

# Note on natural and forced breaking waves in deep water

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## Summary

Two laboratory investigations of natural and forced breaking waves in a two-dimensional wave tank are reported. They are considered to be simple representations of breaking waves that exist in a real ocean, in moderate and rough sea states. The first type (Crescent Waves) is usually observed on a sea surface in relatively calm weather. Initially, for a nonlinear initial wave train of wave steepness  $ak = 0.34$ , a three-dimensional instability dominates the side band instability, and stable Crescent Waves are observed. The direction angle of radiated oblique waves as a most unstable mode, and associated growth rate are found to be in excellent agreement with Mclean's theory. The second type (strongly overturning breaking waves) was generated on the surface of essentially deep water using a wave energy focusing technique. Time evolution of the transient wave was observed using a high speed video image, and the inherent complicated unsteady flow fields was measured by means of a two-dimensional Laser Doppler Anemometer. We found that, 41 % of the wave momentum was lost by the extreme wave plunging breaker. The maximum horizontal velocity at the overturning top of the crest was found to be equal to 1.4 times the phase velocity of the wave, in its most developed stage. The two-dimensional velocity field reveals that the flow is essentially irrotational up to the top of the crest. Surface wave height measurements before and after both an isolated plunging and spilling breaking wave, reveal a frequency downshift phenomenon in the corresponding spectral shapes which seems not to have been previously seen.

## 1. Introduction

Today, ships and fishing vessels capsize and are still lost in steep and breaking seas. Consequently, a better understanding of wave breaking is one of the top goals of the naval architect's research agenda. Thus, the knowledge of the dynamic and kinematic properties of these extreme waves is very important from a practical point of view. Besides that, there are a variety of other reasons why breaking of waves is an important phenomenon. It plays a crucial role in air-sea interactions such as energy transfer from wind to sea surface, in momentum transfer from waves to surface currents, and in the production of air bubbles and sea-water droplets which are important in consideration of the problem of heat and mass exchange across the sea surface.

Surface breaking waves are usually categorized as plunging, spilling and collapsing (or surging) waves. However, in deep water, the two first breaking types are predominant. Plunging breaking represents a powerful phenomenon widely observed on a stormy ocean. It is

one of the most extreme events encountered in the marine environment. It represents a crucial factor in the overturning of ships in rough seas. Spilling breakers are more common in the open ocean and in breaking wave experiments. Breaking may result from intrinsic instabilities of deep-water waves or through wave-wave, wave-current, and wind-wave interactions.

Breaking waves are governed by a nonlinear process because of the free boundary and its nonlinearity. In the last decade, numerical and mathematical analyses to model the unsteady motion of a nonlinear free surface wave have been developed using a variety of boundary-integral methods (Longuet-Higgins and Cokelet<sup>1)</sup>; Vinje and Brevig<sup>2)</sup>; Baker *et al.*<sup>3)</sup>). These time-stepping numerical models attempted to describe the evolution of two-dimensional surface waves up to breaking. However they are unable to continue beyond the point at which the surface impacts on itself (Domermuth *et al.*<sup>4)</sup>). In addition, because observations in the field are difficult (due to the fact that breaking is a strongly intermittent process occurring over a wide range of scales) it is desirable to measure, as an initial step, the dynamic properties of deep water wave breaking in a controlled laboratory experiments. This is the reason why up to now, the study of wave breaking is based in large part on laboratory measurements.

Experimentally, observations have been made on deep water near breaking waves that display a relatively good agreement with theoretical results. Unfortunately, quantitative results are not sufficient in number to make accurate comparison possible. Banner and Peregrine<sup>9)</sup> in their review of deep water breaking waves, assert that the experimental studies fall into several distinct categories. Those focusing on the detection of breaking and the identification of simple breaking criteria (Melville<sup>6,7)</sup>; Koga<sup>8)</sup>, etc...) and those on the kinematics of breaking waves (Bonmarin and Ramamonjjarisoa<sup>9)</sup>; Kjelsen and Myrhaug<sup>10)</sup>). A comprehensive study was reported by Bonmarin<sup>11)</sup> on changes in wave steepness and wave speed as the wave approaches breaking. Few experiments on the estimation of energy dissipation due to breaking, and on field velocity measurements at the crest of a plunging breaker has been made up to now.

Our study aims to fill these gaps by providing a more complete description concerning the energy dissipation due to breaking waves, the evolution of power spectra, and the velocity field measurements of highly unsteady motions. In addition, we also performed some extensive experiments to investigate a natural breaking waves (i. e. weakly nonlinear three-dimensional waves named "Crescent Waves" widely observed in sea or river surface under the action of wind).

The paper is organized as follows. Section 2 introduces the experimental facility. In section 3, three dimensional wave patterns as well as various characteristics of plunging and spilling breaker types are described and investigated. Our conclusions are summar-

ized in section 4.

