



Exploring teachers' eye-tracking data and professional noticing when viewing a 360 video of elementary mathematics

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Abstract

Research incorporating either eye-tracking technology or immersive technology (virtual reality and 360 video) into studying teachers' professional noticing is recent. Yet, such technologies allow a better understanding of the embodied nature of professional noticing. Thus, the goal of the current study is to examine how teachers' eye-gaze in immersive representations of practice correspond to their attending to children's mathematics. Using a mixed methods approach, we incorporated eye-tracking technology embedded within a virtual reality environment to compare novice and expert teachers' gaze duration with quality of professional noticing. Findings and results both corroborate and extend previous research evidence about important differences in professional noticing between expert and novice teachers. Specifically, the amount of experience, and thus familiarity, teachers have with being in a classroom may affect their physical movement in both real and virtual representations of practice. Additionally, findings and results emphasize the importance of teachers' visual focus on students' doing of mathematics across the classroom.

Keywords Professional teacher noticing · Eye-tracking · 360 video · 360-degree video

Introduction

Teachers engaged in professional noticing in mathematics lessons *attend to* students' strategies and descriptions, *interpret* their mathematical thinking, and use these to *shape* the lesson moving forward (Jacobs et al., 2010; van Es & Sherin, 2021). A significant portion of research on professional noticing has focused on the *attending* component (Estapa & Amador, 2023; Santagata et al., 2021; Scheiner, 2016), with scholars investigating differences in attending due to knowledge and experience (Jong et al., 2021; Yang et al., 2021), how attending affects other facets of noticing (Amador et al.,

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2021; Thomas et al., 2022; van den Kieboom, 2021), how to facilitate better attending in teachers (Simpson & Haltiwanger, 2017; van Es et al., 2017), and so forth.

Research on teachers' attending has primarily framed and analyzed it from a cognitive psychological perspective, by examining noticing as informed by knowledge, beliefs, and so forth (König et al., 2022; Scheiner, 2021). However, "noticing includes embodied ways of accessing and exploring the world of classroom events" (Scheiner, 2021, p. 86). Thus, teachers' abilities to attend to pedagogical events are informed not only from psychological sources (knowledge, beliefs, etc.), but also embodied input (i.e., visual, spatial, and auditory sensory & physiological data). Despite the embodied nature of noticing, there is significantly less scholarship studying this facet, leading to a need for further empirical study and theorization on the physiological nature of attending (Kosko et al., 2022b; Walshe & Driver, 2019).

To be clear, various scholars have studied physiological data related to noticing. This includes use of eye-tracking technology to study teachers' noticing of classroom management events when viewing standard video of classrooms (Dessus et al., 2016; Stahnke & Blömeke, 2021) or in a real classroom via mobile eye-tracking glasses (Huang et al., 2021a, 2021b; Keller et al., 2021). Recently, scholars studying more nuanced aspects of noticing have examined teachers' field of view (FOV) when watching 360 videos of classroom practice (Kosko et al., 2021; Gold & Windscheid, 2020; Weston & Amador, 2021). Such 360 videos record omnidirectionally and present a spherical view of the recorded classroom. By examining changes in where in this spherical view teachers focus their attention, scholars have drawn connections between the directionality of FOV and the sophistication in teachers' interpreting students' reasoning (Kosko et al., 2022b; Weston & Amador, 2021).

Despite the revealing findings offered by incorporation of eye-tracking technology and FOV (i.e., virtual reality such as 360 video), studying professional noticing using such approaches is relatively recent (Dindyal et al., 2021; Santagata et al., 2021; Strohmaier et al., 2020). To this end, we consider both technologies as measuring two different aspects of the same physiological perceptual system (Gibson, 1966), with FOV (recorded from viewing 360 video) examining head/body directionality and eye-tracking focusing specifically on the eyes. The current paper sought to bridge the divide between applications of both technologies with teachers' articulated (written) noticing to study the relationship between teachers' physiological perception and their articulated (written) attending. Because there is evidence professional experience affects both embodied (Huang et al., 2021a, 2021b) and psychological (Yang et al., 2021) indicators of attending, we focused our analysis of this phenomenon on experienced and novice elementary teachers. Thus, we conducted an exploratory study using eye-tracking virtual reality (VR) technology to examine experienced and novice teachers' professional noticing of an elementary mathematics lesson on multiplication. In applying such novel technology, we sought to expand scholarship on the embodied nature of teachers' professional noticing of mathematics through analyzing how physiological data (eye-gaze and FOV) corresponds with data traditionally associated with a more cognitive psychological perspective (i.e., written noticing). Therefore, the purpose of this study is to examine how teachers' gaze behaviors in an immersive representation of practice corresponds with their professional noticing of children's mathematics.

Theoretical framework

Professional noticing

Professional noticing involves a teachers' ability to attend to and interpret pedagogical events "that are relevant to the process of learning" (Meschede et al., 2017, pp. 158–159). Many early career teachers struggle "to provide evidence of attending to children's [reasoning]" (Jacobs et al., 2010, p. 194). Rather, noticing develops from less to more specific. Initially, teachers may attend to generic actions of student participation or behavior in the class or, alternatively, focus heavily on the teacher's classroom management or facilitation of the lesson (Jacobs et al., 2010; van Es et al., 2017). As teachers learn to attend more to students' content-related actions, they initially focus on students' ability to produce correct answers and then on students' procedural reasoning. The most sophisticated form of attending involves attending to students' conceptual reasoning (Barnhart & van Es, 2015). Noticing also involves interpreting students' reasoning and shaping the lesson to move forward, with the three components often interacting or even intermingling (van Es & Sherin, 2021). Thus, we use *noticing* to refer to the construct more broadly and *attending* to focus on that particular aspect of noticing. We distinguish attending from *perceiving*, in that attending is an abstraction of perceiving because it involves "selecting certain stimuli" (Scheiner, 2016, p. 231) that are perceived.

Comparisons of novice and experienced teachers have consistently shown that novice teachers (i.e., preservice teachers) initially engage in less sophisticated noticing (i.e., student engagement/behavior), whereas more experienced teachers attend to students' procedural and conceptual reasoning more consistently (Huang & Li, 2012; Wolff et al., 2015; Yang et al., 2021). For example, Krull et al. (2007) compared experienced and novice teachers' professional noticing of a seventh-grade Estonian grammar lesson. They found that experienced teachers attended to more specific pedagogical events and their written descriptive comments were twice as long. Conducting a similar comparison with experienced and novice secondary mathematics teachers in China, Huang and Li (2012) found that experienced teachers "are more sensitive to the student-centered teaching events, while the novice teachers, who may be struggling with effective guidance of students, paid close attention to that aspect" (p. 429). Plöger et al. (2019) compared experienced and novice science teachers in Germany and observed similar trends. Particularly, experienced teachers described more events including effective pedagogy and did so more frequently. Examining a sample of Dutch teachers, Wolff et al. (2015) found that novice teachers attended more to student behavior and the teacher's classroom management while experienced teachers focused more heavily on the content at hand and students' reasoning of it.

Yang et al. (2021) conducted a large-scale study including 152 novice (preservice) teachers, 162 early career teachers (1–5 years of experience) and 123 experienced teachers (15 or more years of experience) in Chinese secondary mathematics classrooms. Using differential item functioning (DIF) via Rasch modeling, Yang et al. (2021) found little difference between novice and early career teachers, but substantial differences between both sets of less experienced teachers to veterans with 15+ years of experience. Indeed, Yang et al. (2021) noted that "almost a linear growth in teacher noticing can be traced as teaching experience increases" (p. 37).

