subseteq \{r_1, r_2, r_3\}$ in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$. If $\mathcal{S} \neq \{r_1, r_2\}$, then preferred and set-stable extensions coincide. However, when $\mathcal{S} = \{r_1, r_2\}$, we note that the preferred extension $\{\}$, which is maximally admissible due to the presence of cyclic attacks, is not set-stable as the empty set of assumptions cannot attack any assumption. When $\mathcal{S} = \{r_1, r_2, r_3\}$, the addition of r_3 breaks the cyclic attacks, making the framework coherent again. Thus, there exists a set $\mathcal{R}$ of rules and three rules r_1, r_2, r_3 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$ iff $\mathcal{S} \neq \{r_1, r_2\}$, for all $\mathcal{S} \subseteq \{r_1, r_2, r_3\}$. Observe that this example can be extended for any $|\mathcal{A}| \geq 3$ by adding redundant assumptions. Thus, P is an implicative in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ for $|\mathcal{A}| \geq 3$.

To show that P is disjunctive, let $\mathcal{A} = \{\alpha, \beta, \gamma\}$, $\mathcal{R} = \{\bar{\alpha} \leftarrow \gamma, \bar{\beta} \leftarrow \alpha, \bar{\gamma} \leftarrow \beta\}$, $r_1 = \{\bar{\gamma} \leftarrow \alpha\}$, and $r_2 = \{\bar{\alpha} \leftarrow \beta\}$, as illustrated in Fig. 12b. Now consider the cases of $\mathcal{S} \subseteq \{r_1, r_2\}$ in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$. If $\mathcal{S} = \{r_1\}$, $\{r_2\}$, or $\{r_1, r_2\}$, the preferred and set-stable extensions coincide. However, if $\mathcal{S} = \{\}$, the preferred extension $\{\}$ (which is maximally admissible due to the presence of cyclic attacks) is not set-stable. Thus the framework is not coherent in this case. Therefore, there exists a set of rules $\mathcal{R}$ and two rules r_1, r_2 such that P holds in $\langle \mathcal{L}, \mathcal{R} \cup \mathcal{S}, \mathcal{A}, ^- \rangle$ iff $\mathcal{S} \neq \{\}$, for all $\mathcal{S} \subseteq \{r_1, r_2\}$. Again, this example can be extended for any $|\mathcal{A}| \geq 3$ by adding redundant assumptions. Hence, P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$ for $|\mathcal{A}| \geq 3$.

As P is proven to be both implicative and disjunctive, then, by Lemma 3, for P to be preserved, the aggregation rule must be dictatorial. □

Theorem 18 *For $|\mathcal{A}| \leq 2$, every quota and oligarchic rule preserves coherence.*

Proof For $|\mathcal{A}| = 1$, let $\mathcal{A} = \{\alpha\}$. If the agents are all individually rational, then $\mathcal{R}_i = \emptyset$ (for all $i \in N$) and hence $\{\alpha\}$ is coherent in each agent as it is closed, conflict-free, preferred and set-stable (as well as admissible, complete, well-founded, and ideal). Therefore, using any quota or oligarchic rule, coherence also holds for the aggregated framework.

For $|\mathcal{A}| = 2$, let $\mathcal{A} = \{\alpha, \beta\}$. By contradiction, suppose coherence does not hold in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle$. As every set-stable extension is preferred, it must be the case that a preferred extension Δ is not set-stable. Thus, there must be some $\gamma \in \mathcal{A} \backslash \Delta$ such that Δ does not attack $Cl(\gamma)$. Suppose then, without loss of generality, that $\Delta = \{\alpha\}$ and $\mathcal{A} \setminus \Delta = \{\beta\}$. As Δ is preferred, it must defend against every potential attack. Thus, we conclude that β does not attack Δ, or, otherwise, Δ would be set-stable. Further, as Δ is preferred, it is maximally admissible. Thus the set $\{\alpha, \beta\}$ must not be admissible. Clearly, this set is vacuously closed

and defends against external attacks. Therefore, it must not be conflict-free. As there are no attacks between α and β, and it holds that $\Delta = \{\alpha\}$ is admissible, it must be the case that $r = \bar{\beta} \leftarrow \beta \in \mathcal{R}_{agg}$. However, as all aggregation rules are grounded (i.e., at least one agent must have accepted r by definition of quota and oligarchic rules), $r \in \mathcal{R}_i$ for some $i \in N$. However, as $\{\beta\}$ is not conflict free in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$, this would contradict the assumption that each of the agent's frameworks is rational. □

Evidently, coherence is hard to preserve. However, note that the impossibility result in Theorem 17 relies on counter-examples where the membership of the empty extension in the set-stable and preferred extensions does not coincide, leading to incoherence.

If we disregard the empty extension in our definition of coherence, we obtain more positive results. This guarantees preservation when the number of assumptions is 3 of fewer (when combining the following Theorem 19 with Theorem 18).