## 2. Experiments

The experiments were conducted in a wave tank, 36 m long, 1 m wide with a water depth,  $h$ , of 2.0 m. The absorbing beach was located 31 m downstream from the wave-maker (see Fig. 1). The experimental setup included a servo-controlled wave-maker, a high speed video image (500 frs/s) for flow visualization, a two-dimensional Laser Doppler Anemometer (LDA) for particle tracking velocimetry, and several wave gauges of servo and capacitance types for surface height measurements. The frequency range of initial uniform waves and of waves that coalesce to form the breaking wave is 0.5 to 2 Hz. According to linear theory, deep water conditions are realized for these experiments since the wavelength at the center (1.25 Hz) is  $\lambda = 0.99$  m so that  $\lambda < h/2$ .

## 3. Natural and forced breaking waves

### 3.1 Natural breaking waves: Crescent Waves

Crescent waves are usually observed on a sea with relatively calm weather, and are the manifestations of a three dimensional instability. A typical manifestation of this phenomenon was found by Su<sup>12)</sup> in a wide basin and was named a "Crescent Wave". Although a Crescent Wave has a three-dimensional structure, it has clear periodic pattern transverse to its direction of propagation, say along the crest lines. Fully utilizing this remarkable property, we expect to generate it in the two-dimensional wave tank shown in Fig. 1. The principal idea is as follows. The width of the tank is 1 m,

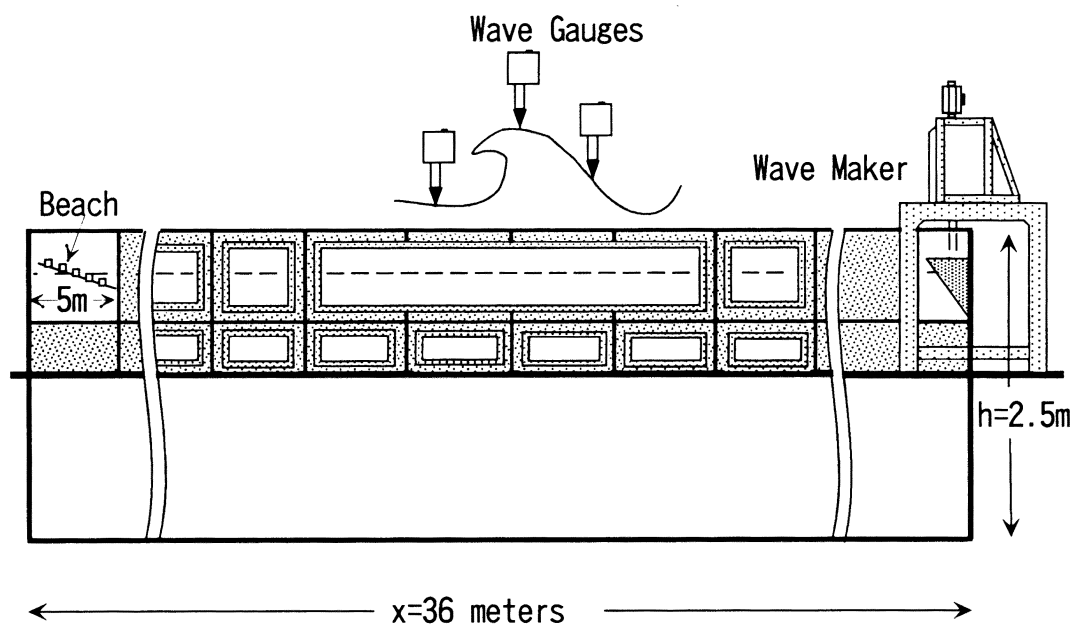


Fig. 1 The experimental apparatus

and the chord length of each crescent depends on the wavelength, and on the steepness of the wave train. Since the transverse fluid velocity component vanishes in the cross sections at the center and at both ends of the crescent, we adjusted the frequency and the stroke of the wave-maker tentatively so as to set the chord length to 1 m. Actually this frequency was 1.16 Hz (1.16 m wavelength by chance) for this case. At a certain combination of the frequency and stroke, we observed a good simulation of a Crescent Wave in the tank. The most enhanced pattern corresponded to a wave steepness of  $ak=0.342$ . The pattern was started between 10 m and 20 m away from the wave-maker. A typical record of surface elevation is shown in Fig. 2. It is remarkable that the crests are doing higher and lower excursion one after another. The former corresponds to the center of the crescent, and the latter to both sides of the chord. An overview photograph is shown in Fig. 3 with a schematic description.

We took advantage of this simulation using a two-dimensional tank, and we made several precise measurements to provide sufficient data for comparison with existing theories.

Here, the work of Mclean<sup>13)</sup>, in which a numerical investigation of the linear instability of the Stokes wave is investigated, is referred to for comparison. McLean<sup>13)</sup> adopted several possible perturbations, that is, obliquely radiated wave components of higher order Stokes wave and calculated their eigenvalues  $\sigma$  as the indices of unstable modes. The growth rate is denoted by  $\text{Im}(\sigma)$  and the phase difference of the mode by  $\text{Re}(\sigma)$ . We expected the most unstable perturbation to appear in our experiments, which we could then estimate it by the following manner. Using the Mclean<sup>13)</sup> method, we put the Stokes wave train as a basic flow such that:

$$\eta = \bar{\eta}(x) = \sum_{n=1}^{\infty} A_n \cos(nx)$$

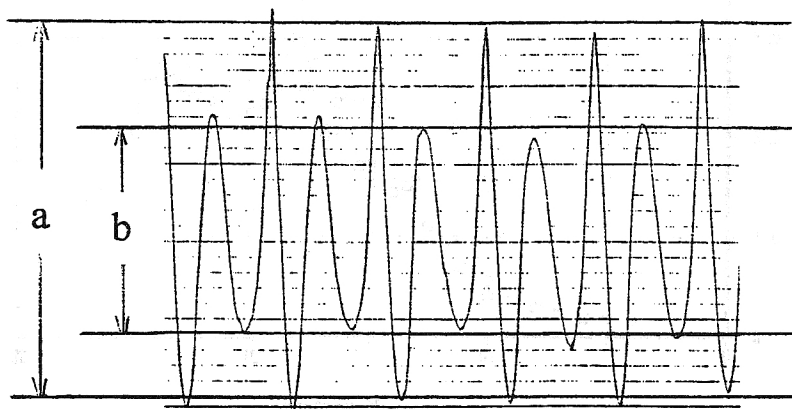


Fig. 2 Time series of surface elevation in Crescent Waves

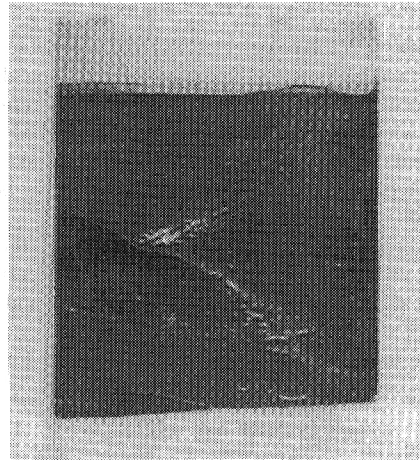
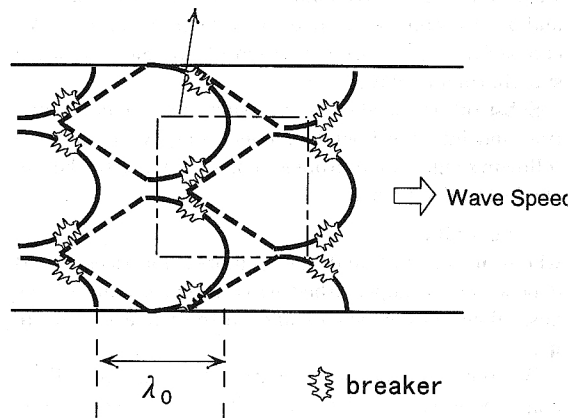


Fig. 3 Overview photo of a Crescent Wave (top) Schematic picture of the wave system with breakers (bottom)

$$\phi = \bar{\phi}(x, z) = -Cx + \sum_{n=1}^{\infty} B_n e^{nz} \sin(nx),$$

where  $\phi(x, z)$  is the velocity potential and  $z = \eta(x)$  is the free surface elevation. Equations are in non-dimensional form and are developed in a frame of reference moving with the phase speed  $C$  of the unperturbed wave. Coefficients  $A_n$  and  $B_n$  and  $C$  are determined from Laplace's equation and from the fully nonlinear boundary conditions applied at the free surface.

The vertical deflection level of the free surface,  $\eta'$ , and the velocity potential,  $\phi'$ , related to the perturbation are superposed to the basic solutions,  $\bar{\eta}$  and  $\bar{\phi}$ , as

$$\eta = \bar{\eta} + \eta', \quad \phi = \bar{\phi} + \phi'$$

Then we look for non-trivial solutions of the form

$$\eta' = e^{-i\sigma t} e^{i(p x + q y)} \sum_{m=-\infty}^{\infty} a_m e^{i m x}$$

$$\phi' = e^{-i\sigma t} e^{i(p x + q y)} \sum_{m=-\infty}^{\infty} b_m e^{i m x} e^{\sqrt{(p+m)^2 + q^2} z}$$

where  $\mathbf{p}$  is the  $x$ -wavenumber and  $\mathbf{q}$  the  $y$ -wavenumber and  $\sigma$  is the complex frequency of the perturbation. We can see that  $\eta'$ ,  $\phi'$  are composed of obliquely radiated superharmonic waves.

Substituting the above expressions into the field equation and into the boundary conditions, we obtain the following eigenvalue problem corresponding to the linear instability theory,

$$\mathbf{A}\mathbf{u} = \sigma \mathbf{B}\mathbf{u}$$

where  $\mathbf{u} = (a_m, b_m)$  and the complex matrices  $\mathbf{A}$ ,  $\mathbf{B}$  depend on the unperturbed wave steepness,  $ak$ , of the basic flow (Stokes wave) and on the wave vector  $(\mathbf{p}, \mathbf{q})$ .