It is worth noting that different scholars caution against viewing years of experience as deterministic in improving one's professional noticing, as there is often variance in the sophistication of professional noticing across experience levels (Jacobs et al., 2010; Yang

et al., 2021). For example, studying primary grades teachers in Germany, Meschede et al. (2017) found that while differences in noticing sophistication between novice and experienced teachers were evident, a portion of such differences was likely due to differences in pedagogical content knowledge (PCK). The interrelationship led Meschede et al. (2017) to conjecture that while PCK appears to facilitate noticing, there is an aspect of noticing that “requires a decisive amount of practical experience” (p. 168). In summary, classroom experience alone does not *determine* the quality of a teacher’s professional noticing, but experience is a significant factor that cannot be discounted.

Physiological indicators of experience

One way the role of experience has been examined is through eye-tracking studies of teachers’ attending behavior. Eye-tracking equipment examines the direction and duration a participant’s pupils gaze down to the millisecond. Nearly all such research in teacher education has focused on a teacher’s ability to notice aspects related to classroom management and student behavior (Dessus et al., 2016; Huang et al., 2021a; Stahnke & Blömeke, 2021; van den Bogert et al., 2014). Such scholarship has consistently shown that experienced teachers’ gaze durations when viewing classroom practice is shorter than novice teachers (Huang et al., 2021a; van den Bogert et al., 2014), but that experienced teachers tend to have a more even gaze duration distribution (Cortina et al., 2015; Dessus et al., 2016; van den Bogert et al., 2014). Moreover, experienced teachers “exhibit fewer task-irrelevant fixations with shorter durations, they also direct more fixations to students” (Huang et al., 2021a, p. 11) and to a wider range of students. Huang et al. (2021a) observed that novices tend to focus more on students proximally closer to them than those further away. Lastly, novice teachers spend substantially more time focusing on the recorded classroom teacher than experienced teachers—and thus, less time focusing on students generally (Huang et al., 2021a; Stahnke & Blömeke, 2021).

Distinct from much of the eye-tracking literature is a recent study by Wyss et al. (2021) who compared teacher educators with student teachers and examined the descriptions of what participants “saw” in detail. Although Wyss et al. (2021) did notice a similar pattern as prior literature related to the gaze time and fixation counts of experienced and novice teachers, they delved deeper in examining whether participants described a particular event in the video used in the eye-tracking study. In doing so, Wyss et al. (2021) found that the statistically significant differences in gaze duration and count were due to a subset of teacher educators who attended to a key pedagogical event during interviews. Those who did not attend to this event had similar eye-gaze statistics as student teachers. Specifically, “the eye tracking proves that all the participants looked at [the key pedagogical event]... only a limited number of persons (without exception teacher educators) talked about the incident” (p. 100). Thus, a teacher may physically *perceive* an event, as measured via eye-tracking technology, but fail to *attend* to it, as assessed linguistically. Such findings also corroborate prior literature that cautions against relying too heavily on experience as a sole indicator of professional noticing (Jacobs et al., 2010; Yang et al., 2021).

A relatively few number of published studies have used eye-tracking technology to examine teachers’ professional noticing beyond the level of classroom management (i.e., Muhonen et al., 2021, 2022; Pouta et al., 2020). However, such scholarship tends to focus on the application of eye-tracking technology in think-aloud interviews related to teachers’ own instruction while wearing mobile eye-tracking glasses. Such scholarship is useful in promoting reflection on the part of participants, but it does not present comparisons of

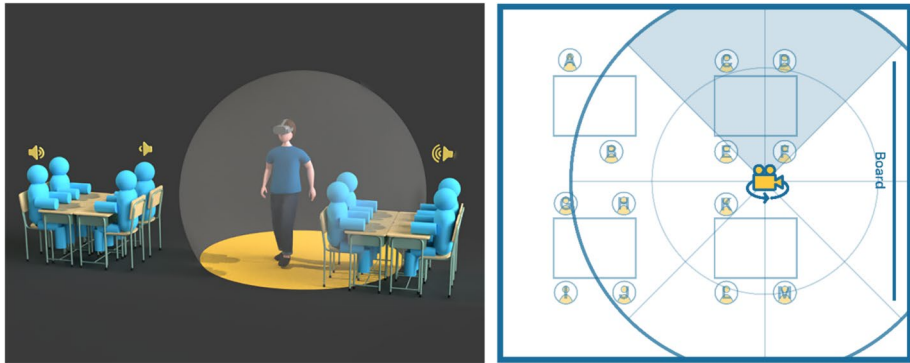


Fig. 1 Illustration of a viewer viewing a 360 video with a VR headset (left) and what is recorded from the camera placement (right) where blue is the direction someone looks of the potential field of view (entire circle)

gaze data or patterns in where participants physically attend (Muhonen et al., 2021, 2022; Pouta et al., 2020). Fortunately, there are some such comparisons regarding teachers' field of view (FOV) when viewing scenarios of instruction in VR and 360 video.

FOV is part of the visual perceptive system as described by Gibson (1966); movement of the eyes and head is a combined system used to consider visual information. Indeed, it is a fundamental feature of how the visual act of focusing is characterized by humans. Various scholars studying professional noticing have taken advantage of this fact by examining variations in teachers' FOV in VR and 360 video. Specifically, 360 video is a medium that records a video omnidirectionally by using two or more camera lenses and then stitching together the video to form a spherical recording from a fixed location. Individuals may then view the recording by turning the perspective in whichever direction they wish to look (see Fig. 1).¹ Thus, two individuals may look at different classroom tables during the same time period of a recording—their FOV is different in such instances. Teachers watching such 360 video can screen record their viewing, and the differences in FOV can be examined.

Pioneering this approach with 360 video, Kosko et al. (2021) partitioned the regions of the classroom (i.e., front-left, front-right, back-left, back-right) and statistically examined differences in viewing habits among novice teachers. Kosko et al. (2022b) drew on this approach to examine novice teachers' FOV in conjunction with their written descriptions of what they noticed. Their results showed that teachers who attended primarily to classroom management demonstrated different viewing patterns than those who attended to children's mathematical reasoning. Specifically, novice teachers who focused on classroom management adjusted their FOV to include the teacher at key points in the lesson, whereas those who attended to children's reasoning had a more student-centered FOV (i.e., spent more time looking at students). Weston and Amador (2021) observed a similar trend when qualitatively examining one student teacher's viewing of their own instruction recorded with 360 video. Different from Kosko

¹ This technology is distinctly different from standard video that tracks individuals, as 360 video allows the viewer to choose what is in the field of view and videos that track individuals include content chosen by the videographer.

et al.'s (2021) analysis, Weston and Amador (2021) examined the student teacher's viewing over time and found that the ability to change their FOV in rewatching their own instruction allowed a shift from teacher-centered to more student-centered noticing. Examinations of teachers' FOV in both 360 video and animated VR environments have yielded useful quantitative and qualitative data in studying what and how teachers notice (Buchbinder et al., 2021; Gandolfi et al., 2021; Gold & Windscheid, 2020; Huang et al., 2021b).

