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Private Data Manipulation in Sponsored Search Auctions

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ABSTRACT

The repeated nature of sponsored search auctions allows the seller to implement Myerson's auction to maximize revenue using past data. But since these data are provided by strategic buyers in the auctions, they can be manipulated, which may hurt the seller's revenue. We model this problem as a Private Data Manipulation (PDM) game: the seller first announces an auction (such as Myerson's) whose allocation and payment rules depend on the value distributions of buyers; the buyers then submit fake value distributions to the seller to implement the auction. The seller's expected revenue and the buyers' expected utilities depend on the auction rule and the game played among the buyers in their choices of the submitted distributions. Under the PDM game, we show that Myerson's auction is equivalent to the generalized first-price auction, and under further assumptions equivalent to the Vickrey–Clarke–Groves (VCG) auction and the generalized second-price auction. Our results partially explain why Myerson's auction is not as popular as the generalized second-price auction in the practice of sponsored search auctions, and provide new perspectives into data-driven decision making in mechanism design.

KEYWORDS

Internet economics; sponsored search auction; Myerson's auction; generalized first-price auction; data-driven decision making

The Sponsored Search Auction (SSA) refers to the commercialization model of a search engine selling advertisement slots to advertisers. When a user searches for a keyword on a search engine, besides the natural search results the search engine sometimes also displays ads to the user. Whenever the user clicks an ad, the corresponding advertiser makes a payment to the search engine. Different ad slots have different click-through rates. An SSA mechanism refers to a mechanism by which the advertisers bid to compete for good ad slots and the seller determine the allocation of the ad slots and the payment of each advertiser based on their bids.

Google, arguably the most successful search engine, has traditionally employed the Generalized Second-Price (GSP) auction as its SSA mechanism. In GSP, bidders are allocated ad slots according to the decreasing order of their bids and charged the bid (per click) one rank lower than their own bid in the sorted bid list[1, 2]. Despite many other SSA mechanisms have been proposed[3–6], GSP has sustained the test of competition to become one of the key factors for Google's success in the sponsored search market in the early years[7]. It has since become the most commonly used mechanism for sponsored search auctions, and can reasonably be named the mother of all sponsored search auctions.

In theory, however, GSP do not always maximize the revenue for the seller in SSA. If the seller knows the distributions of advertisers' values for the ad slots, then the mechanism that maximizes revenue for the seller is a generalization of Myerson's auction[8]. Although advertisers' value distributions are usually private to themselves and unknown to the seller, those distributions can be estimated from historical data, such as the advertisers' bids in previous auctions. Those data are abundant in sponsored search auctions, which are auctions repeated millions of times everyday. By using data to estimate advertisers' value distributions, search engines should be able to implement Myerson's auction and derive more revenue than GSP, at least in theory. The estimation of value distributions and the computation of optimal mechanism can be done automatically by algorithms that make intelligent decisions based on large data. Then, why do search engines not use Myerson's auction in practice?

A possible explanation is that the amount of data needed to implement Myerson's auction may be too large. However, theoretical study has shown that the sample complexity of Myerson's auction is a moderate polynomial $\Theta\left(\frac{n}{\varepsilon^2}\right)$[9, 10]. This amount of samples should be easily obtainable in today's massive-scale SSA. Also, experiments have shown that optimizing revenue using past data is possible in practice[11]. So, it is unclear whether the shortage of data is the real reason why Myerson's auction is unpopular in the SSA market.

In this paper, we propose another explanation for Myerson's auction's unpopularity: advertisers may lie about their value distributions by providing "fake" data. The allocation rule and payment rule of Myerson's auction crucially depend on buyers' value distributions. If advertisers strategically submit bids that are not equal to their values, then the search engine will obtain from these bids some distributions that are different from advertisers' true distributions. These fake distributions may mislead the Myerson's auction, resulting in higher utilities for the advertisers and less revenue for the search engine. GSP, on the other hand, can never be affected by advertisers' strategic bidding behavior because its allocation and payment rules do not depend on value distributions at all. Can this explain why Myerson's auction is not as popular as GSP?

Our results. To know whether advertisers' lying behavior is a

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reason for Myerson's auction's unpopularity, we propose and study a two-stage game between the search engine (seller) and the advertisers (buyers): first, the seller announces an auction, such as Myerson's auction, whose allocation and payment rules may depend on buyer's value distributions. Then, each buyer submits to the seller a distribution, which can be different from their true distribution. After the two steps, the auction is implemented using the submitted distributions, and the utilities of the buyers and the revenue of the seller are realized. As discussed, this two-stage game may be non-truthful for the buyers because they may obtain higher utilities by manipulating the submitted distributions. We call this game a "Private Data Manipulation (PDM) game".

We analyze the buyers' equilibrium in the PDM game, assuming that the seller announces Myerson's (Mye) auction in the first stage. We prove that the equilibrium outcome is equivalent to the equilibrium outcome of the Generalized First-Price (GFP) auction, which is similar to GSP except that bidders pay their own bids per click. Specifically, every equilibrium of the PDM game has a one-to-one mapping to an equilibrium of GFP; the buyers obtain the same utilities and the seller obtains the same revenue under the two corresponding equilibria. This result in particular implies that the PDM game with Mye in the first stage is indeed non-truthful for the buyers. Buyers' strategic behavior causes Myerson's auction to degenerate to a GFP. Thus, our conjecture that buyers' lying behavior is a reason for Myerson's auction's unpopularity is supported.

We then combine the above "Mye = GFP" result with the classical revenue-equivalence theorem to conclude that Mye, GFP, and Vickrey–Clarke–Groves (VCG) auctions are all equivalent if buyers' true values are Independent and Identically Distributed (I.I.D.). Under further assumptions, these three auctions are equivalent to GSP as well (see Fig. 1). This series of equivalence results again partially explain why Myerson's auction is not as popular as GSP in practice—Mye is basically the same as GSP due to buyers' manipulation of value distributions.