We then focused our attention to evaluate: (i) the angle between the direction of the radiated oblique wave and the principal wave train and (ii) the growth rate of the oblique wave perturbation. From Fig. 3, we can easily determine the angle to be 66.9 degree deviat-

ing from the direction of propagation of the train. As for the growth rate  $\text{Im}(\sigma)$ , the ratio  $(a-b)/(a+b)$  (where quantities "a" and "b" are defined in Fig. 2), which represents the experimental growth rate, is measured in many locations from 5 m to 25 m along the tank. Results are shown in Fig. 4. It is evident that the instability begins on the steep regular wave train, grows exponentially and reaches a steady state about 20 m downstream from the wave-maker. The obtained data were transformed to a log-log scale to give the spatial growth rate to be 0.22. For the real part, we concluded that the growth rate equals zero ( $\text{Re}(\sigma) = 0$ ) because the oblique wave pattern propagates with the same speed as a Stokes wave train. This means that there is no phase difference between them.

On the other hand, we extracted from Maclean<sup>13)</sup> paper the most unstable perturbation mode solution. In Maclean's paper variables are non-dimensionalized and equations are formulated as an initial value problem. From Table 1 of his paper, a most unstable mode perturbation solution close to our experimental condition is found at  $ak = 0.349$  where  $p = 0.5$ ,  $q = 1.15$ ,  $\text{Re}(\sigma) = 0$  and  $\text{Im}(\sigma) = 0.041268$ . It means the angle of the oblique radiation wave system is 66.5 degree. Aiming to compare our experimental growth rate to theoretical one, we transformed Mclean's temporal growth rate to a spatial one, and also arranged the dimensionless value to the scale of our experiment so that the theoretical growth rate become  $\text{Im}(\sigma) = 0.223$  and the theoretical phase difference  $\text{Re}(\sigma) = 0$ . In Table 1 given below, our experimental results, corresponding to an initial wave train of wave steepness  $ak = 0.342$ , are compared with theoretical values of Maclean<sup>13)</sup>. From this Table, we can recognize an excellent agreement between the theory and our experiment. Consequently, we have confirmed experimentally that the principal nature of

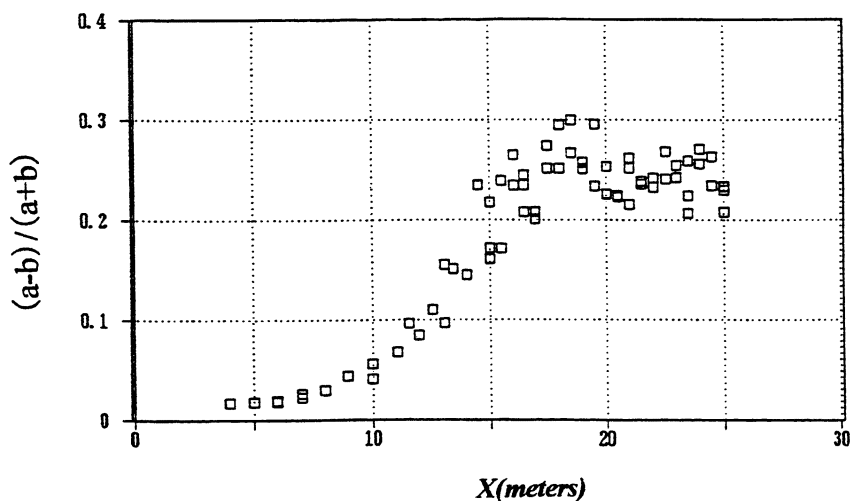


Fig. 4 Measurements of the spatial development of Crescent Waves along the tank

Table 1 Comparison between experiments and theory

|            | wave steepness | propagation angle | phase difference    | growth rate         |
|------------|----------------|-------------------|---------------------|---------------------|
|            | $ak$           | $\theta$          | $\text{Re } \sigma$ | $\text{Im } \sigma$ |
| theory     | 0.349          | 66.5              | 0                   | 0.223               |
| experiment | 0.342          | 66.9              | 0                   | 0.22                |

the three-dimensional Crescent Wave is well explained by the theory of the Stokes wave linear instability.

Furthermore, from Fig. 4, we found that the saturation region of the development of a Crescent Wave occurs between distances of 15 m and 25 m from the wave-maker. In this region, a spilling breaker exists in front of the oblique waves, and this reduces the development of the instability to a steady state (bifurcation) in a short while. This is the reason why we can often see breaking waves even in not so severe sea states.

### 3.2 Forced breaking waves

A very steep transient wave train is formed by sequentially generating series of waves from high to low frequencies that are superposed at a prescribed location. This is done on the basis of the so called 'Storm Building Method', which is a very convenient way for

generating a sufficiently steep gravity wave in a rough sea. In this technique, the stroke and period of the moving partition of the wave-maker are moved slowly, so as to focus the energy of the wave train at a prescribed position in the wave tank. Examples of two types of command input to the wave-maker which result in plunging and spilling breaking waves respectively are shown in Fig. 5.