In sum, the literature on eye-tracking and professional noticing is predominately focused on noticing of events related to classroom management. Such research consistently reports that experienced teachers attend to more events and locations and spend more time focusing on students than the recorded teacher (Dessus et al., 2016; Huang et al., 2021a; Stahnke & Blömeke, 2021; van den Bogert et al., 2014). While such differences are not due to experience alone, they have been primarily attributed to experienced teachers with better demonstrated noticing (Wyss et al., 2021). These findings correspond with the emergent literature incorporating analysis of novice teachers' FOV (Kosko et al., 2021, 2022b; Huang et al., 2021b; Weston & Amador, 2021).

Unfortunately, much of this latter literature does not include any analysis of experienced teachers' noticing behavior and is limited in that eye-tracking data are not incorporated with FOV data. Stated differently, both eye-tracking and FOV approaches are lacking in the absence of one form of data from the other. This is a critical limitation given that the direction the head turns and patterns of eye-gaze are interconnected (Gibson, 1966). Due to the state of technology during times of study, prior eye-tracking scholarship has used standard video and thus given participants a forced choice in their provided FOV, which is known to significantly affect what and how teachers notice (Kosko et al., 2021). In a similar manner, FOV approaches have not included more specific eye-tracking data.

Both sets of research data (eye-tracking and FOV recorded from VR) do an excellent job of addressing *what* they may be perceiving but not necessarily *how* such perceptual focus may inform teachers' attending—even when triangulating with written or spoken data. Knowing *how* is a key factor in understanding *why* noticing happens (or does not) and what is necessary to improve attending. The present study seeks to bridge the gap between such approaches to better understand the underlying nature of professional noticing. We conjecture that incorporating both eye-tracking and FOV data, and triangulating such data with teachers written noticing, will improve the field's underlying understanding of the nature relationship between what teachers perceive and attend.

Context of the present study

The present study integrates eye-tracking technology with VR-based representations of mathematics teaching to study how teachers' gaze behaviors correspond with their professional noticing of children's mathematics. Following Gibson (1966), we conjecture that such integration provides a more complete report of the visual-spatial system. The method was created to examine how teachers' gaze behaviors in a 360 video, where FOV is accounted for, correspond with their professional noticing of children's mathematics. We focus specifically on comparing experienced and novice teachers given prior evidence that teaching experience may affect embodied (Huang et al., 2021a, 2021b) and psychological (Yang et al., 2021) indicators of attending. Thus, we sought to answer the following research questions:

1. Do experienced and novice teachers differ regarding the distribution of gaze duration when viewing the same 360 video of an elementary mathematics classroom?
2. How does distribution of gaze duration correspond with teachers' quality of professional noticing of students' mathematics?

Methods

Participants

Participants represented a convenience sample collected in fall 2021. It included 10 novice, preservice teachers (PSTs) and four experienced teachers ($n = 14$). All participants identified as white with 12 identifying as female and two as male. The novices included nine first-semester freshmen and one first-semester junior enrolled in a teaching licensure program at a Midwest US university. The junior was enrolled in a required educational technology course, and the freshmen were enrolled in a first-year experience course required by the university. All PSTs participated in the first half of the fall semester, and none had yet had a university-assigned field experience in a primary grades classroom. Novice teachers included those majoring preparing to teach primary school for grades pre-K-5, or age 3–11 ($n = 8$), and for grades 4–9, or age 9–14 ($n = 2$). Experienced teachers included three mathematics specialists working in public schools² and one fourth-grade mathematics teacher. Each individual had over 10 years of experience teaching mathematics in the classroom, with the three mathematics specialists having more than five years working in that capacity. One mathematics specialist had earned her doctorate, whereas all other experienced teachers had a master's degree.

Procedure

Participants engaged in a one-hour session during fall 2021 where they were asked to view a series of 360 videos using a Pico Neo 3 Eye VR headset. Following consent, participants were provided a VR face mask (SpiderTech strapless face cover) specially designed so that exhaled air would be redirected so as to not fog up the lenses on the VR headset.³ Next, they put on the eye-tracking headset and were guided through the process of eye calibration and then watched the first 360 video—a tutorial on how to watch a 360 video. The tutorial engages the viewer in turning around and looking at different points in the recorded room as some first-time viewers of 360 video need this brief introduction to engage more normally with the medium.

Following the tutorial, participants kept the headset on and viewed a second 360 video of a third-grade class engaged in an informal introduction to the commutative property of multiplication. The video was 6 min, 59 s long and participants watched without pausing (i.e., as if they were present in the classroom). In this 360 video, the teacher asked students to cover a 7-by-8 array with Cuisenaire rods (a concrete

² Mathematics specialists in this study worked with teachers at multiple schools to provide professional development and support for teaching primary grades mathematics.

³ This was a necessary requirement given mask protocols for the COVID pandemic at time of data collection. Additionally, all equipment was thoroughly sanitized using 75% alcohol wipes.

mathematics manipulative where rods of the same centimeter-based length are the same color, like 1 cm/white, 2 cm/red, 7 cm/black, 8 cm/brown, etc.). After students settled on covering the array with either eight black rods ($8 \text{ rods} \times 7 \text{ cm}$) or seven brown rods ($7 \text{ rods} \times 8 \text{ cm}$), the teacher asked whether it was possible to place the arrangement of black rods on top of an arrangement of brown rods and had students explore whether it was possible and then why it was possible. The video ends with a brief class discussion as students begin to recognize why the commutative property worked with the rods (i.e., $8 \times 7 = 7 \times 8$). The 360 camera was placed between the front two tables, allowing participants to view both these tables and the two tables at the back of the room (see Fig. 2).

After viewing the 360 video, participants then removed their headsets and wrote a response to two prompts:

- Now that you have watched the video, describe any and all pivotal moments regarding students' mathematics that you noticed during the lesson.

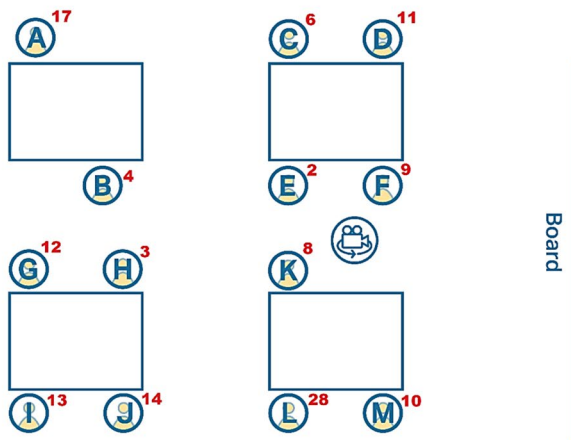


Fig. 2 Machine learning designations for recorded students and teachers, and classroom map

- Review your list of pivotal moments in the prior prompt. Select 2–3 pivotal moments that you believe are most important for an elementary teacher to pay attention. For each of your selected moments, describe:
 - What was the most salient part of this pivotal moment?
 - Why did you select this pivotal moment from your list?

The specific prompts used to solicit written noticing affects what is noticed (Estapa & Amador, 2023). Our wording for the above prompts was crafted to elicit descriptions of students' actions. By asking participants to describe "pivotal moments," we sought to elicit more specificity in detail (i.e., referencing moments) without such descriptions being exhaustive (i.e., the moments are pivotal).