Techniques. At the technical level, our proof of the equivalence between Mye and GFP under the PDM model generalizes the proof by Tang and Zeng[12] who proved that Mye and the first-price auction are equivalent under the PDM model in the single-item setting. Their proof uses the fact that the expected revenue of Mye equals the expected virtual welfare, so each buyer's expected utility equals its welfare minus its virtual welfare. When the buyer is free to submit any distribution to the seller, the virtual value of the submitted distribution can be regarded as the bid in the first-price auction. This establishes the equivalence between the two auctions. Their approach turns out to be generalizable to any single-parameter auction environments, including SSA. We will therefore present and prove the equivalence between Mye and GFP for any single-parameter environments. The further equivalence between Mye, GFP, VCG, and GSP will be presented only for the SSA environment because the GSP auction is specific to SSA.

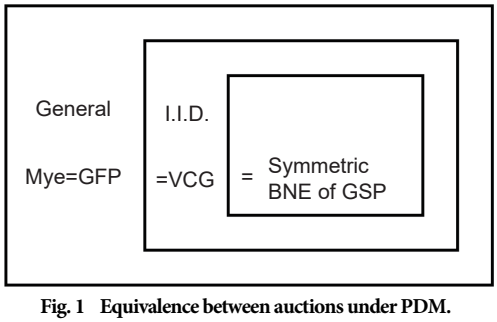


Fig. 1 Equivalence between auctions under PDM.

Main takeaway. One may think that the equivalence between Mye and GFP (and VCG, GSP under further assumptions) means that learning from data is useless for the seller in SSA because buyers can manipulate the data. But there is a caveat: although we show that Myerson's auction is vulnerable to data manipulation, there could be other auctions that are robust against data manipulation and can generate more revenue than Mye in the PDM model. Finding the seller-optimal auction in the PDM model is an interesting open question beyond the scope of this paper. The main takeaway from our paper is: when making decisions based on data, decision-makers should be careful about the possibility of data being manipulated by interested parties. This applies to all data-driven decision-making scenarios, besides sponsored search auctions.

Organization of the paper. We review related works in Section 1. In Section 2, we introduce the sponsored search auction model, the single-parameter auction environment, and the PDM game. In Section 3, we derive the main result that the Myerson's auction and the generalized first auction are equivalent in the PDM game. We present in Section 4 an array of equivalence relationships among Mye, GFP, VCG, and GSP under the PDM model (see Fig. 1). Section 5 includes additional discussions.

1 Related Work

There has been a considerable amount of work on the sponsored search auction; see Ref. [13] for a survey. Among all practically applied auctions for sponsored search, the generalized second-price auction stands out attracting the most attention in Refs. [1, 2, 14]. A significant fraction has focused on the social welfare properties of equilibria where the goal is to measure the price of anarchy[15–18]. Others have studied the revenue properties[19–21] where the goal is to measure the guaranteed revenue out of different equilibrium concepts.

The design of revenue-optimal auctions dates back to the seminal work by Myerson[8], who finds the optimal auction under the Bayesian setting that assumes a prior value distribution for each buyer. In practice, this prior information does not come for free. This initiates the study of sample complexity[9, 22–27], where the seller has some samples from the prior, and the goal is to learn a mechanism from those samples to approximate the optimal revenue. Another line of research focus on repeated auctions[28, 29], with the goal of revenue-maximization or regret-minimization against myopic bidders[30–33] or non-myopic bidders[34–39]. These works have observed that the strategic behavior of non-myopic bidders can reduce the revenue of the seller. Our PDM model captures the limit state of the interaction between a learning seller and non-myopic bidders in repeated auctions, and hence our conclusion that Mye is equivalent to GFP reconfirms the observation by previous works.

The idea of agents' manipulation against learning is not only applicable in multi-agent auction design[40–42]. It has become a significant research topic in many other machine learning problems (e.g., Refs. [43–46]). We expect that this approach will find further applications in the future.

2 Preliminary and Model

In a sponsored search auction, there is a search engine (seller/auctioneer) selling m advertisement (ad) slots to n advertisers (buyers/bidders). Each ad slot j has a click-through rate, or quality, $\gamma_j \geqslant 0$. This means that the probability of an ad

being clicked by a user only depends on the location of the ad slot but not the content of the ad. This assumption was made in the original papers on sponsored search auctions[1, 2]. If the click-through rate can also depend on the ad content then this auction will not fit into the single-parameter environment we will consider, and the revenue-maximizing auction does not have a clean characterization as Myerson's auction. Relaxing this assumption is beyond the scope of our paper. Each bidder i has a value (willingness to pay) $v_i \geqslant 0$ for each click, or each unit quality. Each ad slot can be allocated to at most one bidder and each bidder can be allocated at most one slot. If bidder i is allocated slot j, they obtain profit $v_i\gamma_j$. Without loss of generality we assume that $\gamma_1 \geqslant \gamma_2 \geqslant \cdots \geqslant \gamma_m$. We use $\vec{v}_{-i} = (v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$ to denote the value vector without v_i. At the beginning of the auction, the value per quality v_i of each bidder i is drawn from a distribution F_i by nature. Different from the traditional Bayesian setting where the distributions F_i's are assumed to be common knowledge, here we assume that F_i is known to bidder i itself and unknown to the auctioneer and other bidders. We assume that the distributions F_i's are independent. We use $F = \prod_{i=1}^{n} F_i$ to denote the joint distribution, and $F_{-i} = \prod_{j\neq i} F_j$ to denote the joint distribution without F_i. We also call the distributions F_i's "prior information".

Quantile representation. Value distributions can be represented in two ways: value representation and quantile representation. Suppose value v follows distribution G. We use $G(\cdot)$ to denote the cumulative distribution function, $G(x) = \Pr_G[v < x]$, and $g(\cdot)$ to denote the corresponding probability density function (if exists). We call v the value representation of the distribution G. An alternative representation is to use quantile. Given a distribution G, the quantile $q = q(v) = 1 - G(v)$ is the probability that a buyer has a value of at least v. We regard quantile q as a random variable uniformly distributed in the interval $[0,1]$, and it is mapped to the value by the function $v(\cdot) : [0,1] \to \mathbb{R}_+$ defined by $v(q) = G^{-1}(1-q)$. Clearly, $v(q)$ is monotonically non-increasing in q.