#### 3.2.1 Wave profiles and spectral shapes

Using the command inputs of Fig. 5, plunging and spilling breaker waves take place at fetches between 12 m <  $x$  < 16 m as shown in Fig. 6. Fig. 7 shows a typical example of the observed crest deformation for a plunging breaker obtained from the high speed video images together with the deformation predicted by the numerical study of Vinje and Brevig<sup>2)</sup>. The qualitative similarity between the observed and the calculated deforma-

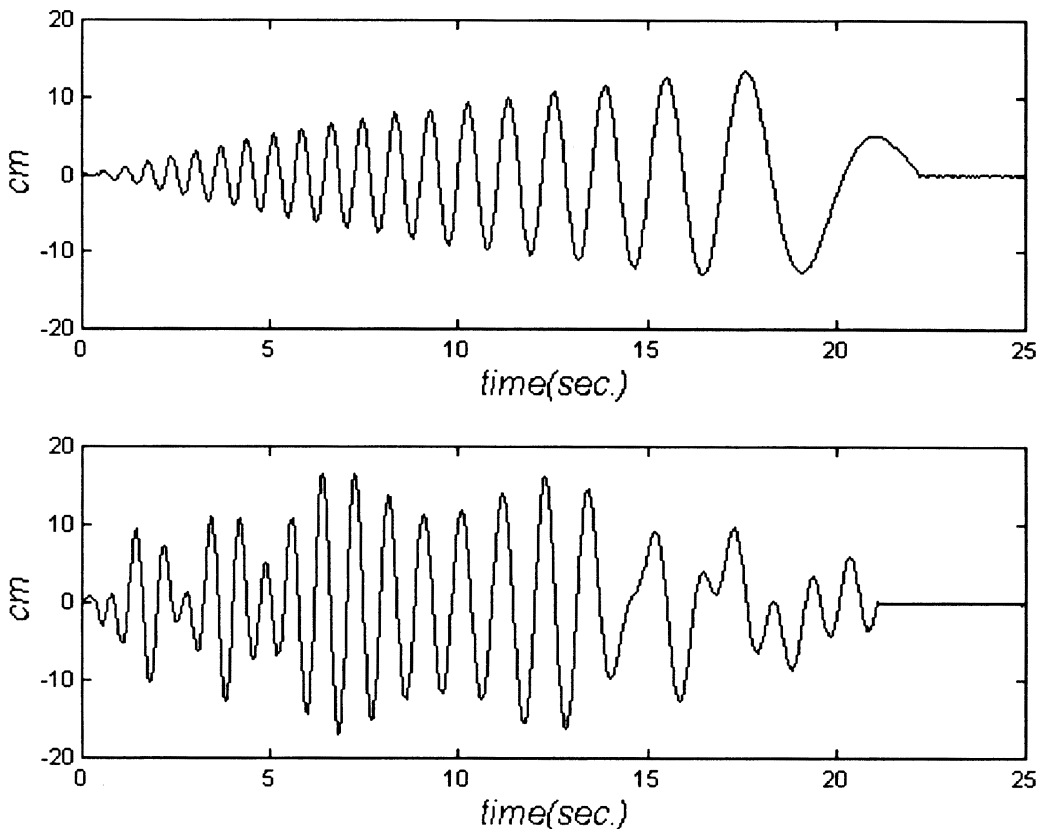


Fig. 5 Plunging breaker (upper) spilling breaker (bottom) command inputs to the wave-maker

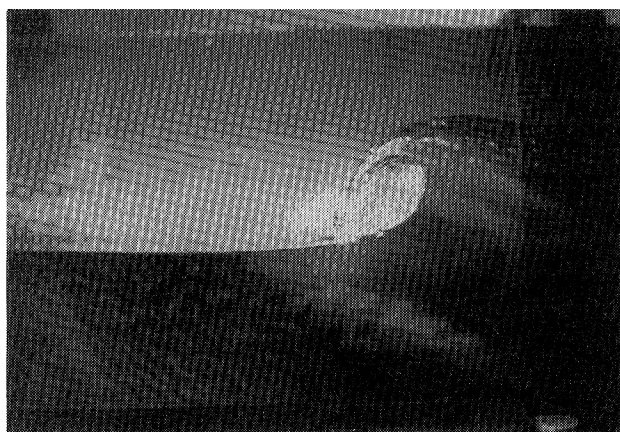
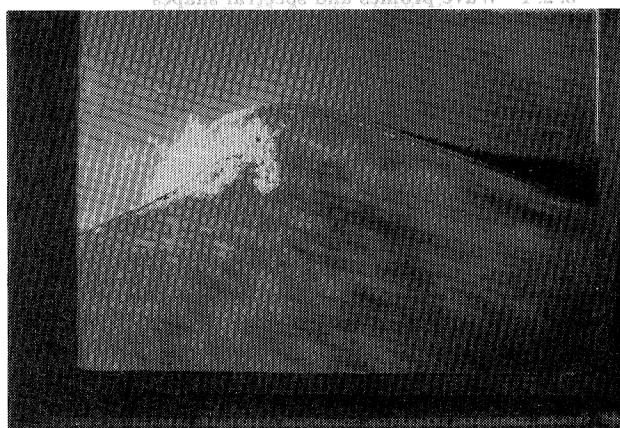


Fig. 6 Plunging (top) and spilling (bottom) breaker wave generated in the two-dimensional tank