Equipment and technology

Participants wore the Pico Neo 2 Eye VR headset and viewed the 360 video using the GazeXR platform (Author3 et al., 2021). The Pico Neo 2 Eye integrates Tobii eye-tracking technology with 90 Hz binocular gaze data output and accuracy within 0.5 degrees. The system uses 5-point calibration to ensure eye-tracking is aligned with the individual user. Once calibrated, participants opened GazeXR. As participants watched the 360 video, GazeXR used a private Wi-Fi network to transfer raw gaze data to a separate computer directly connected to the Wi-Fi router. A machine learning algorithm assigned identifiers to each recorded individual in the 360 video (students and adults). Raw gaze data were aligned with where recorded individuals were at specific times (see Fig. 2) and a report was produced regarding the time point and gaze duration on specific individuals. Illustrated in the top image of Fig. 2 are blue bounding boxes around individuals with numeric identifiers in red, with a corresponding classroom map used for analysis below it. Additionally, these data were used to generate a heat map visually illustrating the distribution of gaze data. The raw gaze data, the machine learning output, and heat map feature were used for analysis in the present study.

Analysis

We incorporated a data transformation variant of a convergent mixed methods design (Creswell & Plano Clark, 2018). As described by Creswell and Plano Clark (2018), a data transformation variant mixed methods design collects both qualitative and quantitative data and performs appropriate analyses on each. However, following qualitative analysis, findings are *quantitized* for incorporation into the quantitative analysis. In the present study, qualitative findings were quantitized into a nominal variable. Although such designs often prioritize quantitative results over qualitative findings, we consider both forms of data to have equivalent weighting in the current analysis. Customarily for mixed methods, research questions will correspond with particular methods, with certain questions focusing on results and findings yielded from the merging of data. In the present study, the first research question was answered using quantitative analysis to determine whether teachers' distribution of eye-gaze duration differ. We used both the qualitative findings and quantitative results, through the data transformation component of our approach, to answer the second research question of how distribution of gaze duration corresponded with teachers' quality of professional noticing.

Qualitative analysis

We adapted van Es et al.'s (2017) coding framework for noticing ambitious pedagogy using systemic functional linguistics (SFL). SFL is a form of discourse analysis that examines how grammar functions to convey meaning (Halliday & Matthiessen, 2004) across a text. In SFL, the clause (i.e., a sentence) includes nominal groups (i.e., nouns or word groupings that function as nouns), transitive processes (i.e., verbs), and various other facets of grammar that convey meaning. Nominal groups often serve as references that point to something previously noted in the discourse or something introduced for the first time (Eggins, 2004), with transitive processes often conveying how these references are picked up and conveyed across a text. As these references are incorporated throughout a discourse, they form a reference chain that serves to build up further meaning regarding what is referenced. Reference chains served as the primary unit of analysis in analyzing a text, with particular focus on how teachers reference students' mathematical thinking. We examined reference chains of participants' entire responses across both prompts (i.e., the text) to allow for a richer examination of how teachers were framing students mathematical thinking. For example, in the first prompt, novice teacher Tina noted:

Most students grabbed blocks // trying to **stack** the blocks on top of the browns// immediately **saying** no they don't fit. Some [students] **chose black** // others **chose brown**. One [student] **noticed** that no matter what it's 7×8 or 8×7 . One student **mentioned** you could do 28×2 .

All nominal groups are underlined with transitive processes in bold. Notably, Tina references students in relation to how they used blocks (i.e., Cuisenaire rods) and how they stacked them and related them to two specific multiplication facts ($7 \times 8 = 8 \times 7$ and 28×2). One of these facts was attended to further in the second prompt where Tina observed:

... one student knew that it was multiplication. // They **knew** that // no matter which way it **was** laid out, it would always be 7×8 or 8×7 . // Again teachers can notice // how some students can put pictures or items toward the bigger picture.

In referencing 7×8 being the same as 8×7 above, Tina points back to her reference in the initial prompt. Her prior reference is framed by how students manipulated the Cuisenaire rods to produce the arrangements, and her reiteration conveys the multiplication facts as answers to the procedure of arranging the rods. Thus, by examining how teachers used and constructed references to the mathematics and students' reasoning, we were able to examine for attending to students mathematical reasoning. Reference chains allowed for us to examine how particular facets were attended and interpreted, as well as the level of specificity to which any such facets were described. For example, a participant could list the answers provided by each student in the classroom, or describe the procedures of each student. Each of these hypothetical scenarios (no participant did this) indicates a pattern of specificity for attending.

In the manner described above, we focused on the specificity to which teachers attended to students' mathematical thinking. Van Es et al. (2017) described three levels ranging from attending primarily to student behavior, participation, and/or engagement

(level 0), attending to students' correct/incorrect answers (level 1), and attending to students' ideas, errors, actions, etc. to draw inferences about their mathematical thinking (level 2). We used SFL to adapt this framework with particular attention to how teachers referenced students' thinking and mathematics. Thus, participants may have conveyed either a single or multiple reference chains that indicate the level of specificity they attended to students mathematics. This revised rubric is presented in Table 1.

The first and fourth authors qualitatively examined participants' writing for referents used and reference chains incorporated. Data were independently coded before examining for agreement and then reconciling our codes. Following independent coding, we calculated a weighted Kappa statistic ($\kappa=0.713$, $p<0.001$) indicating strong agreement (Lanids & Koch, 1977). SFL codings were reconciled through comparison with the rubric, which are described hereafter. We focused primarily on the presence of mathematical referents, their nature (i.e., what aspects of mathematics they reference), and the emphasis of the reference chain across the text.

As an example of this process, we describe how we examined one common referent. Prevalent across all three levels were variations to the reference " $8 \times 7 = 7 \times 8$." However, it became apparent in reviewing participants' written noticing that the meaning of $8 \times 7 = 7 \times 8$ was conveyed differently. In particular, we considered Halliday and Matthiessen's (2004) description of relational processes (e.g., is, are, equal), with specific attention to identifying relational processes. As described by Halliday and Matthiessen (2004), within an identifying relational clause "the two [nominal groups] refer to the same thing; but...there must be some difference between them" (p. 230), thereby establishing one nominal group as the token and the other as the value. This tacit hierarchy corresponds with earlier, presymbolic mathematics in which the transitive process *is* was often utilized. However, as noted by O'Halloran (2005) the emergence of mathematical symbols altered the system of transitivity in mathematics. Interestingly, the English word "is" may still be used when describing 8×7 and 7×8 , but it has a different meaning depending on the implied meaning of the speaker. In one instance, one may say " 8×7 is 7×8 " to convey that 8×7 has the same value as 7×8 (identifying relational process), whereas another person may do so to convey a symbolic process of equivalence (symbolic process); that the two expressions are wholly equivalent in which a token-value relationship is nonsensical because of the reflexive, symmetric relationship between the expressions.