Throughout the paper, we will use either the Cumulative Distribution Function (CDF) $G(\cdot)$ or the value function $v(\cdot)$ to represent a distribution.

Virtual value. The virtual value is defined as follows: given value $v \sim G$, its corresponding virtual value is $v - \frac{1-G(v)}{g(v)}$. Writing virtual value as a function of quantile, we have

$$v(q) - \frac{1-G(v(q))}{g(v(q))} = v(q) - \frac{q}{g(v(q))} = v(q) - \frac{q}{g(G^{-1}(1-q))} = v(q) + q(G^{-1}(1-q))' = v(q) + qv'(q).$$

An alternative definition of virtual value is to first define the revenue curve $R(q) = qv(q)$, then write the virtual value at quantile q as the derivative of $R(q)$:

$$R'(q) = v(q) + qv'(q).$$

So we can also denote the virtual value by $\phi(q) = R'(q)$. A distribution G is regular if $\phi(q)$ is monotonically non-increasing in q, or equivalently, $R(q)$ is concave.

Lemma 1 For any non-increasing function $\phi : [0,1] \to \mathbb{R}$, there exists a unique distribution $v(\cdot)$ whose virtual value function is $\phi(\cdot)$.

Proof Since non-increasing function is Riemann integrable, we can integrate ϕ to get the revenue curve $qv(q) = \int_0^q \phi(x)\mathrm{d}x$. Dividing q gives the desired distribution $v(\cdot)$. (Although $[qv(q)]'$ is undefined at the discontinuous points of $\phi(q)$, we can still define it to be $\phi(q)$ because discontinuous points have zero measure and do not affect the expected utility of the bidder.) □

2.1 Single-parameter environment

For simplicity and generality, we shift from the specific sponsored search setting to the more general single-parameter environment. There are n bidders and a set $A \subseteq \mathbb{R}_+^n$ of feasible allocations $\vec{a} = (a_1, a_2, \ldots, a_n)$. The allocation to bidder i is a_i, and the allocation to other bidders is denoted by $\vec{a}_{-i}$. We assume that A is downward-closed, that is, for each i and any $\vec{a} = (a_i, \vec{a}_{-i}) \in A$, we have $(0, a_{-i}) \in A$. The sponsored search setting is a special case where a_i stands for the quality of ad slot given to i.

Each bidder i has a value $v_i \sim F_i$ for receiving one unit of allocation. A mechanism $\mathscr{M}$ associated with the prior $F = \prod_{i=1}^{n} F_i$ consists of an allocation rule $\vec{X} : \mathbb{R}_+^n \to A$, and a payment rule $\vec{P} : \mathbb{R}_+^n \to \mathbb{R}_+^n$. Bidders submit bids $\vec{b} = (b_i, \vec{b}_{-i})$ to represent their willingness to pay per unit, then bidder i is allocated $X_i(\vec{b})$ units, pays $P_i(\vec{b})$, obtaining a utility of $X_i(\vec{b})v_i - P_i(\vec{b})$. Assuming that other bidders bid truthfully, we denote the interim allocation by $x_i(v_i) = E_{\vec{v}_{-i}\sim F_{-i}}[X_i(v_i, \vec{v}_{-i})]$, and the interim payment by $p_i(v_i) = E_{\vec{v}_{-i}\sim F_{-i}}[P_i(v_i, \vec{v}_{-i})]$.

We say a mechanism $\mathscr{M}$ is Bayesian-Incentive-Compatible (BIC), if the following constraints are satisfied:

$$v_i x_i(v_i) - p_i(v_i) \geqslant v_i x_i(v_i') - p_i(v_i'), \quad \forall v_i, v_i', i.$$

We say a mechanism $\mathscr{M}$ is Bayesian-Individually-Rational (BIR), if the following constraints are satisfied:

$$v_i x_i(v_i) - p_i(v_i) \geqslant 0, \quad \forall v_i, i.$$

In addition, we assume $p_i(0) = 0$.

Lemma 2[47] A mechanism $\mathscr{M}$ is BIC and BIR if the allocation rule and payment rule satisfy:

- $X_i(v_i, \vec{v}_{-i})$ is monotonically non-decreasing in v_i;
- $P_i(\vec{v}) = v_i X_i(\vec{v}) - \int_0^{v_i} X_i(u, \vec{v}_{-i})\mathrm{d}u$.

Sometimes we express allocation and payment using quantile, and use superscript $(\mathscr{M}, F)$ to indicate that they are determined by mechanism $\mathscr{M}$ with F as prior. Thus we have $x_i^{(\mathscr{M},F)}(q) = x_i(v_i(q))$, $p_i^{(\mathscr{M},F)}(q) = p_i(v_i(q))$, etc. Then the expected payments and revenue of a BIC mechanism can be conveniently expressed in terms of virtual value $\phi(q)$:

Lemma 3[47] In a BIC and BIR mechanism $\mathscr{M}$, if bidders bid their true values, then bidder i's expected payment is equal to their expected virtual welfare:

$$E_q[p_i^{(\mathscr{M},F)}(q)] = E_q[\phi_i(q)x_i^{(\mathscr{M},F)}(q)] \tag{1}$$

The expected revenue is thus equal to the expected total virtual welfare:

$$\sum_{i=1}^{n} E_{q_i}[p_i^{(\mathscr{M},F)}(q_i)] = E_{\vec{q}}\left[\sum_{i=1}^{n} \phi_i(q_i)X_i^{(\mathscr{M},F)}(\vec{q})\right] \tag{2}$$

Myerson's auction. As we will show in Section 3.1, if the prior F is known to the seller, then the auction that maximizes the seller's expected revenue (subject to the BIC and BIR constraints with respect to F) in a single-parameter environment is a generalization of the classical Myerson's auction in the single-item setting. For simplicity we also call this auction Myerson's auction. Mye is prior-dependent.