Theorem 19 *For $|\mathcal{A}| = 3$, every quota rule with $q > \frac{n}{2}$ and oligarchic rule preserves coherence between non-empty extension.*

Proof Let $\mathcal{A} = \{\alpha, \beta, \gamma\}$. By contradiction, suppose coherence does not hold in $\langle \mathcal{L}, \mathcal{R}_{agg}, \mathcal{A}, ^- \rangle$. As every set-stable extension is preferred, it must be the case that a preferred extension Δ is not set-stable. Thus, there must be some $\varepsilon \in \mathcal{A} \setminus \Delta$ such that Δ does not attack $Cl(\varepsilon)$. We consider the different possibilities of $\Delta \subseteq \mathcal{A}$ in turn (however note that the cases $\Delta = \{\beta\}$, $\{\gamma\}$, $\{\alpha, \gamma\}$, and $\{\beta, \gamma\}$ are equivalent in reasoning to the cases of the same cardinality of Δ, so it suffices to reason about the following cases).

- $\Delta = \emptyset$. This case is not considered as we ignore coherence of the empty extension.
- $\Delta = \{\alpha\}$. Suppose that $\varepsilon = \beta$ (the same reasoning holds for $\varepsilon = \gamma$). Then, we show that any such $\mathcal{R}_{agg}$ where Δ is preferred but not set-stable must result in some $R \subseteq \mathcal{R}_{agg}$ where either $R = \{\bar{\beta} \leftarrow \gamma,\ \beta \leftarrow \gamma\}$ or $\{\bar{\gamma} \leftarrow \beta,\ \gamma \leftarrow \beta\}$.
  As Δ is preferred, all $\Delta' \supset \Delta$ must be not be admissible. Consider such $\Delta' = \{\alpha, \beta, \gamma\}$. Clearly Δ' is closed and vacuously defends against all external attacks. Thus it must not be conflict-free. So, there exists some attack between the assumptions in Δ, namely at least one of: $\bar{\beta} \leftarrow \alpha$; $\bar{\alpha} \leftarrow \beta$; $\bar{\gamma} \leftarrow \alpha$; $\bar{\gamma} \leftarrow \beta$; $\bar{\alpha} \leftarrow \gamma$; $\bar{\beta} \leftarrow \gamma$.
  Attacks $\bar{\beta} \leftarrow \alpha$ and $\bar{\alpha} \leftarrow \beta$. First, we note that the attacks $\bar{\beta} \leftarrow \alpha$ and $\bar{\alpha} \leftarrow \beta$ cannot be present, as the former would result in Δ being set-stable (contradicting our assumption), and the latter would result in some counter-attack on β due to the fact that Δ is preferred (and thus defends against all attacks), which would also contradict the assumption that Δ is not set-stable.
  Attack $\bar{\gamma} \leftarrow \alpha$. As $\{\alpha, \beta\}$ is not admissible, and we have already reasoned that no attacks between α and β are possible (in the case *Attacks $\bar{\beta} \leftarrow \alpha$ and $\bar{\alpha} \leftarrow \beta$*), this set must not be closed or must not defend against all attacks. In the first case, this implies that $\gamma \leftarrow \beta$ is present ($\gamma \leftarrow \alpha$ cannot be present as $\{\alpha\}$ is assumed to be closed). However, this would imply, in combination with $\bar{\gamma} \leftarrow \alpha$, that α attacks $Cl(\beta)$, which would contradict our assumption that Δ is not set-stable. In the second case, if we consider any attack from γ to Δ, α would defend against this attack by $\bar{\gamma} \leftarrow \alpha$, thus the defensive criteria of admissibility would be met.
  Attack $\bar{\gamma} \leftarrow \beta$. As $\{\alpha, \beta\}$ is not admissible, it must not be closed (this set cannot be conflicting as we have already reasoned in the case *Attacks $\bar{\beta} \leftarrow \alpha$ and $\bar{\alpha} \leftarrow \beta$*). Therefore, the support $\gamma \leftarrow \beta$ must be present (consider that $\gamma \leftarrow \alpha$ cannot be present as $\{\alpha\}$ is closed). Thus if we consider this attack, it implies $R = \{\bar{\gamma} \leftarrow \beta,\ \gamma \leftarrow \beta\} \subseteq \mathcal{R}_{agg}$.
  Attack $\bar{\alpha} \leftarrow \gamma$. As $\{\alpha\}$ is admissible, it must defend against this attack from γ. It could

perform this defence by attacking the closure of γ, such that $\beta \leftarrow \gamma$ and $\bar{\beta} \leftarrow \alpha$ are both present in $\mathcal{R}_{agg}$, but this would cause $\{\alpha\}$ to be set-stable due to the attack on β. Thus we can assume that $\bar{\gamma} \leftarrow \alpha$ is present in $\mathcal{R}_{agg}$. Then, however, as $\{\alpha, \beta\}$ is not admissible, it must not be closed (it cannot be conflicting for the same reasons in the *Attacks* $\bar{\beta} \leftarrow \alpha$ and $\bar{\alpha} \leftarrow \beta$ case, and the defensive condition cannot be violated as $\bar{\gamma} \leftarrow \alpha$ would defend against any potential attack by γ). As $\{\alpha\}$ is closed, the support $\gamma \leftarrow \alpha$ cannot be present. Thus $\gamma \leftarrow \beta$ must be present. However, this would imply that α attacks $Cl(\beta)$, which would contradict our assumption that $\{\alpha\}$ is not set-stable.

