

Decomposing MLP Activations into Interpretable Features via Semi-Nonnegative Matrix Factorization

Or Shafran¹ Atticus Geiger² Mor Geva¹

¹Blavatnik School of Computer Science and AI, Tel Aviv University ²Pr(Ai)²R Group

{ordavids1@mail, morgeva@tauex}.tau.ac.il, atticusg@gmail.com

Abstract

A central goal for mechanistic interpretability has been to identify the right units of analysis in large language models (LLMs) that causally explain their outputs. While early work focused on individual neurons, evidence that neurons often encode multiple concepts has motivated a shift toward analyzing directions in activation space. A key question is how to find directions that capture interpretable features in an unsupervised manner. Current methods rely on dictionary learning with sparse autoencoders (SAEs), commonly trained over residual stream activations to learn directions from scratch. However, SAEs often struggle in causal evaluations and lack intrinsic interpretability, as their learning is not explicitly tied to the computations of the model. Here, we tackle these limitations by directly decomposing MLP activations with semi-nonnegative matrix factorization (SNMF), such that the learned features are (a) sparse linear combinations of co-activated neurons, and (b) mapped to their activating inputs, making them directly interpretable. Experiments on Llama 3.1, Gemma 2 and GPT-2 show that SNMF derived features outperform SAEs and a strong supervised baseline (difference-in-means) on causal steering, while aligning with human-interpretable concepts. Further analysis reveals that specific neuron combinations are reused across semantically-related features, exposing a hierarchical structure in the MLP’s activation space. Together, these results position SNMF as a simple and effective tool for identifying interpretable features and dissecting concept representations in LLMs.

1 Introduction

A central question for mechanistic interpretability is how to decompose hidden representations of large language models (LLMs) into constituent parts (Mueller et al., 2024; Geiger et al., 2024; Sharkey et al., 2025). A natural widely-studied unit is individual neural activations. However, while

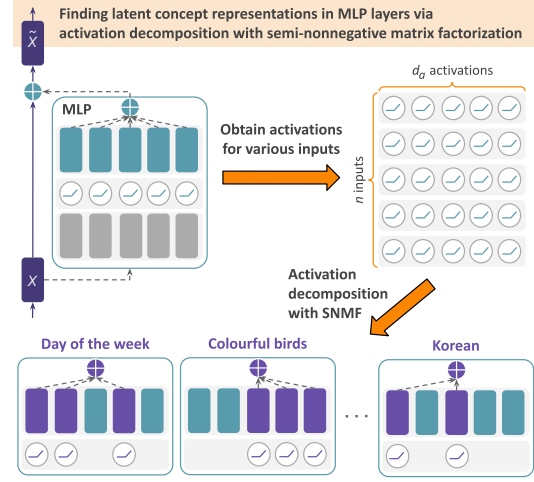


Figure 1: We find latent concept representations by decomposing MLP activations with semi-nonnegative matrix factorization (SNMF). Our method finds sparse compositional MLP features that align with interpretable concepts and outperform existing unsupervised methods and a strong supervised baseline at causal steering.

individual activations often demonstrate human-interpretable concepts (Karpathy et al., 2015; Geva et al., 2021, 2022; Choi et al., 2024), it has long been established that they participate in the representation of multiple intelligible concepts (Smolensky, 1986; Rumelhart et al., 1986; Olah et al., 2020; Bolukbasi et al., 2021; Gurnee et al., 2023).

An alternative unit of analysis that has recently gained consensus is directions in activation space (Elhage et al., 2021; Gurnee et al., 2023; Nanda et al., 2023; Park et al., 2024). A key question is how to find directions encoding interpretable features in an unsupervised manner, namely, without any prior about the concepts they represent. A large body of work has explored dictionary learning methods with sparse autoencoders (SAEs; Bricken et al. 2023; He et al. 2024; Lieberum et al. 2024) to learn directions called *features*. Features are then assigned textual labels with automatic pipelines (Hernandez et al., 2021; Bills et al., 2023; Gur-

Arieh et al., 2025). However, while SAEs can extract interpretable features, they often fail to provide a better unit of analysis in causal evaluations (Huang et al. 2024; Chaudhary and Geiger 2024; Wu et al. 2025; Mueller et al. 2025; cf. Marks et al. 2025). Moreover, the learned features are not constrained to the model’s representation space (Menon et al., 2025; Paulo and Belrose, 2025) nor grounded in specific model mechanisms.

In this work, we tackle the problem of finding interpretable features in LLMs by *decomposing MLP activations*. Instead of learning directions from scratch—as in widely-used SAEs trained on residual stream representations—our method represents features as *sparse linear combinations of existing directions in MLP weight matrices*, defined by groups of neurons (see Figure 1). To decompose the MLP hidden activations, we propose using *semi-nonnegative matrix factorization* (SNMF; Ding et al. 2010). We collect MLP activations with d_a dimensions from a large n -token text, and decompose the matrix $A \in \mathbb{R}^{d_a \times n}$ of data into two matrices: a matrix $Z \in \mathbb{R}^{d_a \times k}$ of MLP features that define sparse neuron combinations, and a nonnegative coefficient matrix $Y \in \mathbb{R}_{\geq 0}^{k \times n}$. A fundamental advantage of this approach over SAEs is that the coefficient matrix Y encodes which tokens contributed to the creation of a feature.

We use texts that strongly activate features to assign textual *concept labels*, and evaluate the features along two axes (Huang et al., 2023; Wu et al., 2025): (1) *concept detection*, measuring the ability of the feature to separate texts exemplifying the concept label and neutral texts, and (2) *concept steering*, evaluating how well feature activation steers generated texts toward the concept label while preserving fluency. We find that features learned with SNMF outperform widely used SAEs (Lieberum et al., 2024; He et al., 2024) and supervised features computed with difference-in-means (Marks and Tegmark, 2024; Rimsky et al., 2024; Turner et al., 2024) on concept steering, while performing comparably on concept detection.