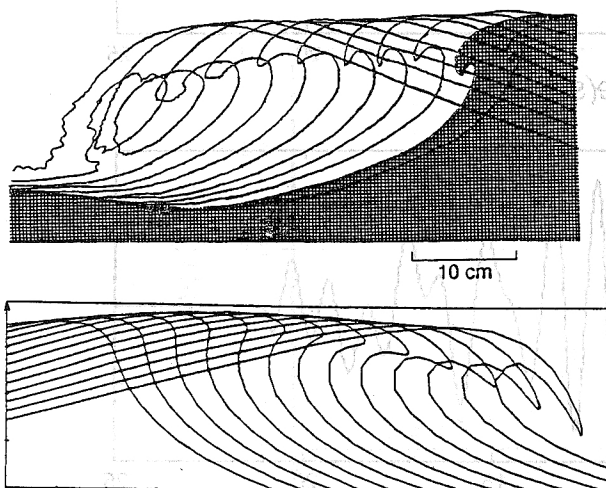


Fig. 7 Successive features of (upper) observed (time between two successive frames  $\Delta t = 0.02$  sec.) and (bottom) computed overturning crest (from numerical study by Vinje and Brevig<sup>2)</sup>)

tion is obvious.

The power spectral densities before and after the wave breaking (plunging and spilling types) are reported in Fig. 8(a, b) respectively. In the case of a plunging breaker, the frequency at the spectral peak before breaking is about 1 Hz and the wave energy beyond the frequency 2 Hz is insignificant. From Fig. 8(a), it can be seen that the wave energy is shifted to a lower

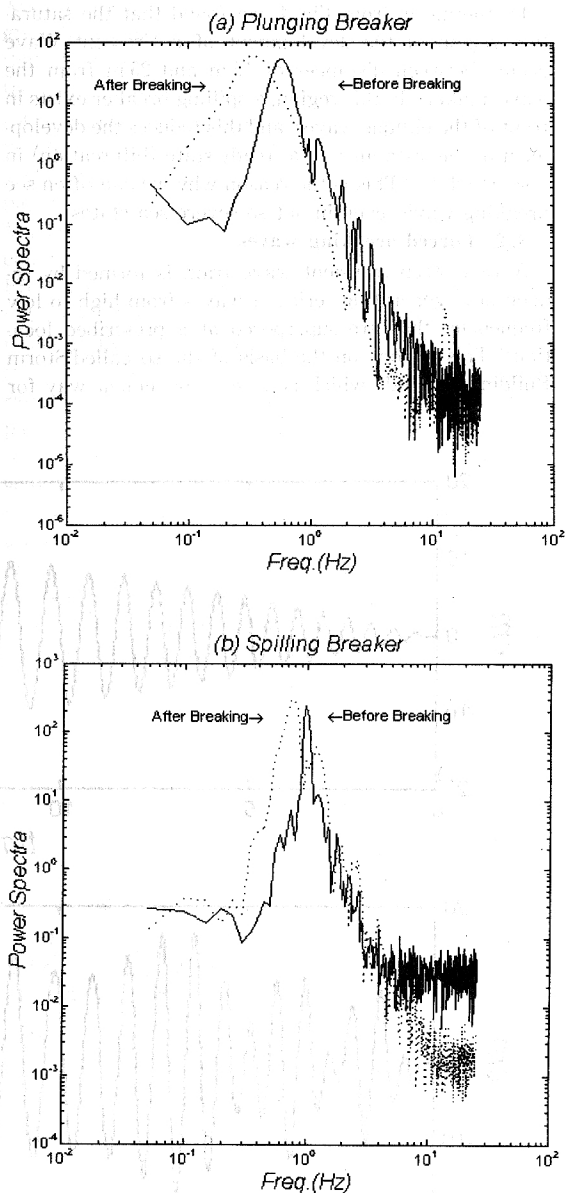


Fig. 8 Spectral shape of wave group before and after a wave (a) plunging breaker (b) spilling breaker

frequency (0.65 Hz) after breaking ( $x=16$  m). This augmentation of energy at low frequency is detected only downstream of the breaking point. In the case of a spilling breaker type, Fig. 8(b) shows that the carrier frequency, 1 Hz before the breaking is also shifted to a lower frequency, 0.78 Hz after breaking ( $x=18$  m).

For the interpretation of such unexpected phenomenon, not previously seen, we consider several possible causes as follows (i) wave reflection from the end of the tank (ii) longitudinal and/or transverse oscillations of water (iii) excitation of waves by impinging of water mass at breaking (iv) nonlinear wave-wave interactions.

By precise examination of the experimental procedure, (i) (ii) and (iii) are confirmed to be negligible in their magnitude. Let us now discuss the possibility (iv). As is well known, radiation stresses in a wave group induce the second order bounded long waves under the carrier wave. In this case, the number of waves in a group reduces gradually to one and a half crests as waves are coalescing at the prescribed position. Around this position, wave set down would be increased to a few centimeters, if we assume that the ordinary second order wave theory is applicable (Longuet-Higgins and Stewart<sup>14</sup>). At this instance (within 0.2 seconds), the coalesced wave crest breaks down in front of itself, and appreciable setup would take place to compensate the loss of momentum in the wave motion. At the same time, long waves would become free from their forcing waves, and propagate in a downstream direction (Molin<sup>19</sup>). We surmise that this mechanism radiates slightly longer waves compared with breaker waves in the downstream direction.