Attack $\bar{\beta} \leftarrow \gamma$. As $\{\alpha, \gamma\}$ is not admissible, it must not be closed, conflict-free or defend against all attacks. Let us consider each case. If $\{\alpha, \gamma\}$ is not closed, it must imply that the support $\beta \leftarrow \gamma$ is present (as $\{\alpha\}$ is closed, $\beta \leftarrow \alpha$ cannot be present). In this case $R = \{\bar{\beta} \leftarrow \gamma,\ \beta \leftarrow \gamma\} \subseteq \mathcal{R}_{agg}$. In the case that $\{\alpha, \gamma\}$ does not defend against all attacks, we note that this is not possible, as the attack $\bar{\beta} \leftarrow \gamma$, which we already assumed to be present, would defend against any potential attack from β. Thus the set must be conflicting. Thus let us consider the inclusion of two potential attacks that would cause this conflict: $\bar{\gamma} \leftarrow \alpha$ and $\bar{\alpha} \leftarrow \gamma$.

Attack $\bar{\gamma} \leftarrow \alpha$. Recall that $\{\alpha, \beta\}$ is not admissible. Therefore the set must not be closed, conflict-free or not defend against all attacks. If the set is not closed, $\gamma \leftarrow \beta$ must be present ($\gamma \leftarrow \alpha$ cannot be present as $\{\alpha\}$ is assumed to be closed). However, this would imply that α attacks $Cl(\beta)$ which would contradict the assumption that $\{\alpha\}$ is not set-stable. For the same reason as in the *Attacks* $\bar{\beta} \leftarrow \alpha$ and $\bar{\alpha} \leftarrow \beta$ case, we know that this set cannot be conflicting, as it would ultimately imply that $\{\alpha\}$ is not set-stable too. Finally, we also know that $\{\alpha, \beta\}$ already defends against all attacks, as the attack $\bar{\gamma} \leftarrow \alpha$ would defend against $\bar{\gamma} \leftarrow \alpha$.

Attack $\bar{\alpha} \leftarrow \gamma$. Consider that $\{\alpha\}$ is assumed to be admissible, so it must defend itself from this attack from γ. It must do so by directly attacking γ, as attacking the closure of γ through attacking β would violate the assumption that $\{\alpha\}$ is not set-stable. Thus we can assume that $\bar{\gamma} \leftarrow \alpha$ is present. Now consider that $\{\alpha, \beta\}$ is inadmissible. Thus it must not be closed, not conflict-free or not defend against all attacks. If the set is not closed, then $\gamma \leftarrow \beta$ must be present ($\{\alpha\}$ is closed so $\gamma \leftarrow \alpha$ cannot be present). However, this would imply that α attacks $Cl(\beta)$, and thus would violate the assumption that $\{\alpha\}$ is not set-stable. If $\{\alpha, \beta\}$ is not conflict-free, this would again violate the assumption that $\{\alpha\}$ is not set-stable. Finally, $\{\alpha, \beta\}$ already defends against any external attack from γ as we have assumed that the attack $\bar{\gamma} \leftarrow \alpha$ is present.

Thus it has been shown in all cases that either a contradiction is derived, or $R = \{\bar{\beta} \leftarrow \gamma,\ \beta \leftarrow \gamma\}$ or $\{\bar{\gamma} \leftarrow \beta,\ \gamma \leftarrow \beta\}$ is present in $\mathcal{R}_{agg}$. In the latter case ($R \subseteq \mathcal{R}_{agg}$), now consider both quota (with $q > \frac{n}{2}$) and oligarchic rules. By definition of the quota rules, each $r \in R$ must be accepted by at least q agents, and so it follows that there are at least $|R| \cdot q$ occurrences of rules R across all the agents in total. By the pigeonhole principle, at least one agent must have accepted at least $\lceil \frac{|R| \cdot q}{n} \rceil$ of the total $|R| \cdot q$ rules. As $|R| = 2$ and $q > \frac{n}{2}$, it holds that $\lceil \frac{|R| \cdot q}{n} \rceil > 1$. Thus at least one agent must have individually adopted both rules in R. Similarly, by definition of oligarchic rules, all agents must have adopted all the rules in R. In whichever case, the framework of the agents who adopted R would be irrational, as either $Cl(\{\gamma\})$ or $Cl(\{\beta\})$ would not be conflict-free.

– $\Delta = \{\alpha, \beta\}$. Here it must be the case that $\varepsilon = \gamma$. Then consider that as Δ is preferred, every $\Delta' \supset \Delta$ must not be admissible. Then $\Delta' = \{\alpha, \beta, \gamma\}$ is not admissible. Clearly this Δ' is closed and vacuously defends against external attacks. Therefore, it must be conflicting. By assumption, Δ is preferred, so it must not be conflicting. Nor does Δ

attack $\{\gamma\}$, as, otherwise, Δ would be set-stable. Similarly, $\{\gamma\}$ does not attack Δ, as the fact that Δ is preferred would also introduce some defensive attack, making Δ set-stable. Thus, for Δ' to be inadmissible, $\{\gamma\}$ must attack $\{\gamma\}$, and, as such, $\bar{\gamma} \leftarrow \gamma \in \mathcal{R}_{agg}$. By definition of quota and oligarchic rules, $\bar{\gamma} \leftarrow \gamma$ must be adopted by some individual agent. However, this contradicts the assumption that each of the agent's frameworks are rational.

– $\Delta = \{\alpha, \beta, \gamma\}$. In this case, clearly no such ε could exist, as $\mathcal{A} \backslash \Delta = \emptyset$.