This has significant implications for the transitive process used by a participant, since commutativity conveyed through an identifying process may indicate a more answer or procedural interpretation and commutativity conveyed through an operational process may indicate a more conceptual interpretation (i.e., reflexive relationship). Thus, interpretation of participants' conveyed meaning depends on what their conveyed definition of *is*

Table 1 Modification of van Es et al.'s (2017) framework for attending to student's mathematical thinking

Level	Description
0	Reference chains focus on student behavior and/or participation, or on the teacher's classroom management
1	References the mathematics at hand but focuses on students' correct or incorrect answers
2	References the mathematics at hand, but reference chains convey a focus on procedural reasoning (i.e., applying the procedure or recognizing another procedure produces the same answer)
3	Reference chains focus on students' conceptual understanding of the mathematics at hand and/or implications of their current way of thinking about the content

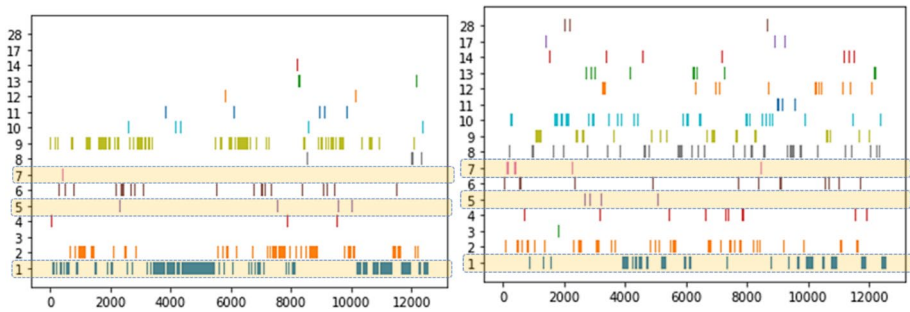


Fig. 3 Graphical illustration of ML output for a novice (left) and experienced (right) teacher

is. The simple presence of “ $8 \times 7 = 7 \times 8$ ” was insufficient alone in making such interpretation. Rather, we examined participants’ reference chains to understand how $8 \times 7 = 7 \times 8$, or similar referents, were operationalized and used throughout participants’ text. In this way, levels 1–3 in Table 1 were conveyed in the following ways:

1. Attending to students’ mathematical answers: $8 \times 7 = 7 \times 8$ is treated as an answer.
2. Attending to students’ mathematical procedures: $8 \times 7 = 7 \times 8$ is treated as a procedure
3. Focuses on conceptual understanding of $8 \times 7 = 7 \times 8$ and attending to students mathematical reasoning of the commutative property.

Quantitative analysis

Gaze data provided through GazeXR included each fixation and its duration that participants gazed at an individual.⁴ Using the ML included in GazeXR, these data were grouped by individual in the recorded scenario with gaze durations recorded in milliseconds (ms). Figure 3 provides a graphical illustration of the output for two participants, with recorded teachers in the 360 video designated by yellow boxes. Specifically, two adults in the room were assisting with instruction during recording (#5 & #7) with there being a primary instructor for the lesson (#1). All other numbered rows are students as indicated in Fig. 2. Data were also used to generate heat maps, which served as visualizations for the examination of raw gaze distribution data. Thus, we viewed such analysis as a visual form of examining descriptive statistics.

Following Cortina et al. (2015), we calculated a statistic to provide an indicator for the variance in gaze time devoted across students and teacher(s). Whereas Cortina et al. (2015) used the Gini coefficient to examine how equitable gaze data were distributed across individuals in the recorded scenario, we used the unalikeability statistic as a more parsimonious indicator of variation in nominal data.⁵ The unalikeability statistic (U_2) ranges from 0 to 1 with higher values representing a more even distribution of gaze time across individuals

⁴ Eye-gaze corresponded significantly with a participant’s FOV to the point that they were often nearly identical. This was to be expected as they are part of the same visual–spatial system. Thus, our analysis of eye-gaze here accounts for FOV in that participants’ FOV was not restricted (whereas it may be when viewing standard video).

⁵ A higher value for the U_2 statistics has a near perfect correlation with a lower value of the Gini coefficient. However, Gini has the risk of overstating inequitable distribution at the extremes.

and a lower value indicating a less equitable distribution (Kader & Perry, 2007). For example, the novice teacher on the left-hand side of Fig. 3 has a U_2 of 0.74, whereas the experienced teacher on the right-hand side has a U_2 of 0.85. The lower U_2 value coincides with a higher concentration of time on the teacher (#1) and little to no gaze time distributed to students toward the back of the room (#11, #12, #13, #14, #17, #28).

U_2 statistics between experienced and novice teachers were compared using a Wilcoxon Mann–Whitney test. The test provides a nonparametric alternative to a t-test, which is appropriate given the smaller sample size in the present study (Seigal & Castellan, 1988). We also analyzed for differences in participants' U_2 statistics in relation to their level of professional noticing. Each level of noticing was quantitized into categorical variables ($1 = \text{math answers}$; $2 = \text{math procedures}$; and $3 = \text{math reasoning}$), and a Kruskal–Wallis test was conducted to examine for differences between these groups. The Kruskal–Wallis one-way analysis of variance by ranks is a nonparametric alternative to ANOVA (Seigal & Castellan, 1988). Indeed, when compared with ANOVA “under conditions where the assumptions associated” (p. 215) are met for each analysis, the Kruskal–Wallis test has asymptotic efficiency of 95.5% (Seigal & Castellan, 1988). Furthermore, the test allows for post hoc analysis for further investigation of comparisons.

Findings and results

Qualitative findings

Of the 14 participants, all referenced the mathematics of the lesson in some capacity. No participant was found to be at van Es et al.'s (2017) level 0 in which reference chains focus on students' behaviors and participation. This is not to suggest that such participation was not referenced, but that the primary focus of reference chains regarding students was not on such generic facets.

Four participants focused on students' correct or incorrect answers (level 1) when asked to describe students' mathematics. Each of these participants were novice teachers. An excerpt of Alli's response is provided to illustrate this level of written noticing. Throughout Alli's written response were references to the teacher, their approaches to classroom management, and students' working in groups. However, Alli's references toward students focused primarily on how they conveyed their answers to the rod-based task. In the excerpt, referents are underlined with transitive processes in bold and clauses separated by // to help illustrate how this reference chain conveyed meaning across the text.

They [students] were able to **understand** that // the **black and brown** were **the same amount**. // Just **one** was 8 rods by 7 and the **other** was 7 rods by 8...

The students being able to **realize** that // they [blocks] just had to **rotate** the **rectangle to fit** on it since // it's **the same amount just written different**. They [students] **came** to the conclusion // they [blocks] were **the same amount**, just **written 8x7 and 7x8**.

Throughout the text, Alli referenced students and the mathematics interchangeably, but in key ways that conveyed a focus on answers as meaning. For example, Alli referenced students as being able to “**understand that // the black and brown were the same amount**.” Here, the use of “were” signifies a relational process as it is pointing to the arrangement of rods having the same value (i.e., “same amount”). This is reinforced with a later

reference to the rods being “the same amount just written different” and “they were the same amount.” We viewed such reference chains as clear evidence of focusing on students mathematical thinking as focusing on correct or incorrect answers (level 1).

The majority of participants ($n=7$) attended to students’ procedural reasoning (level 2). This included one experienced teacher and six novices. An excerpt from Amelia’s written reflection illustrates how such participants used the system of reference to convey a focus on procedures when describing students’ mathematics. For example, Amelia continuously references “different ways to multiply,” “many different ways to correctly solve the problem,” “different thinking process,” and “another way to solve the problem.” She references 8×7 and 7×8 as procedures and enhances this reference by noting that “the blocks all equal the same amount,” that “ 8×7 and 7×8 equal the same amount,” and that both approaches “get the correct answer.” As such, Amelia uses relational processes to convey the relationship between 8×7 and 7×8 and not a symbolic process that would be more appropriate with describing the commutative property (the focus of the 360 video lesson). Indeed, all participants observed to be at level 2 referenced the mathematics similarly.