Further qualitative analysis shows that MLP features are compositional: recursively applying SNMF to the learned MLP features further composes features of fine-grained concepts (e.g., *Javascript* and *Python*) into features corresponding to more high-level concepts (e.g., *programming*). This showcases the inverse of the feature splitting phenomenon observed in SAEs, where larger SAEs have clusters of features corresponding to indi-

vidual features of smaller SAEs (Bricken et al., 2023; Chanin et al., 2024). Moreover, semantically-related features (e.g., the different weekdays) share a core set of neurons and are differentiated by exclusive neurons that control the promotion and suppression of fine-grained concepts.

To conclude, we propose decomposing MLP activations with SNMF for finding interpretable features in LLMs. Our experiments show the effectiveness of this approach compared to existing unsupervised methods and a strong supervised baseline, and our analysis provides insights into how neuron compositionality constructs features in MLP layers. We release our code at <https://github.com/ordavid-s/snmf-mlp-decomposition>.

2 Related Work

Analyzing Neurons in Transformers Geva et al. (2021, 2022) view the MLP output as a collection of updates to the residual stream, where updates correspond to single neurons which often encode human-interpretable concepts. Alternatively, Meng et al. (2022) considers an MLP layer as an associative memory, where all the neural activations form a key that induces the layer’s output. Recently, Cao et al. (2025) used clustering to identify meaningful groups of neurons in CNNs. Dalvi et al. (2018); Gurnee et al. (2023) used supervised methods to isolate groups of neurons in transformers that are responsible for promoting specific concepts or tasks. Unlike prior work, we propose an unsupervised approach that identifies features via patterns in MLP activations. We view neuron combinations as causal mediators reused across inputs to represent similar concepts, extending prior work by showing that such reuse is a natural mechanism the model employs to represent concepts.

Concept Representations in LLMs Recent work (Marks and Tegmark, 2024; Park et al., 2024; Nanda et al., 2023) proposes the linear representation hypothesis; concept representations in language models are approximately linear, aligned with specific directions in the residual stream. However, several works challenge this view (Csordás et al., 2024; Park et al., 2025; Engels et al., 2025; Levy and Geva, 2025), proposing that some concepts have meaning when viewed in multiple dimensions. These works demonstrate the presence of structured, complex concept representations. However, they do not explain how such representations are formed or grounded in the model’s under-

lying mechanisms. Our work applies SNMF to investigate how neurons form complex features, suggesting a complementary view that defines multi-dimensional concepts through their formation.

Finding Latent Concepts Supervised methods have been shown effective in finding directions in activation space that localize (Geiger et al., 2023; Wu et al., 2023) or steer for (Wu et al., 2024; Marks and Tegmark, 2024) predefined concepts. However, fewer works propose unsupervised methods for finding meaningful directions. Early work applied methods such as clustering (Sajjad et al., 2022) and nonnegative matrix factorization (Yun et al., 2021) to the residual stream. These works provided insight into interpretable concepts utilized by the model, but without finding concrete directions in activation space. More recently, SAEs have been used to learn directions that align with interpretable concepts (Bricken et al., 2023; Cunningham et al., 2023). Unlike SAEs, SNMF offers intrinsic interpretability by linearly decomposing activations into sparse features and a nonnegative coefficient matrix that directly attributes each activation to the features it expresses.

NMF and Deep Learning NMF has previously been used to identify interpretable concepts in deep neural networks. Prior work (Lee and Seung, 1999; Melas-Kyriazi et al., 2022; Oldfield et al., 2023) has demonstrated NMF isolates meaningful parts in images, e.g., eyes and noses, by factorizing image data. Yun et al. (2021) applied NMF to activations in the residual stream, showing they encode interpretable concepts. Here, we extend this approach to MLP activations and show that the discovered MLP features have causal roles within the model.

3 Decomposing MLP Activations with Semi-Nonnegative Matrix Factorization

We propose an unsupervised method for finding meaningful directions in LLMs, focusing on their MLP layers. Our approach is motivated by the view that the MLP output to the residual stream can be expressed as a linear combination of underlying features, each formed by a specific group of neurons. Their activations specify how they are linearly combined to construct concepts in a given input. Thus, inputs that express similar concepts should activate similar neurons. This theory is further supported by Yun et al. (2021) who provide evidence of the additive nature of concepts in the

model’s residual stream.

Following this view, we propose to identify groups of neurons representing interpretable features using semi-nonnegative matrix factorization (Ding et al., 2010). SNMF factorizes a data matrix into two matrices: a matrix of *features* and a non-negative *coefficient matrix* that maps between inputs and features. Our choice of SNMF is motivated by Lee and Seung (1999) who demonstrated that the nonnegativity constraints in NMF encourage parts-based, additive representations, which tend to yield interpretable features. SNMF retains this interpretability while relaxing the constraint for nonnegativity in the features, thus being a better fit for MLP activations.

We provide notation and explain how we apply SNMF to find interpretable features in MLP layers.

MLP Activations We assume a transformer-based language model with L layers, a hidden dimension d , and an MLP inner-dimension d_a . An MLP layer is defined with two parameter matrices $W_K, W_V^T \in \mathbb{R}^{d_a \times d}$ and an element-wise nonlinear activation function σ :¹

$$\text{MLP}(\mathbf{h}) = W_V \sigma(W_K \mathbf{h}) := W_V \mathbf{a} \quad (1)$$

where $\mathbf{h} \in \mathbb{R}^d$ is an input hidden representation and $\mathbf{a} \in \mathbb{R}^{d_a}$ is a vector of activations.² Let $\mathbf{k}_i$ be the i -th row of W_K and $\mathbf{v}_i$ the i -th column of W_V , the output of the MLP layer can be cast as a linear combination of W_V ’s columns, each weighted by its corresponding activation:

$$\text{MLP}(\mathbf{h}) = \sum_{i=1}^{d_a} \sigma(\mathbf{k}_i \cdot \mathbf{h}) \mathbf{v}_i = \sum_{i=1}^{d_a} a_i \mathbf{v}_i \quad (2)$$

Decomposing Activations with SNMF Given neuron activations $A := [\mathbf{a}_1 \dots \mathbf{a}_n] \in \mathbb{R}^{d_a \times n}$ obtained from an MLP layer for a set of n corresponding input representations $\mathbf{h}_1, \dots, \mathbf{h}_n$, we apply SNMF to decompose A into MLP features. Specifically, A is factorized into a product of two matrices $Z \in \mathbb{R}^{d_a \times k}$ and $Y \in \mathbb{R}_{\geq 0}^{k \times n}$ such that:

$$A \approx ZY, \quad (3)$$

where k is a hyperparameter defining the number of features.