Therefore, as we derive a contradiction in every case, the theorem holds. □

5.10 Discussion

Our results highlight the difficulty in preserving properties, often imposing significant restrictions. We first focus on positive results, which we consider as those guaranteeing preservation when using the quota rules (which we considered to be fair in accordance with *May's Theorem* [35] as they meet the criteria of neutrality and anonymity). Then, we turn to negative results, where preservation can only be guaranteed by using an oligarchic or dictatorial rule. Although the use of such rules is not ideal (as they ignore most opinions), we deem preservation under these rules better than not being able to preserve the properties at all. Finally, we discuss how our results relate to those in [31].

Positive results. In terms of positive results, closedness is preserved by all quota (and oligarchic) rules (see Theorem 1). If we assume that a particular extension is closed in each agent's framework, we can also guarantee the preservation of conflict-freeness of the extension by the quota rules (see Theorem 3). However, without such an assumption, we see that restrictions on the quota q must be imposed to guarantee preservation, as in Theorem 4, which states that, as the overall number of conflicting arguments increases across agents, the quota will have to be increased accordingly to avoid conflicts in the aggregated framework.

In terms of preserving acceptability (both of an extension and of a single assumption), we can only achieve positive results (w.r.t. the use quota rules) when the number of assumptions is restricted to at most two (such as in Theorems 6, 14 and 10). Similarly, two or less assumptions are required to guarantee the preservation of coherence, as shown in Theorem 18.

Negative results. In terms of negative results, we showed that for a significant number of properties, dictatorship is required to preserve them when the number of assumptions is greater than two (with slight variations in this bound). This includes both the acceptability of assumptions in an extension (Theorems 11, 12 and 13), as well as the preservation of entire extensions (such as in Theorems 5, 8 and 9). In practical terms, this implies that we cannot provide any guarantees about the ability of quota or oligarchic rules to preserve these properties in most cases, as it is often the case that there are more than two alternatives (i.e. assumptions) at play.

We also proved results on the non-emptiness of the well-founded extension and acyclicity. We prove that both properties are preserved when at least one agent has veto power and the number of assumptions is greater than or equal to the number of agents (Theorems 15 and 16). This unique constraint is meant to avoid cyclic relationships.

Finally, we also showed results for the coherence property. The results followed a pattern similar to that before, with dictatorship needed to preserve coherence for more than two assumptions (Theorem 17), but all quota and oligarchic rules suffice when the number of assumptions is limited to at most two (Theorem 18).

Relations with [31]. Our results correct and extend the results in [31]. We stress here the main differences between the results we proved and the ones given therein, following the

Table 3 Comparison of our results with those in [31]

Result	Description
THEOREM 4.1.	Every quota rule and oligarchic rule preserves conflict-freeness
THEOREM 4.2.	Every quota rule and oligarchic rule preserves closedness. (THEOREM 1)
THEOREM 2	Conflict-freeness is preserved by every oligarchic rule
THEOREM 3	If $\Delta \subseteq \mathcal{A}$ is closed in each agent, then conflict-freeness is preserved in Δ by every quota rule
THEOREM 4	Let $\mathcal{E} \neq 0$ be the size of the largest (*wrt* cardinality) minimally (*wrt* to set inclusion) non-empty conflicting subset(s) of $\bigcup_{i \in N} \mathcal{R}_i$. Then every quota rule with $q > n\frac{\mathcal{E}-1}{\mathcal{E}}$ preserves conflict-freeness
THEOREM 4.3.	For $\lvert\mathcal{A}\rvert \geq 4$, nomination is the only quota rule that preserves admissibility
THEOREM 4.4.	For $\lvert\mathcal{A}\rvert \leq 3$, every quota rule and oligarchic rule preserves admissibility
THEOREM 5	For $\lvert\mathcal{A}\rvert \geq 3$, nomination is the only quota rule that preserves admissibility
THEOREM 6	For $\lvert\mathcal{A}\rvert \leq 2$, every quota and oligarchic rule preserves admissibility
THEOREM 4.5.	Nomination is the only quota rule that preserves set-stable extensions. (THEOREM 7)
LEMMA 1.	Let a Bipolar ABA framework property P be implicative in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$, for each $i \in N$. Then, unanimity preserves P
LEMMA 2.	Let a Bipolar ABA framework property P be implicative and disjunctive in $\langle \mathcal{L}, \mathcal{R}_i, \mathcal{A}, ^- \rangle$, for each $i \in N$. Then, the only Bipolar ABA aggregation rule that preserves P is dictatorship
LEMMA 1	Suppose P is implicative in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then P is not preserved by any quota rule with $1 \leq q < n$
LEMMA 2	Suppose P is disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then P is not preserved by any oligarchic rule (except dictatorship)
LEMMA 3	Suppose P is implicative and disjunctive in $\langle \mathcal{L}, _, \mathcal{A}, ^- \rangle$. Then P is preserved by an aggregation rule F iff F is a dictatorship
THEOREM 4.6.	For $\lvert\mathcal{A}\rvert \geq 4$, the only Bipolar ABA aggregation rule that preserves the acceptability of an assumption under preferred, complete, set-stable, well-founded, or ideal semantics is dictatorship
THEOREM 4.7.	For $\lvert\mathcal{A}\rvert = 3$, majority, unanimity, and oligarchic rules preserve assumption acceptability under preferred, complete, set-stable, well-founded, and ideal semantics
THEOREM 14	For $\lvert\mathcal{A}\rvert \geq 3$ no quota rule with $1 \leq q < n$ preserves assumption acceptability under preferred, complete, set-stable, well-founded and ideal semantics
THEOREM 11	For $\lvert\mathcal{A}\rvert \geq 3$ no oligarchic rule (except dictatorship) preserves assumption acceptability under preferred, complete and set-stable semantics
THEOREM 12	For $\lvert\mathcal{A}\rvert \geq 4$, no oligarchic rule (except dictatorship) preserves assumption acceptability under well-founded and ideal semantics
THEOREM 4.8.	For $\lvert\mathcal{A}\rvert \leq 2$, every quota and oligarchic rule preserves the acceptability of an assumption under preferred, complete, set-stable, well-founded, and ideal semantics. (THEOREM 14)
THEOREM 4.9.	For $\lvert\mathcal{A}\rvert \geq 5$, the only Bipolar ABA aggregation rule that preserves preferred, complete, well-founded, and ideal semantics is dictatorship
THEOREM 4.10.	For $\lvert\mathcal{A}\rvert = 3$ and $\lvert\mathcal{A}\rvert = 4$, quota and oligarchic rules do not preserve preferred, complete, well-founded, and ideal semantics
THEOREM 8	For $\lvert\mathcal{A}\rvert \geq 3$, no oligarchic rule (except dictatorship) preserves the preferred, complete, well-founded or ideal semantics