Generalized first-price auction. A generalized first-price auction is the generalization of the single-item first-price auction. Usually, "generalized first-price auction" refers specifically to the auction in the sponsored search setting. Here we use this name for the more general single-parameter environment. The allocation rule of GFP is defined by $\vec{X}(\vec{b}) \in \arg\max_{\vec{a}\in A} \sum_{i=1}^{n} b_i a_i$, with some tie-breaking rule in arg max, and the payments are $P_i(\vec{b}) = b_i X_i(\vec{b})$. Contrary to Mye, GFP is prior-independent and not BIC.

2.2 Private data manipulation game

When a prior-dependent auction such as Mye is used in practice, an issue occurs: the true prior information F is not available to the auctioneer. The auctioneer may try to learn the prior information from data/bids submitted by bidders in history, but those data may be manipulated by bidders if the bidders know that the auctioneer is learning. We interpret the manipulation procedure as follows: any value v_i drawn from F_i is mapped to a bid $\hat{v}_i$, which follows another distribution $\hat{F}_i$. We call the bid distribution $\hat{F}_i$ the manipulated distribution. With enough data, the auctioneer learns the manipulated distributions $\hat{F} = \prod_{i=1}^{n} \hat{F}_i$ and designs auctions based on $\hat{F}$. Throughout the paper, we will use notions with $\hat{\cdot}$ for the manipulated data, (for example, $\hat{F}_i$ for the distribution, $\hat{\phi}_i(\cdot)$ for its virtual value function).

For any mechanism $\mathcal{M}$ that may take prior information into account, we define the following two-stage game, which we call the PDM game:

- Stage 1. The auctioneer announces the mechanism $\mathcal{M}$ before the auction starts.
- Stage 2. Each bidder i reports $\hat{F}_i$ by bidding $\hat{v}_i$ such that $\hat{v}_i \sim \hat{F}_i$.
- Outcome: The auctioneer runs $\mathcal{M}$ with the manipulated distributions $\hat{F} = \prod_{i=1}^{n} \hat{F}_i$ as prior, allocates and charges each bidder according to $\mathcal{M}$.

In the second stage of the PDM game, the bidders strategically respond to the mechanism announced in the first stage. Their strategy sets are defined as follows:

Definition 1 (Strategy Set) Let $S = \prod_{i=1}^{n} S_i$ denote the strategy set of all bidders, where S_i is the strategy set of bidder i. A strategy $s_i \in S_i$ consists of:

- The manipulated distribution, denoted by the CDF $\hat{F}_i(\cdot)$ or the quantile-to-value function $\hat{v}_i(\cdot) : [0,1] \to \mathbb{R}_+$.
- $\sigma_i(\cdot) : [0,1] \to [0,1]$, a permutation of $[0,1]$ satisfying $\sigma_i(\text{Uniform}[0,1]) = \text{Uniform}[0,1]$. Bidder i bids $\hat{v}_i(q)$ when their true value is $v_i(\sigma_i(q))$.

The introduction of $\sigma_i(\cdot)$ is necessary because $\hat{v}_i(\cdot)$ only defines the overall bid distribution but does not describe how values are mapped to bids, and $\sigma_i(\cdot)$ allows a bidder to choose the mapping arbitrarily while keeping the distribution $\hat{v}_i(\cdot)$ unchanged.

Once the auctioneer has announced a mechanism $\mathcal{M}$ and the bidders have chosen their manipulation strategies, we can compute each bidder i's (expected) utility, as follows:

$$U_i(s_1, s_2, \ldots, s_n) = E_q\left[v_i(\sigma_i(q)) \cdot x_i^{(\mathcal{M},\hat{F})}(q) - p_i^{(\mathcal{M},\hat{F})}(q)\right] \tag{3}$$

We emphasize that $x_i^{(\mathcal{M},\hat{F})}(q)$ is the interim allocation when bidder i's bid is $\hat{v}_i(q)$, and others' bids $\hat{\vec{v}}_{-i}$ follow $\hat{F}_{-i}$, under mechanism $\mathcal{M}$ with manipulated distribution $\hat{F} = \prod_{i=1}^{n} \hat{F}_i$ as prior.

We assume that the bidders in a PDM game play a Nash equilibrium, which is defined as follows:

Definition 2 (Nash equilibrium) A profile of bidders' manipulation strategies $\vec{s} = (s_1, s_2, \ldots, s_n)$ is a Nash equilibrium if $U_i(s_i, s_{-i}) \geqslant U_i(t_i, s_{-i})$ for any strategy $t_i \in S_i$, any i.

We show in the following lemma that for prior independent auctions, whose allocation and payment rules do not depend on prior distributions, the behaviors of bidders under the PDM setting and the traditional Bayesian setting are strategically equivalent. In the traditional Bayesian setting, the strategy of a bidder is a value-to-bid mapping $\pi_i : \mathbb{R}_+ \to \mathbb{R}_+$. A profile of strategies $\vec{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$ is called a Bayes–Nash Equilibrium (BNE) of mechanism $\mathcal{M}$ if each bidder i has no incentive to unilaterally deviate from their strategy π_i:

$$E_{\vec{v}\sim F}[X_i(\vec{\pi}(\vec{v}))v_i - P_i(\vec{\pi}(\vec{v}))] \geqslant$$
$$E_{\vec{v}\sim F}[X_i(\pi_i'(v_i), \vec{\pi}_{-i}(\vec{v}_{-i}))v_i - P_i(\vec{\pi}(\vec{v}))], \quad \forall \pi_i', \forall i.$$

Let $\pi_i^s : \mathbb{R}_+ \to \mathbb{R}_+$ denote the value-to-bid mapping induced by the strategy s_i in the PDM game, namely, i bids $\pi_i^s(v_i(q)) = \hat{v}_i(\sigma_i^{-1}(q))$ when their true value is $v_i(q)$.

Lemma 4 For any prior-independent auction $\mathcal{M}$, $(s_1, s_2, \ldots, s_n)$ is a Nash equilibrium of $\mathcal{M}$ under PDM if and only if $(\pi_1^s, \pi_2^s, \ldots, \pi_n^s)$ is a BNE of $\mathcal{M}$ under the traditional Bayesian setting.

Lemma 4 holds since manipulating distributions does not affect the allocation and payment rules of $\mathcal{M}$.