The columns of Z define the representations of the *MLP features*. Each MLP feature $\mathbf{z}_i \in \mathbb{R}^{d_a}$ lies

¹Bias terms are omitted for brevity.

²In modern LLMs, activations often go through additional gating before the output projection (Liu et al., 2021).

in the MLP activation space and specifies a linear combination of neurons. Since the rows of A are neuron activations, each MLP feature $\mathbf{z}_i$ captures a co-activation pattern across neurons, so multiplying by W_V maps it to the residual stream defining a *residual stream feature* $\mathbf{f}_i \in \mathbb{R}^d$:

$$\mathbf{f}_i = W_V \mathbf{z}_i = \sum_{j=1}^{d_a} z_{i,j} \mathbf{v}_j \quad (4)$$

We encourage these linear combinations to be *sparse*, as typically observed in MLP layers in LLMs (Geva et al., 2021) (see details below). The *coefficient matrix* Y is restricted to nonnegative values, which specify how each input representation $\mathbf{h}_i$ is reconstructed using the MLP features. Each entry $Y_{i,j}$ indicates how strongly MLP feature i contributes to sample j . Together, Y and Z provide a parts-based decomposition, where the original activations are reconstructed as additive combinations of the MLP features.

Optimization To learn the decomposition, we initialize the matrices Z and Y such that the entries of Z are drawn from a uniform distribution $\mathcal{U}(0, 1)$, and the entries of Y are drawn from a standard normal distribution $\mathcal{N}(0, 1)$. Then we use the Multiplicative Updates scheme of Ding et al. (2010), which alternates between a closed-form update of the matrix Z and a multiplicative update of the nonnegative matrix Y in order to minimize the Frobenius norm between the matrix A and its reconstruction ZY . Lastly, we impose sparsity by applying a hard winner-take-all (WTA) operator to every column of Z , keeping only the largest $p\%$ of entries (by absolute value) and zeroing the remainder as suggested in Peharz and Pernkopf (2012). For full details, see §A.

4 Concept Detection Evaluations

We start by evaluating if the features learned by SNMF capture generalizable input-dependent patterns. We show that the MLP features consistently activate in response to concept-related inputs and remain inactive for neutral inputs, suggesting that a stable group of neurons underlies each concept. These features perform comparably to those identified by SAEs trained on the MLP output and better than equally sized SAEs trained on the same data, supporting their relevance to the model’s input.

4.1 Experiment

We follow previous work (Paulo et al., 2024; Gur-Arieh et al., 2025; Wu et al., 2025) and measure the ability of MLP features to discern between contexts that exemplify the associated concept and those that do not. Concretely, we first describe the concept captured by an MLP feature based on inputs that activate it, and then evaluate if the feature activates on inputs that were generated based on the description.

To describe the concept captured by MLP features, we leverage GPT-4o-mini (OpenAI et al., 2024) and prompt it to describe prominent patterns in the inputs that most strongly activate it (Bills et al., 2023; Bricken et al., 2023; Choi et al., 2024; Paulo et al., 2024) (see §G.2 for the prompt). Then, for every description we generate five activating sentences that express the concept and five neutral sentences that do not. Next, we run each sentence through the model and record the MLP activations for every token during the forward pass.

To evaluate how well an MLP feature aligns with its description, we compute the cosine similarity between the feature and each token activation in a sentence, and take the maximum similarity per sentence.³ We then average these maximum scores across the activating sentences to obtain $\bar{a}_{\text{activating}}$, and across the neutral sentences to obtain $\bar{a}_{\text{neutral}}$. Lastly, we measure the log-ratio between the mean scores for the activating and neutral sentences to obtain the Concept Detection score (S_{CD}):

$$S_{CD} := \log \frac{\bar{a}_{\text{activating}}}{\bar{a}_{\text{neutral}}} \quad (5)$$

We use logarithm to normalize the scale for comparability across methods. A positive log-ratio indicates greater similarity with activating sentences, where larger values indicate better separability.

We apply this evaluation to SNMF-learned MLP features with $k = 100, 200, 300, 400$ on multiple layers of Gemma-2-2B (Team et al., 2024), Llama-3.1-8B (Grattafiori et al., 2024), and GPT-2 Small (Radford et al., 2019). Additional details on the training of SNMF are in §D. As a baseline, we use publicly available SAEs (He et al., 2024; Lieberum et al., 2024; Gao et al., 2025) trained on MLP outputs (SAE_{out}), i.e. $W_V \mathbf{a}$ (Equation 3), and SAEs trained on MLP activations (SAE_{act}) with the same dataset and number of features as SNMF

³We use cosine similarity as opposed to projection to make the scales of different methods comparable (see details in §B).

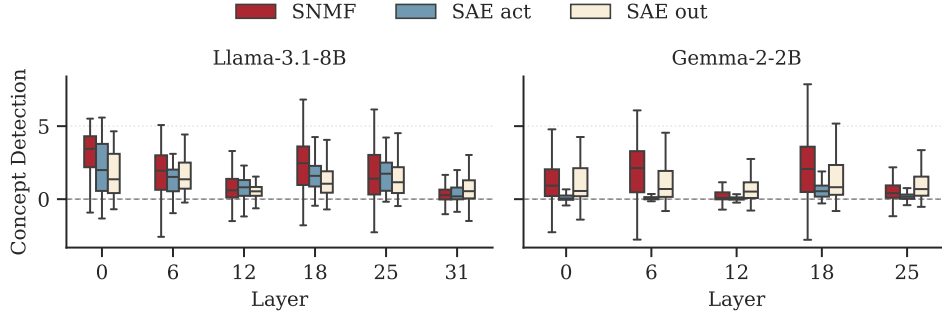


Figure 2: The distribution of concept detection scores across selected layers in Llama-3.1-8B (left) and Gemma-2-2B (right) for SNMF ($k=100$), SAE trained on MLP activations with the same dataset as SNMF (SAE act) and publicly-available SAEs trained on MLP outputs (SAE out).