Table 3 continued

Result	Description
THEOREM 9	For $\|\mathcal{A}\| \geq 3$, the quota rules do not preserve preferred, complete, well-founded or ideal semantics
THEOREM 4.11.	For $\|\mathcal{A}\| \leq 2$, every quota and oligarchic rule preserves preferred, complete, well-founded, and ideal semantics. (THEOREM 10)
LEMMA 3.	Let P be a k-exclusive property of Bipolar ABA framework. For $k \geq N$, where N is the number of agents, P is preserved if at least one of the N agents has veto power
LEMMA 4	Let P be a property of a Bipolar ABA framework which is k-exclusive in $\langle \mathcal{L}, _, \mathcal{A}, \overline{\ } \rangle$. For $k \geq n$, P is preserved by F only if at least one of the n agents has veto power under F
THEOREM 4.12.	For $\|\mathcal{A}\| \geq N$, at least one agent must have veto power to preserve the non-emptiness of the well-founded extension
THEOREM 15	If $\|\mathcal{A}\| \geq n$, then any aggregation rule F that preserves non-emptiness of the well-founded extension must give at least one agent veto powers
COROLLARY 1.	For $\|\mathcal{A}\| \geq N$, at least one agent must have veto power to preserve acyclicity. (THEOREM 16)
THEOREM 4.13.	For $\|\mathcal{A}\| \geq 4$, the only aggregation rule preserving coherence is dictatorship
THEOREM 17	For $\|\mathcal{A}\| \geq 3$, the only aggregation rule preserving coherence is dictatorship
THEOREM 4.14.	For $\|\mathcal{A}\| = 2$ or $\|\mathcal{A}\| = 3$, the unanimity rule is the only quota rule that preserves coherence
THEOREM 4.15.	For $\|\mathcal{A}\| = 1$, every quota rule and oligarchic rule preserve coherence
THEOREM 18	For $\|\mathcal{A}\| \leq 2$, every quota and oligarchic rule preserves coherence
THEOREM 19	For $\|\mathcal{A}\| = 3$, every quota rule with $q > \frac{n}{2}$ and oligarchic rule preserves coherence between non-empty extension

Here we indicate (with colours) the results in [31] that are valid, invalid, valid with inconsistencies (in proof), in conjunction with theorems in this paper that are novel in comparison (Color table online)

same structure as in [31]. The differences are summarised in Table 3. In a nutshell, we rectified inconsistencies in the proofs of some results in [31]; we dropped some results that we proved invalid by identifying counter-examples; we introduced some novel results, some of which by refining the invalid results in [31] with further restrictions. Related theorems are grouped together, such that revised theorems are listed after corresponding theorems that were identified as incorrect or inconsistent. Theorems that are stated in the same wording in this paper as in [31], reference the corresponding theorem at the end of the description. For instance, Theorem 4.5. in [31] corresponds to Theorem 7.

6 Illustrative example: treating long COVID

In this section we illustrate how the notions introduced in this paper may be used in a real world setting using an example in the medical domain. In this example a General Practitioner (GP) is attempting to determine how to treat an outpatient with long COVID (post-COVID syndrome), a multifaceted, post-viral condition with a variety of effects on different bodily systems, e.g. the respiratory and cardiovascular systems. The GP thus considers the advice of six specialists, along with the outpatient's own opinion, on six possible treatments. Thus, there are seven agents: the outpatient and the specialists. For each treatment, the GP aggre-