### 3.2.2 Quantification of breaking energy loss

Denoting the surface elevation by  $\eta$ , momentum flux is expressed using the basic theory of hydrodynamics by :

$$\frac{1}{2} \rho g \overline{\eta^2}$$

which is evaluated only by the measurements of surface elevation. In this experiment, we examine how much wave momentum is transferred to mean flow and/or turbulent motions through breaking. To estimate this quantity, we calculate the power spectra of each surface displacement record by determining the root mean square using the well known Parseval's equality. The ratio of  $\overline{\eta^2}$  for a wave group at every location downstream against distance from the wave-maker is shown in Fig. 9. In this figure, spectral components are divided into two parts, say less than 0.8 Hz and greater than 0.8 Hz. The reduction of higher frequency components means the loss of momentum by breaking. Let  $M_a$  to be the wave momentum flux at fetch  $x=am$ . The quantification of breaking energy loss between  $14 \text{ m} \leq x \leq 17 \text{ m}$  can be evaluated using the following expression

$$\mathcal{L} = (M_{14} - M_{17}) / M_{14}$$

In this experiment, the momentum loss is  $\mathcal{L}=0.41$ . That is 41% of the wave energy is lost in an extreme plunging breaker. This result does not disagree with that of Rapp & Melville<sup>16</sup> for the plunging breakers of milder cases. It gives an extreme limit of their result.

### 3.2.3 Measurements of the velocity field

It is generally recognized that an individual wave breaking event starts when the velocity of water particles near a wave crest exceeds the wave phase speed itself. Plunging breakers which are the objective of our experiments are highly unsteady flow, so that instrumentation of sufficiently high response is necessary. In this experiment, a two-dimensional Laser Doppler Anemometer (LDA) was used to detect such transient flows. The main specifications of our LDA system are reported in Table 2.

Measurements were performed through the tank side

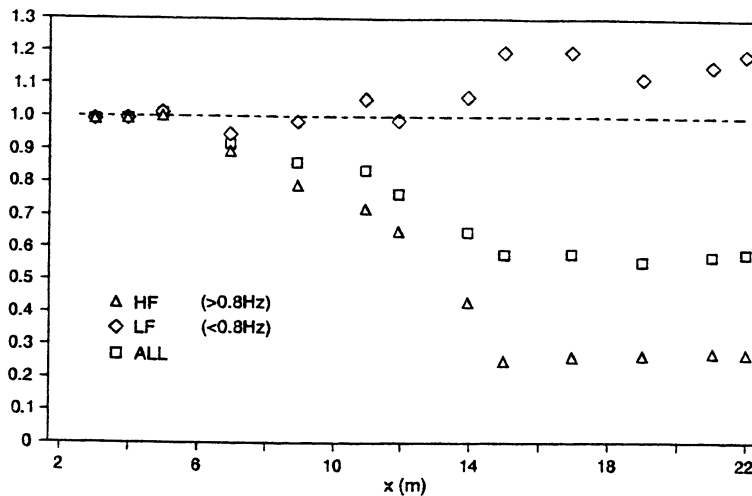


Fig.9 Ratio of momentum fluxes due to breaking wave

Table 2 The main technical specification of our LDA system

| Laser     | Wavelength  | Power | Mode   | Focal lenght | Spot volume  |
|-----------|-------------|-------|--------|--------------|--------------|
| Argon-ion | 488/514.5nm | 100mW | 3 beam | 160/400mm    | 0.076/0.64mm |

-wall, which was made of glass, every 5 cm mesh (1 cm near the surface). About 90 measurements were made for each case. The wave-making command inputs (introduced in subsection 3.2) were improved to get a maximum crest height up to 20 cm by not taking strict coalescence into account. Several examples of measured data at different depth (under and over still water levels) show that the data rate is sufficiently high enough (10 KHz) for analyzing such a rapidly varying phenomenon (Fig. 10).

The vertical distributions of maximum horizontal velocities for different phases of development of breaking waves are reported in Fig. 11.  $x/D=0$  means that measurements were made at the position of breaking onset, while  $x/D=1$  refers to the position where the top of an overturning crest impinges onto the water surface. The horizontal velocity in the vicinity of the crest increases rapidly, up to 1.41 times greater than the wave speed in the maximum developed stage ( $x/D=0.6$ ). The two-dimensional velocity field at the fully developed stage is shown in Fig. 12. For this type of breaker intruding into still water, the assumption of potential