(see details in §C). For SNMF, we generate concept labels by analyzing the input samples that most strongly activate it, using the coefficient matrix Y . For SAE act, we take the feature descriptions from Neuronpedia, which are derived using autointerp (Paulo et al., 2024) and similarly rely on the highest-activating examples. For SAEs trained on the SNMF dataset, we generate descriptions based on their top-activating contexts using the same prompts used for generating descriptions for SNMF.

4.2 Results

Figure 2 shows the Concept Detection score for SNMF MLP features, SAE act MLP features and SAE out features in Llama-3.1-8B and Gemma-2-2B. For SNMF we present the results for $k = 100$, which shows similar trends to other values of k . Additional results showing similar trends for GPT-2 Small and comparisons across different k values are provided in §D. Across different layers and models, most of the SNMF MLP features (>75%) obtain a positive concept detection score, indicating that the majority of SNMF-discovered features activate more for concept-related inputs than for neutral ones. This suggests that the majority of MLP features are meaningfully tied to the appearance of specific concepts in the input. Interestingly, we observe comparatively high scores in the first layer of both Llama-3.1-8B and GPT-2. We hypothesize that this is because the activation representations have passed through fewer attention layers so they encode fewer entangled concepts, making them easier to decompose.

When comparing methods, SNMF generally achieves concept detection scores that are comparable to or exceed those of existing SAEs trained on MLP outputs and a better performance compared

to SAEs trained on the SNMF dataset. A minority of MLP features receive negative scores, indicating they respond more strongly to neutral contexts, possibly reflecting sensitivity to noise or layer-specific biases. The lower scores of SAE act may partly reflect challenges in achieving sparsity with less features and smaller datasets. For Gemma in particular, training was less stable as we observed that increased sparsity often led to a large number of dead features. We believe that with additional tuning and broader parameter searches, smaller SAEs have the potential to perform more competitively than the versions evaluated here.

These results show that SNMF reliably identifies sets of neurons that correspond to human-interpretable concepts. We note that SNMF’s higher concept detection scores may stem from the improved interpretability of SNMF provided by the coefficient matrix Y , which gives a straightforward association between features and their most activating contexts. In Table 1, we present examples of identified concepts along with the tokens most strongly associated with them according to the coefficient matrix Y .

5 Concept Steering Evaluations

In this section, we turn to measure the steering effectiveness of MLP features while ensuring that model fluency is preserved. We show that MLP features match and often exceed the steering performance of strong unsupervised and supervised baselines trained on MLP outputs. Together with the interpretability results, these findings show the success of our approach in isolating meaningful neuron combinations used by the model.

Layer	Concept	Top Activating Contexts
0	The word “resonate” and variations	captivating blend of nostalgia and innovation that resonates Pont Mirabeau captivates the eye, resonating with the a new advertising campaign that would resonate more strongly
12	Actions related to implementing or establishing	the police enacted curfews and employed sound cannons In JavaScript, establishing a coherent code layout schools must implement a comprehensive framework that supports the evaluation
23	Historical documentation	stone carvings, striving to reveal the profound stories etched by generations the rich historical narratives preserved by the ancients forgotten texts found in the dusty archives

Table 1: Example concepts discovered by SNMF across different layers of Llama-3.1-8B, along with their top activating contexts extracted from the coefficient matrix Y .

5.1 Experiment

We evaluate whether MLP features discovered by SNMF causally influence the model’s output in a concept-consistent manner (Paulo et al., 2024; Gur-Arieh et al., 2025; Wu et al., 2025). We benchmark SNMF against SAE out, SAE act, and a strong supervised baseline, Difference-in-Means (DiffMeans) (Wu et al., 2025; Rimsky et al., 2024; Marks and Tegmark, 2024; Turner et al., 2024).

We feed the prompt “<BOS> I think that” to the model while steering with a feature during inference. To control the steering’s strength, we perform a grid search over seven target values of KL divergence between the output logits from a standard forward pass and from an intervened pass where we amplify the feature (Paulo et al., 2024). As some features may suppress a concept rather than promote it, we consider both the feature $\mathbf{f}$ and its negation $-\mathbf{f}$. For each KL divergence value and sign, we sample 8 generations, to obtain an overall of 112 completions.

Next, we prompt GPT-4o-mini (OpenAI et al., 2024) to score each generated continuation along two axes: *concept expression* (concept score) and *fluency* (fluency score).⁴ We use the evaluation prompts by Wu et al. (2025), where each axis is scored from 0-2. For each feature, we take the best result over KL/sign combinations. We report two scores: (a) the concept score, which evaluates how well the generated completions align with the target concept, and (b) the harmonic mean of the concept and fluency scores, which quantifies how effectively a feature can steer the model’s generation toward the target concept while preserving output coherence. We exclude fluency scores from the main figure as they are less informative in iso-

lation. Fluency scores are provided in §E.

Concept scores are computed with respect to a reference feature description. Gur-Arieh et al. (2025) has shown that input-centric descriptions are better for describing features in early layers, while output-centric descriptions are better for describing features in upper layers. Therefore, we generate descriptions for early layer features using activating inputs (as explained in §4). For later layers, across all methods, we use an output-centric description. For an MLP feature (SNMF and SAE act), $\mathbf{z}_i$, we calculate its projection onto the residual stream via the MLP out weights,

$$\mathbf{f}_i := W_V \mathbf{z}_i \quad (6)$$

For features of SAE out and DiffMeans we directly use $\mathbf{f}_i$ as they were trained on MLP outputs. Next, we project the feature to the model’s vocabulary space using the unembedding matrix, and generate the description based on the set of tokens corresponding to the top and bottom logits in the projection (Gur-Arieh et al., 2025).

We apply this evaluation to SNMF features with $k = 100$ across multiple layers in LLaMA-3.1-8B and Gemma-2-2B. For SAE act we use all 100 features. For SAE out, we randomly sample 100 features from Gemma Scope and LLaMA Scope and use their corresponding decoder rows. Since DiffMeans is supervised, we apply it to learn features for the concepts identified by SNMF and SAEs. For each concept, we use its description to generate 72 activating and 72 neutral sentences (as done in §4) and define the feature as the difference between the average token representations for each set (Wu et al., 2025).