They [students] came up with different ways to multiply 8×7 (8×7 or 7×8). [Students] found out that // the blocks all equal the same amount. [Students] showed // how they did the problem using the blocks. Described // why 8×7 and 7×8 equal the same amount. The students showed different ways to multiply (8×7 , 7×8) using blocks. This stood out // because it shows us how there are many different ways to correctly solve the problem. I chose this moment because // it reminds us that everyone has a different thinking process, and that doesn’t make their work wrong... They [students] learned another way to solve the problem and get the correct answer.

The three participants who referenced students’ conceptual understanding were all experienced teachers. Van’s excerpt is useful in seeing how both the mathematics and the nature of students’ thinking are referenced fundamentally different than in the other levels. Specifically, Van references that students “made connections to $8 \times 7 = 7 \times 8$,” and distinguishes between an understanding of the commutative property and “fact knowledge.” He expands upon this reference to the commutative property noting that many students did not believe the two arrangements of rods “would fit on each other” and that this “shows a misconception.” Rather, by having students look at the relationship between the two arrangements of rods, students could “make a mathematical connection way different than saying $7 \times 8 = 8 \times 7$.” Whereas participants at lower levels of noticing emphasized a relational understanding between the two multiplication facts, referencing that the answer was the same as justification, Van and others at level 3 focused on the sameness of the expressions both symbolically and physically. Teachers at levels 1 and 2 did not focus on the overlay of the rods, but rather focused on students seeing that they were the same total. Thus, the continued reference toward commutativity and facets within the recorded lesson that supported this property was a distinguishing feature in level 3 participants’ writing.

They made connections to $8 \times 7 = 7 \times 8$, // but it was ironic the fact knowledge did not equal understanding of commutative.

The most pivotal moment was the follow-up challenge. [The Teacher] asked // if they [rods] would fit on each other. // I heard and saw mostly “No’s” – This is pivotal because it [students saying no] shows a misconception, // but even better, [students saying no] creates a motivation to make a mathematical connection way different than saying $7 \times 8 = 8 \times 7$. // Secondly, I thought it was pivotal to model the mathematics

on the board. // **Helping young people describe their pictures using math.** // Finally, **giving tasks** that **keep students** actively **thinking about the mathematics** and **not [thinking] only a procedure.**

Across the 14 participants, there was a clear difference in how reference was used to convey the nature of students' mathematical thinking, as well as the mathematics itself. Participants attending to children's answers in the 360 video referenced $7 \times 8 = 8 \times 7$ as producing the same value. Although participants attending to procedural reasoning also referenced the produced value, these participants also focused on multiplication and the various approaches to representing the correct answer. There was a marked difference in participants attending to students' conceptual reasoning as both 7×8 and 8×7 were considered as wholly equivalent. As noted earlier in the paper, this is indicative of applying a symbolic process, even via English words, where a token–value relationship was either not present or conveyed explicitly as inappropriate by participants (i.e., Van's description of such as a misconception on the part of students). In the next section, we sought to understand these patterns in reference use and attending through participants' embodied actions via eye-tracking data.

Quantitative results

Quantitative analysis focused primarily on two comparisons. The first was to compare variance in eye-gaze duration of focus (i.e., U_2 statistics) between novice and experienced teachers. The second was to compare this variance by the level of noticing assessed in the qualitative stage of this study. To do this, qualitative themes, in the form of level of noticing, were quantitized into a categorical variable (*attend to answers* = 1; *attend to procedures* = 2; and *attend to concepts* = 3).

The first statistical comparison was between novice and experienced teachers. Results from the Mann–Whitney U test indicated a statistically significant difference between experienced teachers and novice teachers regarding distribution of focus ($U = 35$, $p = 0.036$). Specifically, experienced teachers had statistically significant and higher U_2 statistics ($M = 0.86$, $SD = 0.07$) than novice teachers ($M = 0.82$, $SD = 0.04$). Figure 4 provides

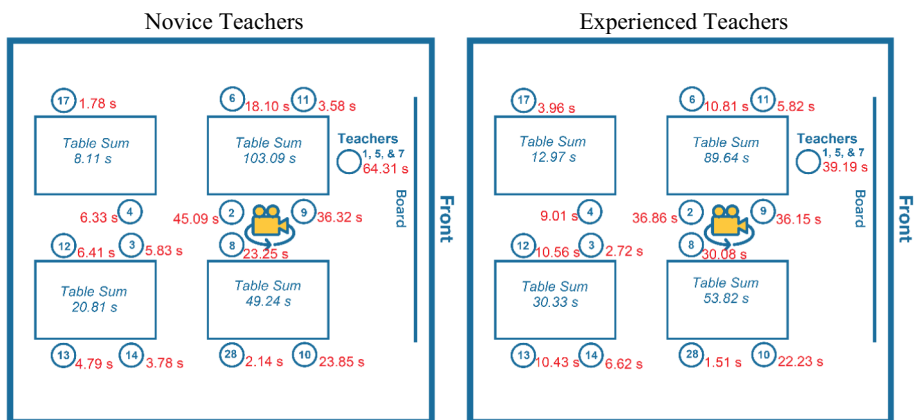


Fig. 4 Average duration of focus in seconds for novice and experienced teachers. Machine learning identifiers are indicated within each circle/student location

a graphical illustration of the average duration of focus per individual in the 360 video, which was calculated across participants in each group based on raw gaze duration data. To aid the reader, we included the sum of average seconds for students at each table, as this allows for a simpler side-by-side illustration to explain the statistically significant differences in duration of focus. For example, according to percentages calculated from data in Fig. 4, experienced teachers tended to focus on the classroom teachers 17.3% of the time, whereas novices had 26.2% of their gaze time on classroom teachers. Additionally, novice teachers spent more time focusing on students proximally closer to the 360 camera than experienced teachers, with a particularly heavy emphasis on the front-left table (see Fig. 4). This also corresponded with experienced teachers demonstrating shorter average durations of gaze on students ($M=0.308$ ms) than novice teachers ($M=0.336$ ms).

In considering these statistics regarding experienced teachers' eye-gaze, it is important to note some important observations of participant movement. For example, data from observation notes and video recordings of the sessions themselves confirmed that experienced teachers physically walked and turned their bodies significantly more than novice teachers. These observations were reflected in both the eye-gaze and FOV data. Specifically, novice teachers turned their head less, but also moved their body less (torso and walking) than experienced teachers.

Next, we examined for differences in gaze behavior in association with teachers' written professional noticing. Results from the Kruskal–Wallis test were statistically significant (H ($df=2$) = 8.184, $p=0.017$). The Dunn post hoc analysis indicated a statistically significant difference between attending to students' mathematical answers and their conceptual reasoning ($Z=9.083$, $p=0.004$), and between attending to students' mathematical answers and procedures ($Z=4.607$, $p=0.079$) at the 0.10 level.⁶ However, the difference between attending to children's procedures and conceptual reasoning was not statistically significant ($Z=4.76$, $p=0.121$).

To better understand the differences in why the observed differences between experienced and novice teachers emerged, we also examined variations in heat map data, data regarding head positionality (center of FOV) and corresponding eye-gaze data. We noticed that experienced teachers tended to look at locations physically lower, and we tested for differences in the y-coordinates of participants' gaze. The difference was found to be statistically significant ($t(df=9444)=12.718$, $p<0.001$) with experienced teachers demonstrating gaze behavior when looking at individuals at 41.88 pixels lower.⁷ In reviewing heat maps and video of participants' viewing sessions (see Fig. 5), it appears that this coincided with focusing on students' hands (using manipulatives) and table work (i.e., writing and counting). Figure 5 illustrates this with higher duration of focus (red) on more areas of the table surface where students worked for experienced teachers' eye-tracking data than novice teachers. This is highlighted with a visual examination of a magnified (zoomed-in) portion of the heat map in which novice teachers have less gaze duration than experienced teachers at the front-right table proximally closest to the 360 camera (see bottom row of images in Fig. 5).