3 Equivalence between Auctions in Private Data Manipulation Model

In this section, we show that Myerson's optimal auction and the generalized first-price auction are equivalent when bidders are able to manipulate their value distributions. We consider Myerson's auction in PDM because it is natural for the auctioneer to choose it, given the reported value distributions, since it extracts the most revenue based on all available information. However, it turns out that in the PDM game the originally optimal auction degenerates into a generalized first-price auction in which no private data is used at all. We start by reviewing how the optimal auction works.

3.1 Myerson's auction

Myerson's optimal auction in the single-parameter environment is the generalization of Myerson's auction in the single-item setting[8]. Denote the given prior by $F = \prod_{i=1}^{n} F_i$.

If F_i's are regular, by Lemma 3, the revenue can be maximized by choosing an allocation that maximizes the virtual welfare for each bid vector $\vec{v}(\vec{q})$. Formally,

$$\vec{X}^{(\text{Mye},F)}(\vec{q}) \in \arg\max_{\vec{a}\in A} \sum_{i=1}^{n} a_i \phi_i(q_i).$$

Payments are calculated according to Lemma 2. Such a mechanism is BIC by Lemma 2, because when v_i increases, its virtual value $\phi(q_i(v_i))$ does not decrease by definition, so the allocation X_i is monotonically non-decreasing.

In addition, in order to maximize the virtual welfare, Mye will never allocate positive units to a bidder with non-positive virtual value; otherwise, setting their allocation to be zero weakly increases the virtual welfare, without violating the feasibility constraint since A is downward-closed. That is, we set a reserve price r_i at which the virtual value of bidder i is zero, and only allocate to bidders whose bids are above their respective reserve

prices. We choose the maximal reserve price r_i when there are multiple solutions to $\phi_i^v(r_i) = 0$. As a result,

$$\phi_i(q_i) \leqslant 0 \implies X_i^{(\mathrm{Mye},F)}(q_i, \vec{q}_{-i}) = 0 \ \forall \vec{q}_{-i} \tag{4}$$

For irregular distribution F_i, Myerson's auction will first "iron" the distribution[8], transforming it into a regular distribution $\overline{F}_i$, and then run the auction with respect to $\overline{F}_i$.

3.2 Equivalence between Mye and GFP

Now we analyze how bidders will behave under the Nash equilibrium of Mye in the PDM game. Recall that the true distribution F is not available, and Mye has to rely on bidders' reported distribution $\hat{F}$ to allocate items and charge prices.

First we argue that without loss of generality, the set of best responding strategies of the bidders can be limited as follows:

Lemma 5 For any i, any s_{-i}, choosing the strategy $s_i = (\hat{v}_i(\cdot), \sigma_i(\cdot))$ that satisfies the following properties maximizes bidder i's utility:

1. Identity permutation: $\sigma_i(q) = q$.
2. Monotonicity (of virtual value): $\forall q_1 < q_2, \hat{\phi}_i(q_1) \geqslant \hat{\phi}_i(q_2)$.
3. Non-negative virtual value: $\hat{\phi}_i(q) \geqslant 0$.

Proof of Properies 1 and 2 For Property 1, since the payment term in Eq. (3) does not depend on σ_i, to maximize utility we only need to maximize the first term, which is $\int_0^1 \left[v_i(\sigma_i(q)) \cdot x_i^{(\mathrm{Mye},\hat{F})}(q)\right] \mathrm{d}q$ in integral form. According to the allocation rule of Mye, $x_i^{(\mathrm{Mye},\hat{F})}(q)$ is monotonically non-increasing in q, so by the rearrange inequality, we have:

$$\int_0^1 \left[v_i(\sigma_i(q)) \cdot x_i^{(\mathrm{Mye},\hat{F})}(q)\right] \mathrm{d}q \leqslant \int_0^1 \left[v_i(q) \cdot x_i^{(\mathrm{Mye},\hat{F})}(q)\right] \mathrm{d}q.$$

The identity permutation $\sigma_i(q) = q$ maximizes this integral.

For Property 2, if $\hat{F}_i$ is regular, then the monotonicity of $\hat{\phi}_i$ follows from the regularity of $\hat{F}_i$. If $\hat{F}_i$ is irregular, recall that the ironing procedure of Myerson's auction will first iron $\hat{F}_i$ to a regular distribution $\overline{F}_i$, and then run the Myerson's auction on $\overline{F}_i$. Note that here the virtual value function for bidder i is no longer defined on $\hat{F}_i$, but defined on $\overline{F}_i$, which is a regular distribution. Thus, the monotonicity property still holds. □

Before proving Property 3, we characterize the strategy set of bidders and simplify some notations. By Property 1, we can omit $\sigma_i(\cdot)$ and assume that s_i is uniquely determined by $\hat{v}_i(\cdot)$, that is, bidders will always bid $\hat{v}_i(q)$ for value $v_i(q)$. By Property 2, we only need to consider strategies with monotone virtual value function $\hat{\phi}_i$. Lemma 1 tells us that any monotone virtual value function $\hat{\phi}_i$ determines a unique distribution $\hat{v}_i(\cdot)$. So, we can use the virtual value function $\hat{\phi}_i$ only to represent a strategy s_i. Lemma 1 also implies that any "strategy $\hat{\phi}_i$" is valid as long as $\hat{\phi}_i$ is monotone. Using Lemma 3 to write expected payment as expected virtual welfare, the expected utility of bidder i becomes:

$$U_i(\hat{\phi}_i, \hat{\phi}_{-i}) = E_q\left[x_i^{(\mathrm{Mye},\hat{F})}(q)\left(v_i(q) - \hat{\phi}_i(q)\right)\right] \tag{5}$$

Proof of Property 3 Assume that $\hat{\phi}_i(\cdot)$ is monotonically non-increasing (Property 2). Suppose $\hat{\phi}_i(\cdot)$ takes a negative value in $[w_i, 1]$ (or $(w_i, 1]$). Then, we define another virtual value function $\widetilde{\phi}_i(\cdot)$, which takes the following value:

$$\widetilde{\phi}_i(q) = \begin{cases} \hat{\phi}_i(q), & \text{if } q < w_i; \\ \hat{\phi}_i(q) \cdot \mathbb{I}[\hat{\phi}_i(q) > 0], & \text{if } q = w_i; \\ 0, & \text{if } q > w_i. \end{cases}$$

Note that $\widetilde{\phi}_i(\cdot)$ is also monotonically non-increasing.