5.2 Results

Figure 3 presents the results, showing varying performance across the layers of both models. Early

⁴We omit the instruct score used in Wu et al. (2025) as we evaluate base models rather than instruction tuned.

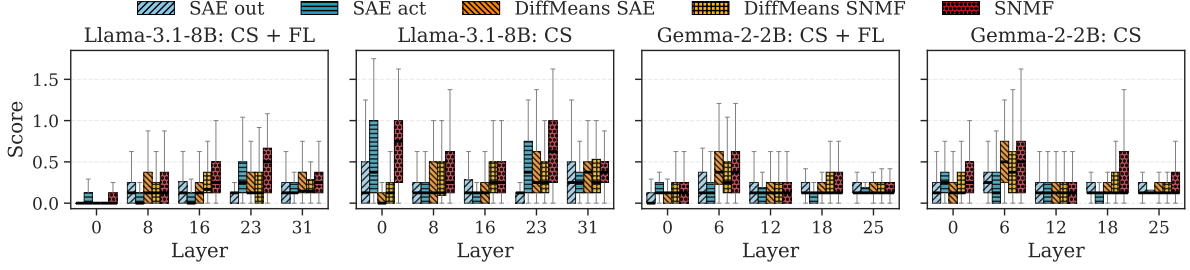


Figure 3: Causality evaluation results across layers in Llama-3.1-8B and Gemma-2-2B, using publicly-available SAEs (SAE out), SAE trained on MLP activations (SAE act), SNMF, and DiffMeans trained on concepts detected by SAE or SNMF. Concept Steering + Fluency (CS + FL) measure steering performance while preserving coherence, and Concept Steering (CS) quantifies how strongly the target concept is steered. Across both models SNMF scores often exceed both DiffMeans and SAEs, showing strong causal influence.

layers tend to show lower final scores but relatively high concept scores. This discrepancy arises from the sensitivity of early-layer steering. Modifications at these layers propagate through the network, often strongly influencing model fluency and as such impacting the final score.

Notably, SNMF consistently outperforms SAEs across all layers and often matches or exceeds the performance of DiffMeans, a supervised baseline. Since DiffMeans computes feature directions via subtraction of positive and negative representations, it is susceptible to noise by unrelated concepts present in the positive samples. This issue is especially prominent in earlier layers, where limited contextualization leaves token representations within the positive sample distinct, making their average less meaningful. In contrast, SNMF produces a parts-based decomposition that captures structure consistently associated with the target concept, leading to improved performance across all layers.

These results demonstrate that SNMF is able to isolate groups of neurons encoding concepts, supporting its viability as an interpretable unsupervised method for discovering meaningful directions through the MLP. Furthermore, the ability of MLP features produced by SNMF to reliably steer model outputs implies that the MLP operates using additive updates composed of interpretable neuron sets, which extends the findings by Geva et al. (2022) that single MLP vectors promote concepts in the residual stream. Each MLP feature corresponds to a distinct combination of neurons whose activations affect the residual stream in ways aligned with human-interpretable concepts. Considering that neuron combinations activate to represent a concept, it follows naturally that a single neuron may participate in multiple concepts, and that a

single concept may activate multiple neurons.

6 Analyzing Neuron Compositionality

Our experiments show that SNMF finds groups of neurons that represent interpretable features. A natural question that arises is *how are different neurons combined to form features?* Here, we investigate this question using the properties of SNMF.

Feature Merging via Recursive SNMF We apply SNMF recursively with progressively smaller values of k , which encourage more generic patterns (Song and Lee, 2013). Namely, we start by decomposing the activations and then recursively decompose the generated MLP features. This process reveals subsets of neurons that are shared across features, forming a hierarchy that shows how the model additively combines groups of neurons to represent concepts. We run this experiment on a dataset of sentences involving the concept of time units (e.g., months, weekdays, minutes) and use the coefficient matrices Y from each step to identify relationships between MLP features. More details on the recursive application of SNMF are in §F.

Figure 4 illustrates the resulting hierarchy of concepts corresponding to the MLP features learned at each step. Concept labels were derived from the top activating contexts of the MLP feature according to the coefficient matrix. For example, the *week-end* node was activated for contexts mentioning Saturday, Sunday and the word “weekend”. Overall, we see that the resulting structure progresses from groups of neurons that represent fine-grained concepts (i.e., specific weekdays) to more high-level concepts (middle/end of the week and “day of week”). This feature-merging behavior is the inverse of what has been described as feature splitting in SAEs (Chanin et al., 2024; Bricken et al.,

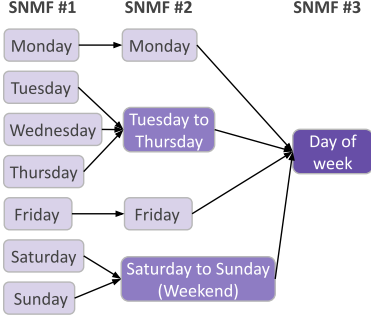


Figure 4: Example of feature merging via recursive SNMF, showing the concepts corresponding to the features identified at each step. Features were obtained for activations in GPT-2 Small on the weekdays dataset.

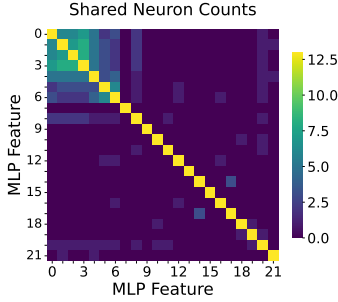


Figure 5: Shared neuron counts between MLP feature pairs. Features 0–6 correspond to weekdays (Monday–Sunday). the rest are randomly sampled from Z . All days share core neurons, suggesting a general day representation, while weekdays and weekends share distinct subsets, forming specialized neuron cores.

2023), where a single feature in a smaller SAE corresponds to multiple features in a larger SAE. Additional recursive decomposition examples learned from a more general dataset are provided in §F.