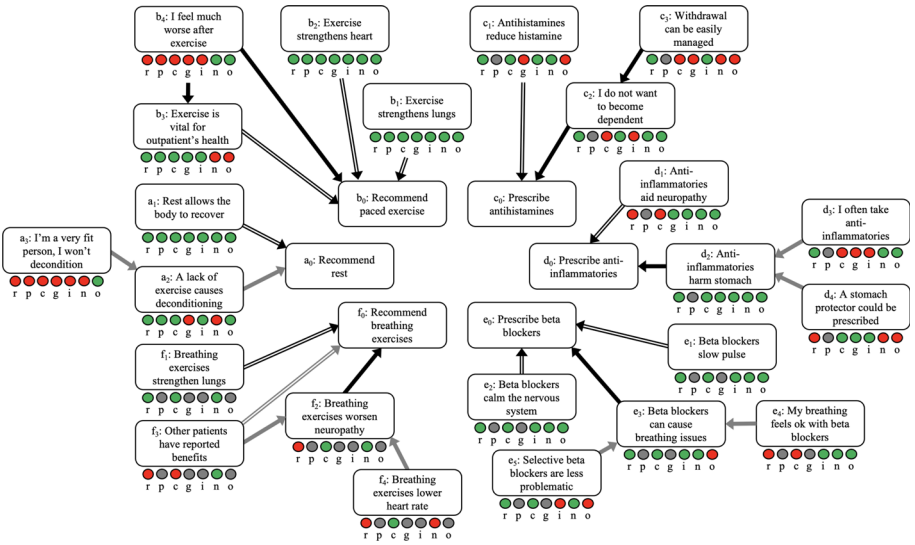


Fig. 13 A combined representation of seven Bipolar ABA frameworks (one for each agent), for six treatment options (a_0–f_0). The Bipolar ABA frameworks are represented as graphical BAFs (see Notation 1). The endorsement/non-endorsement/lack of an opinion from an agent on an argument's outgoing attacks/supports are represented by green/red/grey, respectively, circles underneath the nodes, where the agents are: respiratory specialist, physiotherapist, cardiologist, gastroenterologist, immunologist, neurologist and the outpatient. Note that in this simple example, for clarity, we assume that agents agree with either all or none of the outgoing attacks/supports from an argument, though this may not be the case in general. Note also that black (grey) edges represent attacks/supports that are present (not present, respectively) in aggregated frameworks (presented and discussed throughout Sect. 6) (Color figure online)

gates the seven agents' Bipolar ABA frameworks representing the debate in order to make a decision on whether to recommend undertaking any of the treatments. A bipolar argumentation representation of the combined seven Bipolar ABA frameworks is given in Fig. 13. In each of the following six subsections we consider an aggregated Bipolar ABA framework for each treatment in turn.

We note that this section serves as an illustrative example of how our aggregation method might be applied in specific scenarios. The situations described here rely on particular assumptions, such as the number of arguments and the specific aggregation rules used. While this example demonstrates potential applications of our theoretical framework, it represents a simplified version of real-world decision-making processes. Any application of this aggregation method, regardless of the domain, would necessarily be quite specific and dependent on the particular context and constraints of the situation at hand.

6.1 Recommending rest

The first proposal from the specialists is that of the physiotherapist, suggesting that rest will allow the body to recover, giving the rule $a_0 \leftarrow a_1$ which all agents endorse. The cardiologist puts forward the argument that a lack of exercise causes deconditioning, giving $\bar{a_0} \leftarrow a_2$, which some specialists do not endorse (as they believe that the required period of rest will not be lengthy, and thus will not lead to deconditioning). The outpatient then claims they

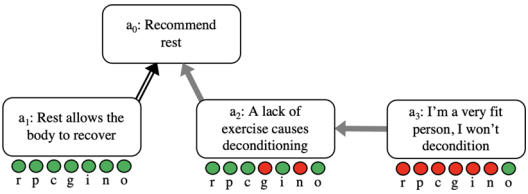


Fig. 14 The (combined) BAF representing the debate on whether to recommend rest (Sect. 6.1). Here and in the figures which follow, black (grey) edges represent attacks/supports that are present (not present, respectively) in the considered aggregated frameworks (Color figure online)

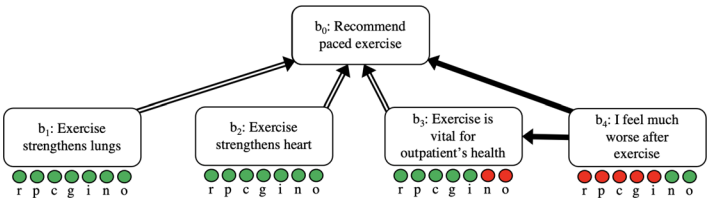


Fig. 15 The BAF representing the debate on whether to recommend paced exercise (Sect. 6.2) (Color figure online)

will not decondition since they are very fit, giving $\bar{a_2} \leftarrow a_3$, which none of the specialists endorse.

The GP decides to determine whether to recommend rest based on whether the set $\{a_0, a_1, a_3\}$ is admissible in the aggregated framework. Obviously this admissibility depends on the aggregation rule. If unanimity is used, then rules $\overline{a_0} \leftarrow a_2$ and $\overline{a_2} \leftarrow a_3$ do not belong to the aggregated framework, as shown in Fig. 14 (where the corresponding attacks are grey). Thus, the set $\{a_0, a_1, a_3\}$ is admissible and rest is recommended. However, if a quota rule with $1 < q < 6$ is used, the set is not admissible in the aggregated framework. Different results occur with the oligarchic rules depending on the chosen allocation of veto powers.