When $\hat{\phi}_i(q_i) \leqslant 0$, both $X_i^{(\mathrm{Mye},\hat{F})}(\vec{q})$ and $X_i^{(\mathrm{Mye},\widetilde{F}_i \times \hat{F}_{-i})}(\vec{q})$ are 0 according to the allocation rule of Mye (Formula (4)). The two allocations are also the same for all bidders when $\hat{\phi}_i(q_i) = \widetilde{\phi}_i(q_i) > 0$. This implies that no bidder gets an altered allocation when bidder i switch strategy from $\hat{\phi}_i(\cdot)$ to $\widetilde{\phi}_i(\cdot)$. Bidders' expected utilities are not altered, either. Thus, bidder i can switch to strategy $\widetilde{\phi}_i(\cdot)$, which is non-negative. □

Now, we can assume without loss of generality that $\hat{\phi}_i(\cdot)$ always takes a non-negative value. This enables us to relate $\hat{\phi}_i(\cdot)$ with the bidding strategy in a generalized first-price auction, as discussed next.

Recall the allocation rule $X_i^{\mathrm{GFP}}(\vec{b})$ in Section 2.1 for the GFP, which maximizes $\sum_{i=1}^n a_i b_i$. Let

$$x_i^{\mathrm{GFP}}(b_i(q_i)) = E_{\vec{q}_{-i}} X_i^{\mathrm{GFP}}\left(b_i(q_i), \vec{b}_{-i}(\vec{q}_{-i})\right)$$

denote the corresponding interim allocation.

Then we write down the utility of bidder i in GFP. Since GFP is not BIC, we use a quantile-to-bid mapping $b_i : [0,1] \to \mathbb{R}_+$ to represent bidder i's strategy. With value $v_i(q_i)$, bidder i bids $b_i(q_i)$, obtaining an allocation of $X_i\left(b_i(q_i), \vec{b}_{-i}(\vec{q}_{-i})\right)$ units, paying $b_i(q_i)$ for each unit. Thus, the expected utility of bidder i is

$$\begin{aligned} U_i^{\mathrm{GFP}}(b_i, b_{-i}) =& E_{q_i, \vec{q}_{-i}}\left[X_i^{\mathrm{GFP}}\left(b_i(q_i), \vec{b}_{-i}(\vec{q}_{-i})\right)\left(v_i(q_i) - b_i(q_i)\right)\right] = \\ & E_{q_i}\left[x_i^{\mathrm{GFP}}(b_i(q_i))\left(v_i(q_i) - b_i(q_i)\right)\right] \end{aligned} \tag{6}$$

A key observation is: the expected utility (Eq. (5)) of bidder i in a PDM with Mye can be written exactly in the same way:

$$U_i^{\mathrm{Mye}}(\hat{\phi}_i, \hat{\phi}_{-i}) = E_{q_i}\left[x_i^{\mathrm{GFP}}(\hat{\phi}_i(q_i))\left(v_i(q_i) - \hat{\phi}_i(q_i)\right)\right] \tag{7}$$

This is because $X_i^{(\mathrm{Mye},\hat{F})}(\vec{q}) = X_i^{\mathrm{GFP}}(\hat{\phi}(\vec{q}))$ by definition (assuming that ties in GFP are broken in the same way as ties are broken in the virtual welfare maximization of Mye).

We are ready to show the main result of this section. We say a strategy s_i of PDM is normal if it satisfies the three properties in Lemma 3.1 (identity permutation, monotonicity, and non-negative virtual value).

Theorem 1 (Mye = GFP) Myerson's auction under PDM is equivalent to the generalized first-price auction. Specifically, there is a bijection between all normal strategy profiles $\vec{s} = (s_i, s_{-i})$ of PDM and all non-increasing strategy profiles $\vec{b} = (b_i, b_{-i})$ of GFP, such that $\vec{s}$ and $\vec{b}$ produce the same expected utility and expected payment for each bidder, and the same revenue for the auctioneer.

Proof We can equate s_i and $\hat{\phi}_i$ because: each normal strategy s_i has a non-increasing and non-negative virtual value function $\hat{\phi}_i$; and by Lemma 2.1, each non-increasing and non-negative $\hat{\phi}_i$ determines a unique normal strategy s_i. Thus by setting $\hat{\phi}_i = b_i$ for each i, the bijection between normal $\vec{s}$ and monotone $\vec{b}$ is established.

Then by the above observations (Eqs. (6) and (7)), we have

$$U_i^{\mathrm{GFP}}(b_i, b_{-i}) = U_i^{\mathrm{Mye}}(\hat{\phi}_i, \hat{\phi}_{-i}).$$

Furthermore, for any $\vec{v}(\vec{q})$, we have the same allocation in the two auctions: $X_i^{(\mathrm{Mye},\hat{F})}(\vec{q}) = X_i^{\mathrm{GFP}}(\vec{b}(\vec{q}))$. By subtracting the value term from the expected utility, we conclude that the expected payment (and thus the expected revenue) is the same in the two auctions. □

Corollary 1 A normal strategy profile $\vec{s}$ is a Nash equilibrium

of Mye under PDM if and only if the corresponding monotone strategy vector $\vec{b}$ is a BNE of GFP.

4 Equivalence in Sponsored Search Auctions

In this section, we focus our attention on the sponsored search auction. We first introduce some necessary notions.