Semantically-Related MLP Features Share Common Neuron Structures The previous analysis suggests that certain groups of neurons contribute to semantically-related MLP features. Here, we look into these shared structures using the matrix Z . We binarize Z with a per-feature threshold to produce $\bar{Z}$, where the top activating neurons of each MLP feature are set to 1 and the rest to 0. Then, we calculate $M := \bar{Z}\bar{Z}^T$, where each entry $M_{i,j}$ is the number of overlapping, top activating neurons between the MLP features i and j .

Figure 5 visualizes M , where the first seven rows correspond to MLP features associated with Monday through Sunday and the rest MLP features are randomly sampled from Z . The heatmap reveals that the model represents the concept of weekdays using a common set of neurons, as indicated by the higher shared neuron counts among the first

Neuron group	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Monday	2.0	-0.8	-1.0	-1.2	-0.8	-1.1	-0.1
Tuesday	-3.3	0.5	-2.2	-2.4	-2.4	-3.2	-2.8
Wednesday	-2.7	-3.2	2.5	-2.7	-4.3	-3.2	-2.7
Thursday	-0.9	-1.0	0.6	4.5	-0.1	-1.7	-0.5
Friday	-2.9	-2.8	-2.7	-1.6	1.3	-1.9	-2.6
Saturday	-0.2	-0.7	-0.1	-0.7	-0.5	2.4	0.7
Sunday	-0.4	-1.7	-1.6	-2.0	-0.8	-0.8	2.6
Core weekday	5.8	5.7	5.4	5.8	6.0	5.4	4.7

Table 2: Change in logits for weekday tokens (columns) when amplifying exclusive neurons tied to different weekday concepts (rows). Positive values (black) indicate promotion; negative (red) indicate suppression. Exclusive neurons promote their own token (diagonal) and suppress others, while core neurons (bottom row) raise all logits, encoding a general “weekday” concept.

seven rows and columns. Given the increased diagonal values, we understand that each day also activates its own additional distinct subset of neurons. Furthermore, the MLP features representing the weekend and those representing weekdays each share more neurons within their respective groups. We can see this in rows 5 and 6, which have higher shared neuron counts with each other than in rows 0 to 4 (and vice versa). Meanwhile, there is little overlap with the remaining MLP features. These patterns show that the model utilizes sets of neurons as building blocks to represent complex and hierarchical concepts. Specifically, a core set of neurons encodes the concept of a day, while additional neurons refine the representation into weekend or weekday, and further neurons produce representations for individual days.

We test this via causal interventions. Let the *neuron base* be the neurons shared across all weekday MLP features, and the *exclusive neurons* the unique neurons to each MLP feature. We feed the prompt “I think that” to GPT-2 Large, while amplifying the neuron base and, separately, the exclusive neurons of each weekday. We measure the effect of each intervention on the model’s output distribution by calculating the change in logits for tokens associated with each day of the week.

Table 2 shows that amplifying the neuron base promotes all weekday tokens, suggesting it encodes a high-level weekday concept. Conversely, amplifying a day’s exclusive neurons promotes its associated token while suppressing the others. For example, activating Sunday’s exclusive neurons

suppresses the tokens for “Monday” through “Saturday”. These findings demonstrate that the model builds increasingly specific concepts by using sets of neurons as compositional building blocks.

7 Conclusion

We introduce an unsupervised method to decompose MLP activations into MLP features, which represent how groups of neurons contribute to encoding concepts. The MLP features often align with human-interpretable concepts, performing on par with SAEs on interpretability benchmarks, while achieving better steering performance than both SAEs and a strong supervised baseline, DiffMeans. We further demonstrate that the model composes groups of neurons additively to represent a wide range of concepts, revealing a hierarchical organization in the hidden activation space. This behavior helps explain why the same sets of neurons contribute to multiple features, extending the functional role of neurons in the MLP. It also supports the view that the feature splitting observed in SAEs reflects the model’s use of neuron composition to construct more specific features, rather than necessarily being a byproduct of SAE training. Overall, our findings provide a new lens on how MLPs form features in LLMs and we introduce an interpretable unsupervised approach for identifying and interpreting MLP features in transformers.

Limitations

While our work introduces a promising method for analyzing MLP activations, our benchmarks were limited to number of MLP features of $k < 500$. Analyzing the granularity of features that may emerge at larger values of k and whether the method scales effectively to thousands of MLP features are interesting directions for future work.

In terms of optimization of SNMF, we did not perform a rigorous comparison of optimization methods or initialization strategies. We chose sampling-based initializations (uniform and normal) for simplicity. However, given that SNMF is non-convex, different initializations of the matrices Y and Z may converge to different local minima. Previous papers have suggested initializations such as K-means (Ding et al., 2010) or SVD based (Boutsidis and Gallopoulos, 2008) that may provide improved concept detection and steering. Moreover, using projected gradient descent rather than Multiplicative Updates may allow for regularization, further

improving performance.

Acknowledgments

We thank Aaron Mueller for his valuable feedback and guidance on training sparse autoencoders. This work was supported in part by the Gemma 2 Academic Research Program at Google, a grant from Open Philanthropy, and the Israel Science Foundation grant 1083/24.