6.2 Recommending paced exercise

All agents take part in the next debate regarding paced exercise. The respiratory specialist and the cardiologist put forward arguments that exercise strengthens the lungs and heart, respectively, giving $b_0 \leftarrow b_1$ and $b_0 \leftarrow b_2$, respectively, which all agents endorse. The physiotherapist then states that exercise is vital for the outpatient's health, giving $b_0 \leftarrow b_3$, which the neurologist does not endorse as they are concerned about aggravating the outpatient's condition. The outpatient also disagrees with this and states that they feel much worse after exercise, giving $\bar{b_3} \leftarrow b_4$ and $\bar{b_0} \leftarrow b_4$, which only the neurologist endorses.

The GP decides to determine whether to recommend paced exercise based on the acceptability of b_0 under complete semantics. They opt for the quota rule with $q = 2$, which results in the argument not being considered to be acceptable, as shown in Fig. 15, despite the minimal support for its sole attacker. On the other hand, had the GP opted for a more stringent aggregation method such as weak majority, then the argument would have been accepted.

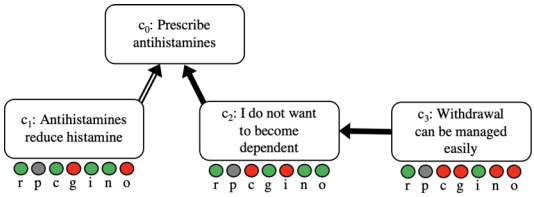


Fig. 16 The BAF representing the debate on whether to prescribe antihistamines (Sect. 6.3)

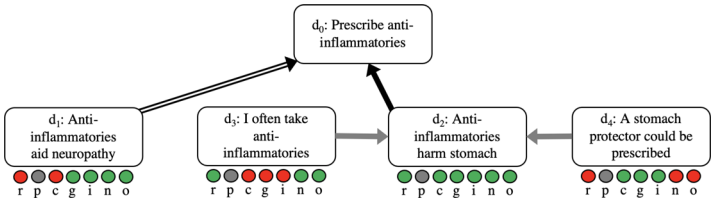


Fig. 17 The BAF representing the debate on whether to prescribe anti-inflammatories (Sect. 6.4)

6.3 Prescribing antihistamines

The immunologist claims the outpatient could benefit from reducing histamine in the body, proposing the rule $c_0 \leftarrow c_1$, which the outpatient and the gastroenterologist do not endorse as there is no definitive proof this is the problem. The outpatient states that they do not want to become dependent on the medication, giving $\bar{c_0} \leftarrow c_2$, to which the immunologist responds that withdrawal can be managed easily, giving $\bar{c_2} \leftarrow c_3$. Both rules receive mixed endorsements. The physiotherapist takes no view on this matter.

Note that despite the diverse opinions on this subject, as shown in Fig. 16, conflict-freeness and closedness in combination are preserved by all aggregation measures, e.g. $\{c_0, c_1\}$ will always be conflict-free, in line with Theorem 3. The GP decides to use the oligarchic rule with the outpatient given veto power. The outpatient thus determines that they would prefer not to be prescribed the drug because $\{c_0, c_2\}$ is not conflict-free in the aggregated framework and they do not want to take unnecessary risks.

6.4 Prescribing anti-inflammatories

The neurologist would like to prescribe anti-inflammatories to aid the neuropathy they believe to be the main issue, and so puts forward the rule $d_0 \leftarrow d_1$. The physiotherapist takes no view on this matter. Some specialists are not convinced of the neurologist's theory and so do not endorse this rule. The gastroenterologist points out that anti-inflammatories may harm the stomach and so puts forward the rule $\bar{d_0} \leftarrow d_2$, which all agents other than the physiotherapist endorse. Two considerations supporting $\bar{d_2}$ are proposed: the outpatient states that they often take anti-inflammatories without issues and the gastroenterologist begrudgingly states that they could prescribe a stomach protector, which result in the two rules $\bar{d_2} \leftarrow d_3$ and $\bar{d_2} \leftarrow d_4$, respectively, which are each endorsed by 3 agents.

The GP decides to use the admissibility of $\{d_0, d_1, d_3, d_4\}$ to determine whether the anti-inflammatories should be prescribed. The GP adopts the quota rule with $q = 4$, which means that $\{d_0, d_1, d_3, d_4\}$ is admissible in all agents' individual frameworks but not in the aggre-

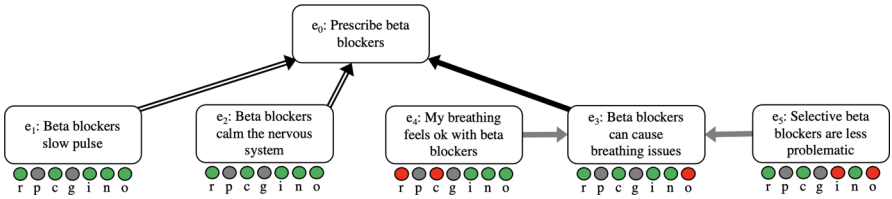


Fig. 18 The BAF representing the debate on whether to prescribe beta blockers (Sect. 6.5)

gated framework (in line with Theorem 5), as shown in Fig. 17, and so anti-inflammatories are not prescribed.

Note that, if instead the quota rule were used with $q < 4$ or the oligarchic rule were used with the outpatient given veto power, the set would be found to be admissible.