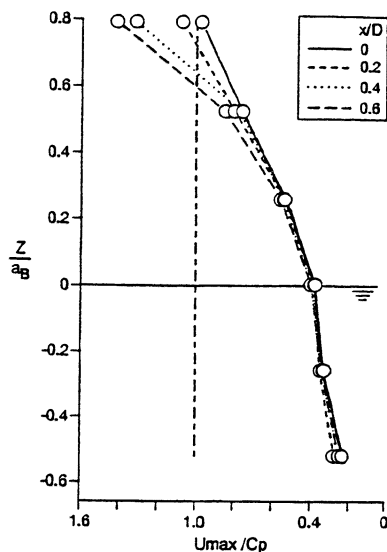


Fig. 11 Vertical profiles of maximum horizontal velocities

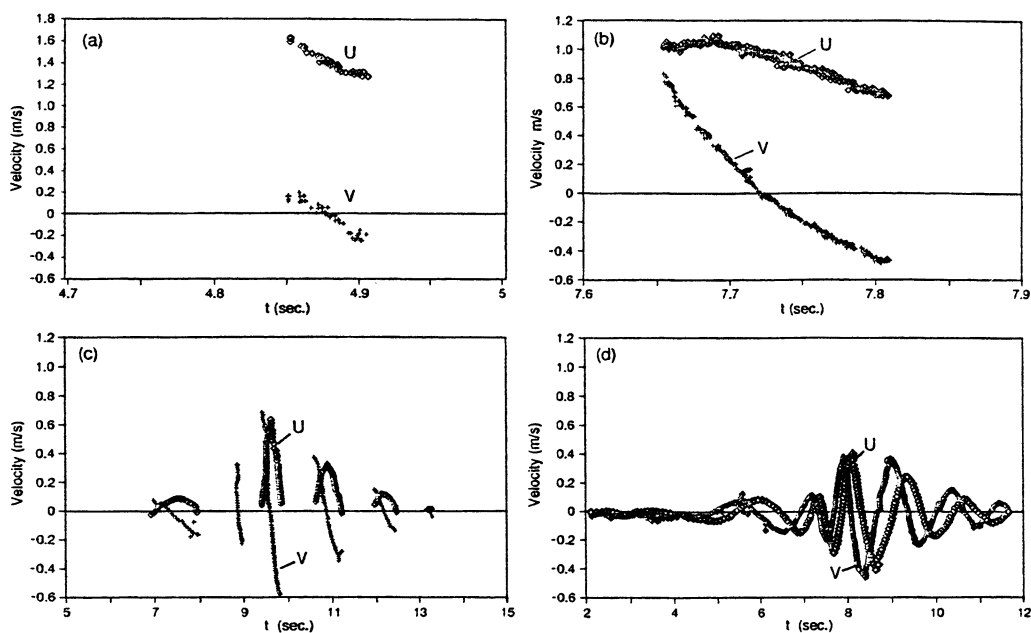


Fig. 10 Examples of internal velocities (a) 15 cm above still water level; (b) 10 cm above still water level; (c) at still water level; (d) 10 cm below still water level

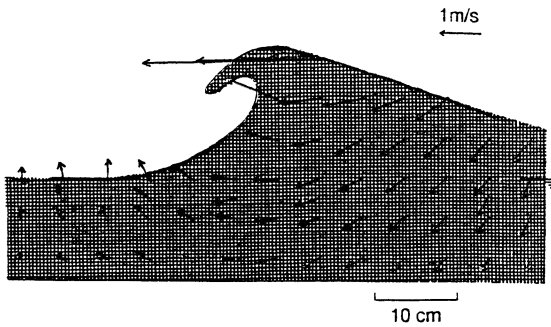


Fig. 12 An example of the velocity field at and around the overturning crest

flow seems valid except for the top of the crest. This is in agreement with the recent work of Perlin et al.<sup>17)</sup> At the wave crest, water mass (in less than two centimeters thickness) is accelerated horizontally up to 1.41 times greater than the wave speed, and has a slight downward velocity due to gravity. These flows converge to form a strong jet layer impinging forward with a thin vortex sheet underneath.

#### 4. Concluding remarks

Two types of experiment have been performed. First, a regular wave train was produced with a paddle. At a certain combination of the frequency and stroke, a clearly defined Crescent Wave was observed in the tank. The most enhanced pattern corresponds to wave steepness  $ak=0.342$ , where the long lived quasi stationary spilling breaker appears in front of the crescent shaped crest. This means that an energy balance occurs between the spontaneous radiation of oblique waves and its actual breaking. This mechanism is quite different from the strong plunging breaker caused by the superposition of many wave systems observed in rough seas. Comparison between experimental results and theoretical work of Maclean<sup>13)</sup> shows that a very good agreement is achieved between the two approaches. Consequently, we can conclude from our experiments that, the principal nature of the Crescent Waves is well explained by the linear instability theory of the Stokes wave.

From the second experimental investigation, driving the wave-maker sequentially from high to low frequencies, "clean" plunging and spilling breaking waves were generated in the two-dimensional wave tank. An asymmetric wave profile was captured during pre-breaking. A small jet was seen at the crest of the wave front, accelerating forward at a more rapid rate than the wave itself, overturning, becoming incident on the water surface, and finally creating a splash zone.

From the measurements of the wave momentum transfer, we found that 41% of wave energy was lost by the most energetic plunging breaking waves. A careful look at the wave spectra before and after the

wave breaking revealed an interesting frequency down-shift phenomenon. An explanation has been given in terms of the already developed theories associated with the nonlinear wave-wave interactions.

Field velocity measurement was achieved with an LDA system. Water particles near a wave crest were found to develop a velocity sufficiently large to cause these particles to fall down the front of the wave. It was found that the horizontal velocity in the vicinity of the crest is 1.41 time larger than the wave speed itself. The two-dimensional velocity field reveals that the flow is essentially irrotational except for the top of the crest, and consequently we concluded that potential theory is valid until breaking ensues.

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