References

- Steven Bills, Nick Cammarata, Dan Mossing, Henk Tillman, Leo Gao, Gabriel Goh, Ilya Sutskever, Jan Leike, Jeff Wu, and William Saunders. 2023. Language models can explain neurons in language models. <https://openaipublic.blob.core.windows.net/neuron-explainer/paper/index.html>.
- Tolga Bolukbasi, Adam Pearce, Ann Yuan, Andy Coenen, Emily Reif, Fernanda Viégas, and Martin Wattenberg. 2021. An interpretability illusion for bert. *arXiv preprint arXiv:2104.07143*.
- C. Boutsidis and E. Gallopoulos. 2008. [Svd based initialization: A head start for nonnegative matrix factorization](#). *Pattern Recognition*, 41(4):1350–1362.
- Trenton Bricken, Adly Templeton, Joshua Batson, Brian Chen, Adam Jermy, Tom Conerly, Nick Turner, Cem Anil, Carson Denison, Amanda Askell, Robert Lasenby, Yifan Wu, Shauna Kravec, Nicholas Schiefer, Tim Maxwell, Nicholas Joseph, Zac Hatfield-Dodds, Alex Tamkin, Karina Nguyen, Brayden McLean, Josiah E Burke, Tristan Hume, Shan Carter, Tom Henighan, and Christopher Olah. 2023. Towards monosemanticity: Decomposing language models with dictionary learning. *Transformer Circuits Thread*. <https://transformer-circuits.pub/2023/monosemantic-features/index.html>.
- Tue Minh Cao, Nhat Hoang-Xuan, Hieu Pham, Phi Le Nguyen, and My T. Thai. 2025. [Neurflow: Interpreting neural networks through neuron groups and functional interactions](#). In *The Thirteenth International Conference on Learning Representations*.
- David Chanin, James Wilken-Smith, Tomáš Dulka, Hardik Bhatnagar, and Joseph Isaac Bloom. 2024. [A is for absorption: Studying feature splitting and absorption in sparse autoencoders](#). In *Interpretable AI: Past, Present and Future*.
- Maheep Chaudhary and Atticus Geiger. 2024. Evaluating open-source sparse autoencoders on disentangling factual knowledge in gpt-2 small. *arXiv preprint arXiv:2409.04478*.
- Dami Choi, Vincent Huang, Kevin Meng, Daniel D Johnson, Jacob Steinhardt, and Sarah Schwettmann.

2024. Scaling automatic neuron description. <https://transluce.org/neuron-descriptions>.
- Róbert Csordás, Christopher Potts, Christopher D Manning, and Atticus Geiger. 2024. [Recurrent neural networks learn to store and generate sequences using non-linear representations](#). In *Proceedings of the 7th BlackboxNLP Workshop: Analyzing and Interpreting Neural Networks for NLP*, pages 248–262, Miami, Florida, US. Association for Computational Linguistics.
- Hoagy Cunningham, Aidan Ewart, Logan Riggs, Robert Huben, and Lee Sharkey. 2023. [Sparse autoencoders find highly interpretable features in language models](#). *Preprint*, arXiv:2309.08600.
- Fahim Dalvi, Nadir Durrani, Hassan Sajjad, Yonatan Belinkov, Anthony Bau, and James R. Glass. 2018. [What is one grain of sand in the desert? analyzing individual neurons in deep NLP models](#). *CoRR*, abs/1812.09355.
- Chris Ding, Tao Li, and Michael Jordan. 2010. [Convex and semi-nonnegative matrix factorizations](#). *IEEE transactions on pattern analysis and machine intelligence*, 32:45–55.
- Nelson Elhage, Neel Nanda, Catherine Olsson, Tom Henighan, Nicholas Joseph, Ben Mann, Amanda Askell, Yuntao Bai, Anna Chen, Tom Conerly, Nova DasSarma, Dawn Drain, Deep Ganguli, Zac Hatfield-Dodds, Danny Hernandez, Andy Jones, Jackson Kernion, Liane Lovitt, Kamal Ndousse, Dario Amodei, Tom Brown, Jack Clark, Jared Kaplan, Sam McCandlish, and Chris Olah. 2021. A mathematical framework for transformer circuits. *Transformer Circuits Thread*. <https://transformer-circuits.pub/2021/framework/index.html>.
- Joshua Engels, Eric J Michaud, Isaac Liao, Wes Gurnee, and Max Tegmark. 2025. [Not all language model features are one-dimensionally linear](#). In *The Thirteenth International Conference on Learning Representations*.
- Leo Gao, Tom Dupre la Tour, Henk Tillman, Gabriel Goh, Rajan Troll, Alec Radford, Ilya Sutskever, Jan Leike, and Jeffrey Wu. 2025. [Scaling and evaluating sparse autoencoders](#). In *The Thirteenth International Conference on Learning Representations*.
- Atticus Geiger, Duligur Ibeling, Amir Zur, Maheep Chaudhary, Sonakshi Chauhan, Jing Huang, Aryaman Arora, Zhengxuan Wu, Noah Goodman, Christopher Potts, and Thomas Icard. 2024. [Causal abstraction: A theoretical foundation for mechanistic interpretability](#). *Preprint*, arXiv:2301.04709.
- Atticus Geiger, Zhengxuan Wu, Christopher Potts, Thomas Icard, and Noah D. Goodman. 2023. [Finding alignments between interpretable causal variables and distributed neural representations](#). Ms., Stanford University.
- Mor Geva, Avi Caciularu, Kevin Wang, and Yoav Goldberg. 2022. [Transformer feed-forward layers build predictions by promoting concepts in the vocabulary space](#). In *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing*, pages 30–45, Abu Dhabi, United Arab Emirates. Association for Computational Linguistics.
- Mor Geva, Roei Schuster, Jonathan Berant, and Omer Levy. 2021. [Transformer feed-forward layers are key-value memories](#). In *Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing*, pages 5484–5495, Online and Punta Cana, Dominican Republic. Association for Computational Linguistics.
- Google. 2025. Gemini 2.0 Flash. <https://cloud.google.com/vertex-ai/generative-ai/docs/models/gemini/2-0-flash>.
- Aaron Grattafiori, Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Alex Vaughan, Amy Yang, Angela Fan, Anirudh Goyal, Anthony Hartshorn, Aobo Yang, Archi Mitra, Archie Sravankumar, Artem Korenev, Arthur Hinsvark, Arun Rao, Aston Zhang, Aurelien Rodriguez, Austen Gregerson, Ava Spataru, Baptiste Roziere, Bethany Biron, Binh Tang, Bobbie Chern, Charlotte Caucheteux, Chaya Nayak, Chloe Bi, Chris Marra, Chris McConnell, Christian Keller, Christophe Touret, Chunyang Wu, Corinne Wong, Cristian Canton Ferrer, Cyrus Nikolaidis, Damien Al-lonsius, Daniel Song, Danielle Pintz, Danny Livshits, Danny Wyatt, David Esiobu, Dhruv Choudhary, Dhruv Mahajan, Diego Garcia-Olano, Diego Perino, Dieuwke Hupkes, Egor Lakomkin, Ehab AlBadawy, Elina Lobanova, Emily Dinan, Eric Michael Smith, Filip Radenovic, Francisco Guzmán, Frank Zhang, Gabriel Synnaeve, Gabrielle Lee, Georgia Lewis Anderson, Govind Thattai, Graeme Nail, Gregoire Mialon, Guan Pang, Guillem Cucurell, Hailey Nguyen, Hannah Korevaar, Hu Xu, Hugo Touvron, Iliyan Zarov, Imanol Arrieta Ibarra, Isabel Kloumann, Ishan Misra, Ivan Evtimov, Jack Zhang, Jade Copet, Jaewon Lee, Jan Geffert, Jana Vranes, Jason Park, Jay Mahadeokar, Jeet Shah, Jelmer van der Linde, Jennifer Billock, Jenny Hong, Jenya Lee, Jeremy Fu, Jianfeng Chi, Jianyu Huang, Jiawen Liu, Jie Wang, Jiecao Yu, Joanna Bitton, Joe Spisak, Jongsoo Park, Joseph Rocca, Joshua Johnstun, Joshua Saxe, Junteng Jia, Kalyan Vasuden Alwala, Karthik Prasad, Kartikeya Upasani, Kate Plawiak, Ke Li, Kenneth Heafield, Kevin Stone, Khalid El-Arini, Krithika Iyer, Kshitiz Malik, Kuenley Chiu, Kunal Bhalla, Kushal Lakhotia, Lauren Rantala-Yeary, Laurens van der Maaten, Lawrence Chen, Liang Tan, Liz Jenkins, Louis Martin, Lovish Madaan, Lubo Malo, Lukas Blecher, Lukas Landzaat, Luke de Oliveira, Madeline Muzzi, Mahesh Pasupuleti, Mannat Singh, Manohar Paluri, Marcin Kardas, Maria Tsimpoukelli, Mathew Oldham, Mathieu Rita, Maya Pavlova, Melanie Kam-badur, Mike Lewis, Min Si, Mitesh Kumar Singh, Mona Hassan, Naman Goyal, Narjes Torabi, Nikolay Bashlykov, Nikolay Bogoychev, Niladri Chatterji,