6.5 Prescribing beta blockers

The cardiologist claims that beta blockers could help by slowing the outpatient's pulse, giving $e_0 \leftarrow e_1$, while the neurologist states that they could also calm the nervous system, giving $e_0 \leftarrow e_2$. The physiotherapist and gastroenterologist take no view on this matter, but all participating agents endorse these rules. However, the respiratory specialist states that beta blockers can cause breathing issues, giving $\bar{e_0} \leftarrow e_3$, which all participating agents except the outpatient endorse. The outpatient then states that, in the past, their breathing has felt okay with beta blockers, giving $\bar{e_3} \leftarrow e_4$. This is enough reasoning for two specialists but not the others. The respiratory specialist then reasons that selective beta blockers are less problematic, giving $\bar{e_3} \leftarrow e_5$, which some agents endorse but some are unsure about.

In this debate, the GP opts to determine whether to prescribe beta blockers using the acceptability of e_0 under complete semantics using a quota rule with $q = 4$, as shown in Fig. 18. It can be seen that despite e_0 being acceptable in the individual frameworks, this status is not preserved under this aggregation, in line with Theorem 11, and beta blockers are not prescribed.

6.6 Recommending breathing exercises

The respiratory specialist advocates breathing exercises as they believe that they will strengthen the outpatient's lungs, and so puts forward the rule $f_0 \leftarrow f_1$, which two agents, the cardiologist and the neurologist, also endorse. Only these three specialists feel they have the expertise to take part in this debate. The neurologist claims that breathing exercises could worsen the neuropathy they believe the patient has, proposing $\bar{f_0} \leftarrow f_2$, which the respiratory specialist does not endorse. The neurologist admits that other patients have reported benefits after trying breathing exercises, proposing the rule $\bar{f_2} \leftarrow f_3$, though the other specialists are skeptical about reading too much into this in such a complicated condition and so do not endorse the rule. This also results in the rule $f_0 \leftarrow f_3$. Finally, the cardiologist reports that breathing exercises have been shown to lower patients' heart rates, which they believe indicates that they do not worsen neuropathy, giving $\bar{f_2} \leftarrow f_4$, while the other specialists do not believe there is a link here.

In this case, the GP decides to determine whether to recommend breathing exercises using the acceptability of f_0 under preferred semantics using the quota rule with $q = 2$, i.e. two

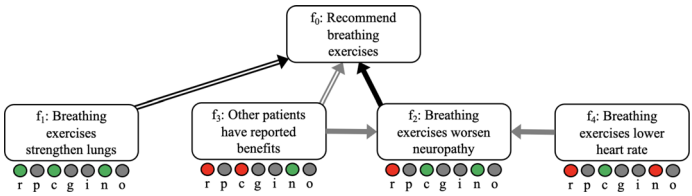


Fig. 19 The BAF representing the debate on whether to prescribe breathing exercises (Sect. 6.6)

agents are required to support a rule for it to be included in the aggregated framework. This means that, although f_0 is acceptable under preferred semantics in each agent's individual framework, it is not in the aggregated framework, in line with Theorem 9, and thus breathing exercises are not recommended.

Overall, this illustration shows the power of our framework to support debate and decision-making.

7 Conclusion

We have considered Bipolar ABA Frameworks [19] to account for both attack and support relationships between arguments in Bipolar Argumentation, as it allows to uniformly capture different interpretations of support. The aggregation of Bipolar ABA Frameworks combines the rules of all agents into a collective set of rules. We made use of the aggregation rules from judgement aggregation [26, 33], specifically, quota and oligarchic rules, to combine these agents' rules and extended results (on abstract argumentation) from [11] by considering more properties, some relevant to bipolar argumentation only (e.g. closedness) others also relevant to abstract argumentation (e.g. ideal semantics); we also provide preservation results for corner cases.

As discussed in Sect. 5.10, the preservation results show that most properties can only be preserved under significant restrictions. The results assume agreement among the agents on language, assumptions, contraries, and assume that agents accept the same properties. While many of the results are negative (higlighting the difficulty in preserving properties during aggregation), they offer motivation for future research in this field. By precisely identifying the limitations of our approach, our results establish a foundation for developing novel aggregation methods that can overcome these restrictions. In particular, by considering the specific scenarios in which properties are not preserved (as highlighted in our proofs), new aggregation rules can be developed accordingly in order to avoid them.

There are several possible directions for future work. First of all, here the preservation of properties relies on the agreement on which properties matter by all agents. However, in real applications it is likely that some agents have different opinions, e.g., some of them might disagree on which semantics should be preserved. Thus, it would be interesting to study preservation when a number of agents disagree. Another path to work on in the future is to have agents with different knowledge about the environment, meaning that they might have different languages, assumptions, or contraries.

Additionally, it would be of interest to explore a wider range of aggregation rules beyond the quota and oligarchic rules considered in this paper. As mentioned in Sect. 4, quota and oligarchic rules satisfy the axioms of unanimity, groundedness, neutrality, independence, and

monotonicity (with quota rules also fulfilling anonymity). By considering rules that satisfy different sets of axioms, more positive results may be obtained.

Furthermore, it would be interesting to identify and study properties of the argumentation frameworks resulting from aggregation as in this paper, along the lines of the postulates for merging of abstract argumentation frameworks, e.g., [20]. Finally, it would be worth generalising this study for the more general ABA Frameworks of [3, 14], as well as for other forms of structured argumentation, such as ASPIC [36], DeLP [28], or logic-based argumentation [5].

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