Recall that γ_j is the quality of slot/item j, v_i is the value per quality of bidder i. Let b_i be the bid by bidder i. Let $\mathrm{X}_{ij}(\vec{b})$ denote the probability that item j is allocated to bidder i, and let $P_i(\vec{b})$ denote the payment for bidder i given bid vector $\vec{b}=(b_1,b_2,\ldots,b_n)$. We use $\mathscr{M}=(X,\vec{P})$ to denote a mechanism $\mathscr{M}$ that consists of an allocation matrix and payment vector (both as functions of the bid vector), where $X=(X_{ij})_{n\times m}$ and $\vec{P}=(P_1,P_2,\ldots,P_n)$. The allocation vector to bidder i is $\vec{X}_{i\cdot}$. We assume here that a bidder's value across items is additive. Let $X_i=\sum_j X_{ij}\gamma_j$ be the total quality allocated to bidder i. Let $v_i(\mathrm{X})=v(\vec{X}_{i\cdot})=\sum_j X_{ij}\cdot v_i\gamma_j=v_i\cdot\sum_j X_{ij}\gamma_j=v_iX_i$. Basically, $v_i(\mathrm{X})$ is the total value of i in the auction outcome specified by X.

In a sponsored search auction, we say the auction is feasible, if each bidder is allocated at most one ad slot, and each slot is allocated to at most one bidder. This can be interpreted by the following constraints:

$$\begin{aligned}&\text{Unit demand}: \sum_{j=1}^{m} X_{ij}(\vec{b}) \leqslant 1, && \forall \vec{b}, i;\\ &\text{Avoid over allocation}: \sum_{i=1}^{n} X_{ij}(\vec{b}) \leqslant 1, && \forall \vec{b}, j;\\ &\text{Indivisible ad slots}: X_{ij}(\vec{b}) \in \{0,1\}, && \forall \vec{b}, i, j.\end{aligned}$$

We will call them feasibility constraints for the rest of the paper.

It is not hard to verify that the sponsored search auction setting defined above is a single-parameter environment: $X_i(\vec{b})=\sum_j X_{ij}\gamma_j$ is the equivalent allocation rule based on quality; the feasibility constraints form a feasible domain that is downward-closed. Thus, the prior-dependent revenue-optimal auction in the sponsored search auction setting is a special case of the (generalized) Myerson's auction in Section 3.1: bidders with the i-th highest positive ironed virtual value gets the i-th ad slot, and payments follow from the payment rule of Myerson's auction (Lemma 2).

4.1 Prior-independent auctions

Here we introduce some prior-independent sponsored search auctions that are widely used in practice. Sort the bids of bidders non-increasingly, $b_1\geqslant b_2\geqslant\cdots\geqslant b_n$. Let $\gamma_1\geqslant\gamma_2\geqslant\cdots\geqslant\gamma_m$ denote the qualities of ad slots.

- GFP: In a GFP, each bidder $i\in[m]=\{1,2,\ldots,m\}$ gets slot i, pays $b_i\cdot\gamma_i$.
- GSP: In a GSP, each bidder $i\in[m]$ gets slot i, pays $b_{i+1}\cdot\gamma_i$.
- VCG auction: In a VCG auction, each bidder $i\in[m]$ gets slot i, pays $\sum_{j=i+1}^{m+1} b_j\cdot(\gamma_{j-1}-\gamma_j)$. (Here we denote $\gamma_{m+1}=0$.) VCG auction is incentive-compatible: bidders bid their values truthfully at equilibrium.

4.2 PDM game in sponsored search auction

Now we focus on the PDM game played in sponsored search auction, and study the equivalence phenomenon concerning various sponsored search auctions: Mye, GFP, GSP, and VCG.

We say a BNE is efficient if for any value vector $\vec{v}$ with corresponding bid vector $\vec{b}=\vec{b}(\vec{v})$, the resulting allocation $\mathrm{X}(\vec{b})$ is feasible and maximizes the social welfare $\sum_i v_iX_i$. When buyers have I.I.D. value distribution, we say a BNE is symmetric if buyers use a same bidding strategy. We first introduce some useful lemmas.

Lemma 6[48] In a GFP, when there are n bidders with I.I.D. continuous distribution on values, there is only one BNE, and this BNE is symmetric and efficient.

Lemma 7[15] In a GSP, when there are n bidders with I.I.D. continuous distribution on values, if a symmetric BNE exists, then it is efficient.

Lemma 8 (Classical revenue equivalence theorem[49]) For any two mechanisms that both have efficient Bayes-Nash equilibria, if bidders pay 0 when having value 0 in both mechanisms, then the two mechanisms have the same expected revenue.

We are now ready to show our equivalence results. First, since the sponsored search auction is a single-parameter environment, by Theorem 1 we immediately conclude that:

Theorem 2 In the PDM game under the sponsored search auction scenario, Mye is equivalent to GFP.

Then, we show that when buyers have I.I.D. values, Mye, GFP, and VCG are always equivalent, while GSP are equivalent to them in some cases.

Theorem 3 When buyers have I.I.D. continuous values, Mye, GFP, and VCG are revenue-equivalent in their respective equilibria under PDM.

Proof We just need to show the equivalence between GFP and VCG. By Lemma 4, we know that the equilibrium of GFP under PDM is exactly the same as the traditional BNE. By Lemma 6, we know that the equilibrium of GFP is efficient. Since VCG is also efficient, GFP and VCG are revenue-equivalent by Lemma 8. □

GFP and VCG may not be equivalent if buyers have independent but non-identical distributions.

Example 1 (Example 2 in Ref. [50]) Suppose there is one item and two buyers. Buyer 1 has a uniformly random value v_1 in $\left[0,\frac{1}{1+z}\right]$, buyer 2 has a uniformly random value v_2 in $U\left[0,\frac{1}{1-z}\right]$, for $z\geqslant 0$. In equilibrium, the bidders' inverse bidding functions are

$$v_1=b_1^{-1}(b)=\frac{2b}{1+z(2b)^2},\qquad v_2=b_2^{-1}(b)=\frac{2b}{1-z(2b)^2}.$$

The CDF $G_{\mathrm{FP}}(b)$ of the larger bid is:

$$G_{\mathrm{FP}}(b)=\Pr[b_1(v_1)\leqslant b]\cdot\Pr[b_2(v_2)\leqslant b]=\frac{(1-z^2)(2b)^2}{1-z^2(2b)^4},$$

which is decreasing in z. Thus the revenue of GFP (the first-price auction in this case) is increasing in z. However, for the VCG auction (the second-price auction in this case), the CDF $G_{\mathrm{SP}}(b)$ of the second value is

$$G_{\mathrm{SP}}(b)=1-\Pr[v_1>b]\cdot\Pr[v_2>b]=2b-(1-z^2)b^2,$$

which increases with z. Since the two auctions have the same revenue when $z=0$ (the I.I.D. case), the revenue of the first-price auction is strictly better than that of the second-price auction when $z>0$.