⁶ Given the small sample size in the present study ($n=14$), a p -value of less than .10 was considered appropriate.

⁷ An important note in this observation is that eye-gaze tended to stay closer to the center of one's FOV. Thus, teachers' heads pivoted downward in a way that corresponded with other observed body movements).

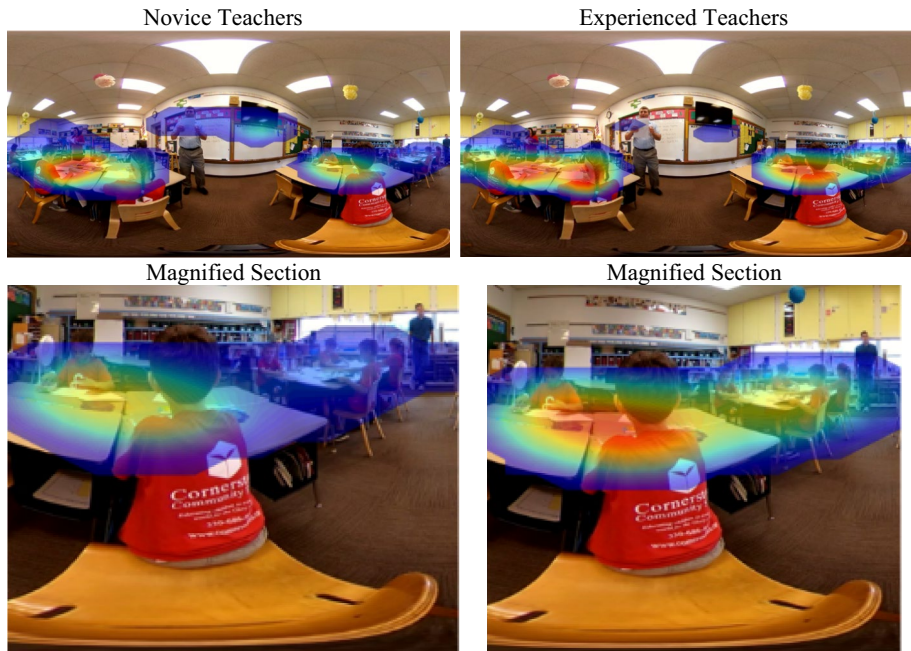


Fig. 5 Heat map for duration of focus for novice and experienced teachers, with image magnified beneath for comparison

Discussion

The purpose of this study was to examine how experienced and novice teachers' gaze behaviors in a 360 video, where FOV was accounted for, corresponded with their professional noticing of children's mathematics. As previously noted, scholars have used eye-tracking to examine teachers' gaze behaviors in viewing standard video or with self-worn eye-tracking glasses while teaching (Huang et al., 2021a; Stahnke & Blömeke, 2021; Wyss et al., 2021). The former approach presents a forced choice FOV in which participants perceived positionality in viewing the recorded scenario is decided for them. The second option of using self-worn eye-tracking glasses is more authentic, but it presents issues for comparison since most teachers in such studies are viewing different classrooms. The current study's application of eye-tracking to examining noticing in viewing 360 video provides a means for comparing more embodied forms of noticing across multiple teachers. Two research questions were created to address this gap in the literature.

The first research question set out to explore whether experienced and novice teachers differed regarding the distribution of gaze duration when they viewed the same 360 video of classroom practice. Experienced and novice teachers were found to differ in their gaze duration in several ways. First, as reported in the results, novice teachers focused on the classroom teacher for a larger portion of time (26.2%) than experienced teachers (17.3%). Second, experienced teachers demonstrated shorter average durations of gaze on students (0.308 ms) than novice teachers (0.336 ms). While such findings have been proposed by others (i.e., Huang et al., 2021a; Stahnke & Blömeke, 2021; van den Bogert et al., 2014), findings of the current study suggest it was the accumulation of these gaze

durations that were most noticeable (see Fig. 4). This finding contributes to the few studies examining classroom arrangement and eye-tracking data (Smidekova et al., 2020). Rather, both the location of students in the classroom and within groups working on a common task appeared to influence whether and for how long individual students were looked at by teachers. Finally, confirming such prior research, we observed that novices focused most of their attention on students proximally closer, and predominately at a single table.

Experienced teachers have been observed to focus on more student-centered events and attend to more content specific pedagogy (Huang & Li, 2012; Wolff et al., 2015; Yang et al., 2021). Analysis of teachers' written noticing in the present study corroborates prior research, while eye-gaze data extends it. Rather, their student-centered focus in writing corresponded with more focused attention on what students were doing mathematically (see Fig. 5). As highlighted in the findings, experienced teachers were observed to physically move (walking and turning their torsos) while wearing VR headsets significantly more than novice teachers. Further, observational data corresponded with eye-gaze and FOV data. Using FOV data, Huang et al. (2021b) observed that increased cognitive load in a VR-based environment decreased novice teachers' physical attending to events in the scenario. Thus, because 360 video presents a more immersive and realistic representation of a classroom (Ferdig & Kosko, 2020), we conjecture the amount of experience, and thus familiarity, teachers have with being in a classroom may affect their physical movement in both real and virtual representations of practice. In other words, the physical movements demonstrated by experienced teachers may represent an anticipatory scheme for attending to students in the classroom. Future research is needed to better understand the nature of this relationship, but results from the present study indicate that the relationship may be of some significance for explaining aspects of the embodied nature of noticing.

The second research question was created to explore how distribution of gaze duration corresponded with teachers' quality of professional noticing. As with the first research question, results presented here corroborate prior scholarship on teachers' eye-gaze behavior, while expanding upon interpretations of these patterns. The Kruskal–Wallis statistic indicated that across experience level (novice or not), teachers who attended to children's mathematics with a higher degree of sophistication demonstrated key differences in their eye-gaze behavior. First, teachers with higher variance in where they focused attention (i.e., U_2 statistic) tended to provide more sophisticated written descriptions of children's mathematics. Stated differently, teachers with more evenly distributed eye-gaze across children also described children's mathematics with more specificity and sophistication. This corresponds with prior scholarship using FOV to examine differences in the region of a classroom novice teachers focused attention (Kosko et al., 2021, 2022b). However, the eye-gaze data in the present study provided more specificity for why this difference is observed. Specifically, teachers with more distributed focus also attended more to students' tablework than their faces (see Fig. 5). This distinction in focusing either on students' faces or tablework is not trivial, as it corresponds with the written sophistication of teachers' noticing and appears to be associated with other aspects of physical movement beyond eye-gaze and FOV.

We observed this pattern not only between novice and experienced teachers, but also between the levels of written noticing. Notably, the post hoc analysis for the Kruskal–Wallis statistic did not find a statistically significant difference in eye-gaze behavior when teachers attended to children's procedures versus conceptual reasoning. This result may be due to sample size that the difference between these levels of reasoning is closer than the lower levels examined, or some combination. Future research is needed. Regardless, more sophisticated attending corresponded with more visual focus on students' doing of

mathematics. Again, this particular finding may sound superficial. In other words, looking at students' mathematical actions (working with manipulatives, writing mathematics, etc.) corresponds with more sophisticated noticing. Yet, abundant literature suggests attending to particular aspects of children's mathematics is quite difficult for some teachers to learn to do (Jacobs et al., 2010; Santagata et al., 2021; Simpson & Haltiwanger, 2017; van Es et al., 2017; Weston & Amador, 2021). As explained by Jacobs et al. (2010), "to interpret children's understandings, one must not only attend to children's strategies but also have sufficient understanding of the mathematical landscape to connect how those strategies reflect understanding of mathematical concepts" (p. 195). The physical act of looking at the section of desk/table in front of the child, and not merely at the child's face, seems simple. Yet, as Jacobs et al. (2010) suggested, it is likely intertwined with various other factors including experience within the classroom, with the content itself, as well as the ways that teachers move about within the classroom.

Expanding upon the above points, analysis of teachers' written attending indicated key differences in how they conveyed the meaning of the commutative property of multiplication. The three teachers who conveyed the expressions 8×7 and 7×8 as mathematically equivalent were also those observed to physically look more closely at students' deskwork for both students proximally closer and further away from them. The other teachers, including one experienced teacher, conveyed the expressions as having the same value (i.e., yielding the same answer), had less gaze time on students' deskwork, and had more uneven distribution of gaze time across students. These findings suggest a complex interplay between teachers' professional experience, mathematical knowledge for teaching, physical behaviors when observing students, and sophistication in attending to students' mathematics. However, more study is needed to further unpack these interrelationships.

Limitations

Limitations for this study include our small sample size and the fact that it was a convenient sample. A larger, more random sample of novice and experienced teachers would have allowed for both more generalizable results and more robust analysis of statistical effects. The study itself was exploratory in its nature given the relative novelty of eye-tracking applications with VR headsets. As such, other limitations include forms of data not collected in this iteration of analysis that, upon inspection of our results and related procedures, would have significantly enhanced this study. For instance, there are forms of physiological data we did not collect that appeared to have been important including tracking teacher movement (such as with a pedometer). Other physiological data such as heart rate and galvanic skin response may also have been of importance.

There are various other factors not assessed in the present study that may be of significance in explaining teachers' embodied noticing and eye-gaze data. For example, Kosko et al. (2023) observed a relationship between teachers' pedagogical content knowledge and eye-gaze data. Thus, teachers' content and pedagogical content knowledge may be important factors when examining physiological data. Additionally, affective factors such as anxiety, motivation, and self-efficacy have been tied to how teachers attend to and interpret students' reasoning (Cross Francis et al., 2022; Jong et al., 2021). Lastly, although we focused on differences between novice preservice teachers and experienced teachers, there are differences in the nature and quality of teachers' professional experience(s) that was not investigated. Not assessing these various factors (knowledge, affect, and nuances

of experience) is a limitation of the present study, but also suggests avenues for future research.

Implications for research and practice

There are several implications for this work. Theoretically, this research has provided continued evidence for the role of embodied cognition (Scheiner, 2016, 2021) in understanding professional noticing. Embodiment here refers to experienced teachers being comfortable enough to be able to walk around the room. It also refers to knowing not just what table to look at (FOV), but also where to focus attention while looking at the table (eye-tracking), and such patterns correspond to the sophistication of teachers' written noticing. Additionally, distinctions made in the present paper between perception and attending may be useful to further theorization of professional noticing and allow for better incorporation of physiological data when triangulated with teachers' written and spoken descriptions. Thus, a clear implication is that there is a need for more research on professional noticing that either incorporates or acknowledges the role that physiological data plays in what and how teachers attend to pedagogical events.

Innovative technologies such as those used in this study have examined both eye-tracking and FOV as corresponding parts of the same visual-spatial system. However, other technologies can also be applied to examine various forms of physiological data connected to teachers' noticing to help then answer *why* this noticing occurs (or does not). Such physiological data may include heart rate (Kosko & Ferdig, 2023; Huang et al., 2022; Lamb & Etopio, 2020), galvanic skin response (Lake & Nardi, 2023; Lamb & Etopio, 2020) and examination of blood flow in cortical regions of the brain (Lamb & Etopio, 2020). Such work would combine innovative data collection from perceptual systems within the context of more traditional approaches to studying noticing (written noticing, stimulated recall, etc.).

The present study has also provided evidence that this visual-spatial system is underdeveloped in some teachers, most likely those with less experience and/or mathematical knowledge of teaching. Such evidence raises additional questions about eye-tracking, FOV, and VR as instructional tools rather than simply diagnostic ones. For instance, novice teachers and those with lower understandings in written noticing focused more on the teacher, had longer gaze duration, and directed their attention proximately closer than experienced teachers and/or those with sophisticated noticing. Yet, teacher educators could use representations like 360 video to scaffold where and how novice teachers should focus in the classroom, and use recordings or self-reports of such viewings, with or without eye-tracking, to provide feedback on where they looked (Kosko et al., 2022a; Zolfaghari et al., 2020).

Future research could examine whether and how such tools, paired with current best practices for facilitating teacher noticing, could be used to help teacher educators diagnose PSTs' needs and deficits in attention earlier in the educative process. For example, a report detailing to PSTs how distributed their gaze was across students in a scenario could be paired with focused discussions on the children's mathematical thinking in that scenario. Additionally, because experience alone does not guarantee sophisticated noticing (Jacobs et al., 2010; Wyss et al., 2021), future research could also examine how such technology could be used in K-12 schools for professional development. Rather, such research might include using reflective feedback through visual displays such as those represented in Fig. 5 and used with both preservice and inservice teachers.

Conclusion

The present study integrated data from eye-tracking and VR-based viewing of a mathematics lesson to examine how teachers attend to students' mathematics. Results confirmed many previous studies' findings, including those showing that novice teachers focused more on the teacher, directed their attention to proximally closer students, and had a longer average gaze duration than experienced teachers. Additionally, teachers who attended to children's mathematics had a higher degree of sophistication in written interpretations of that work and focused their attention more on students' deskwork than their faces. In other words, where they looked was more tied to their understanding of children's mathematical reasoning (via their written noticing) than it was simply a function of their experience.

Beyond confirming prior scholarship, our original conjecture held true. Namely, our approach to integrating both FOV and eye-tracking approaches extended prior scholarship with a deeper exploration of how this happened, specifically examining where teachers looked. For example, by attending to FOV when examining eye-gaze we were able to discern distinctions between teachers who focused on students' faces versus their written work—a nuance we have not observed in other research using eye-tracking technology to study professional noticing. Thus, the approach to data collection itself allowed for a change in perspective for more precise assessment of what mathematics teachers attend. This study was one of the first to do this procedurally around teacher noticing; our hope is that mathematics teacher education researchers will build upon these methods to determine continued affordances and constraints of such a methodology and expand theory on noticing.

Jacobs et al. (2010) suggested that mathematics teacher educators should “be watchful for...topics that are important to discuss but that may also derail an explicit conversation about children's understandings” (Jacobs et al., 2010, p. 195). Thus, it is important to discuss classroom management (level 0), whether students got correct/incorrect answers (level 1), and their procedures for doing so (level 2). However, a lack of attention to deeper conceptual aspects may suggest a need for further professional development. Findings here suggest VR-based eye-tracking may serve as an additional way of diagnosing this beyond analyses of student writing or an examination of their experience. Additionally, by examining different physiological resources used in noticing, mathematics educators may improve our own understanding of how to notice teacher noticing.

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