Theorem 4 When the I.I.D. continuous value distribution of buyers and the qualities of slots satisfy certain conditions such that GSP has a symmetric BNE, Mye, GFP, VCG, and GSP are revenue-equivalent in their respective symmetric equilibria under PDM.

Proof We only need to show the equivalence between GSP and VCG. By Lemma 4, we know that the equilibrium of GSP under PDM is the same as the traditional BNE. By Lemma 7, the symmetric equilibrium of GSP must be efficient. Then by Lemma 8, we know that GSP is revenue-equivalent to VCG. □

Finally, we demonstrate by two examples that GSP may not have a symmetric BNE, and may admit an asymmetric BNE, even if buyers have I.I.D. values. Our equivalence results concerning GSP do not hold in these two situations.

Example 2 (Example 1 in Ref. [15]) Consider three buyers with I.I.D. values from Uniform$[0,1]$ and two slots with qualities $(1,\gamma_2)$. A symmetric BNE exists if and only if $\gamma_2 \leqslant 0.75$.

Example 3 (Section 3.1 in Ref. [21]) Consider three buyers with I.I.D. values from Uniform$[1,2]$ and three slots with qualities $\left(1,\frac{1}{2},\frac{1}{2}\right)$. There is an asymmetric equilibrium: $b_1(v_1)=v_1$, $b_2(v_2)=b_3(v_3)=0$. Clearly buyer 1 is in equilibrium. To see that buyers $i=2,3$ are in equilibrium, suppose i has value $v_i>0$. If i bids $b\leqslant 1$, then their utility is $\frac{1}{2}v_i-0$, and if they bids $b>1$, their utility would

$$v_i\Pr[v_1\leqslant b]+\frac{1}{2}v_i\Pr[v_1>b]-\int_1^b v_1 \mathrm{d}v_1=\frac{1}{2}bv_i-\frac{b^2-1}{2}\leqslant\frac{1}{2}v_i.$$

5 Summary and Discussion

In this paper, we studied a repeated auction scenario where a seller sells multiple items to several buyers, with the sponsored search auction as an example. The seller uses a mechanism whose allocation and pricing policy may depend on the value distributions submitted by the bidders. Bidders best respond by choosing some distributions to submit. We showed that when the seller uses the Myerson's auction, which is truthful and revenue-maximizing with respect to the submitted distribution, bidders will not be truthful and will submit a manipulated distribution such that they obtain the same utilities and the seller obtains the same revenue as the generalized first-price auction. In the sponsored search auction scenario specifically, Mye and GFP are further equivalent to VCG and GSP under additional assumptions.

Our results could partially explain why Myerson's auction is not as popular as GFP and GSP in practice. Advertisers' manipulation of data/distributions eliminates Myerson's auction's power of revenue maximization.

With this work, we provide a new perspective on mechanism design and data-driven decision making. In general, we believe that this is a direction of great importance, with lots of challenges and opportunities. We conclude by discussing some possible extensions and open questions:

- **Extension to correlated values**. In this work we assumed that bidders have independent values. When bidders' values are correlated, there exists an auction by Crémer and McLean[51] that can extract full surplus from the bidders, if bidders' correlated value distribution is discrete and satisfies the condition that the matrices of conditional probabilities have full rank. If we use the Crémer–McLean auction in the PDM game and impose the requirement that bidders' reported correlated distribution must satisfy the Crémer–McLean condition, then we can prove that the Crémer-McLean auction is also equivalent to the first-price auction. However, since the Crémer–McLean auction is even more complicated and less practical than the Myerson's auction, we think that exploring other simpler and more practical auctions for correlated values in the PDM game is more meaningful.
- **Revisit the notion of "truthfulness" under the PDM setting**. We have shown that traditionally truthful auctions (like Mye) can be non-truthful under PDM, if the auction is prior-dependent. On the other hand, traditionally truthful prior-independent auctions like VCG are still truthful under PDM. Can we characterize the set of all truthful mechanisms under the PDM setting? Is there a truthful mechanism that achieves better revenue than Mye/GFP? To optimize revenue in the PDM setting, can we still "without loss of generality" restrict attention to truthful mechanisms according to the revelation principle?
- **Semi-prior-dependent auction**. One possible way to design auctions with higher revenue than Mye/GFP in the PDM setting is to design "semi-prior-dependent" auctions whose allocation and payment rules for a bidder i do not depend on i's reported distribution but may depend on other bidders' reports. For example, a second-price auction with personalized reserves where each bidder's reserve price is determined from other bidders' distributions coincides with the original Myerson's auction and is hence revenue-optimal if bidders have I.I.D. regular distributions. For general distributions what is the optimal semi-prior-dependent auction?
- **Parameterized learning**. If the seller can reconstruct the whole bid distribution, and use Mye on that, then the PDM game becomes equivalent to the (generalized) first-price auction. If the seller can only do parameterized learning, and use Mye, and bidders respond by parameterized distributions, then the revenue can also be less than Mye[40]. Whether it is higher than the revenue of first-price auction is open.
- **Connections with more mechanisms**. Can we find more equivalence/comparison results for prior-dependent mechanisms beyond Mye, in particular more practical mechanisms, e.g., anonymous pricing, anonymous reserve, and sequential posted pricing?
- **Apply the PDM framework to multi-parameter auction environments and to other Bayesian games**. Recent works[52, 53] have made such attempts.
- **Discrete value space and bid space**. Our "Mye=GFP" result holds for arbitrary value space but only for continuous bid space. The "Mye=GFP=VCG=GSP" result relies on continuous value space and bid space. Discrete bid space is practically motivated and worth studying.

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