Ning Zhang, Olivier Duchenne, Onur Çelebi, Patrick Alrassy, Pengchuan Zhang, Pengwei Li, Petar Vasic, Peter Weng, Prajjwal Bhargava, Pratik Dubal, Praveen Krishnan, Punit Singh Koura, Puxin Xu, Qing He, Qingxiao Dong, Ragavan Srinivasan, Raj Ganapathy, Ramon Calderer, Ricardo Silveira Cabral, Robert Stojnic, Roberta Raileanu, Rohan Maheswari, Rohit Girdhar, Rohit Patel, Romain Sauvestre, Ronnie Polidoro, Roshan Sumbaly, Ross Taylor, Ruan Silva, Rui Hou, Rui Wang, Saghar Hosseini, Sahana Chennabasappa, Sanjay Singh, Sean Bell, Seohyun Sonia Kim, Sergey Edunov, Shaoliang Nie, Sharan Narang, Sharath Rapparthi, Sheng Shen, Shengye Wan, Shruti Bhosale, Shun Zhang, Simon Vandenhende, Soumya Batra, Spencer Whitman, Sten Sootla, Stephane Collot, Suchin Gururangan, Sydney Borodinsky, Tamar Herman, Tara Fowler, Tarek Sheasha, Thomas Georgiou, Thomas Scialom, Tobias Speckbacher, Todor Mihaylov, Tong Xiao, Ujjwal Karn, Vedanuj Goswami, Vibhor Gupta, Vignesh Ramanathan, Viktor Kerkez, Vincent Gouget, Virginie Do, Vish Vogeti, Vitor Albiero, Vladan Petrovic, Weiwei Chu, Wenhan Xiong, Wenyin Fu, Whitney Meers, Xavier Martinet, Xiaodong Wang, Xiaofang Wang, Xiaoqing Ellen Tan, Xide Xia, Xinfeng Xie, Xuchao Jia, Xuewei Wang, Yaelle Goldschlag, Yashesh Gaur, Yasmine Babaei, Yi Wen, Yiwen Song, Yuchen Zhang, Yue Li, Yuning Mao, Zacharie Delpierre Coudert, Zheng Yan, Zhengxing Chen, Zoe Papakipos, Aaditya Singh, Aayushi Srivastava, Abha Jain, Adam Kelsey, Adam Shajnfeld, Adithya Gangidi, Adolfo Victoria, Ahuva Goldstand, Ajay Menon, Ajay Sharma, Alex Boesenberg, Alexei Baevski, Allie Feinstein, Amanda Kallet, Amit Sangani, Amos Teo, Anam Yunus, Andrei Lupu, Andres Alvarado, Andrew Caples, Andrew Gu, Andrew Ho, Andrew Poulton, Andrew Ryan, Ankit Ramchandani, Annie Dong, Annie Franco, Anuj Goyal, Aparajita Saraf, Arkabandhu Chowdhury, Ashley Gabriel, Ashwin Bharambe, Assaf Eisenman, Azadeh Yazdan, Beau James, Ben Maurer, Benjamin Leonhardi, Bernie Huang, Beth Loyd, Beto De Paola, Bhargavi Paranjape, Bing Liu, Bo Wu, Boyu Ni, Braden Hancock, Bram Wasti, Brandon Spence, Brani Stojkovic, Brian Gamido, Britt Montalvo, Carl Parker, Carly Burton, Catalina Mejia, Ce Liu, Changan Wang, Changkyu Kim, Chao Zhou, Chester Hu, Ching-Hsiang Chu, Chris Cai, Chris Tindal, Christoph Feichtenhofer, Cynthia Gao, Damon Civin, Dana Beaty, Daniel Kreymer, Daniel Li, David Adkins, David Xu, Davide Testuggine, Delia David, Devi Parikh, Diana Liskovich, Didem Foss, Dingkan Wang, Duc Le, Dustin Holland, Edward Dowling, Eissa Jamil, Elaine Montgomery, Eleonora Presani, Emily Hahn, Emily Wood, Eric-Tuan Le, Erik Brinkman, Esteban Arcaute, Evan Dunbar, Evan Smothers, Fei Sun, Felix Kreuk, Feng Tian, Filippas Kokkinos, Firat Ozgenel, Francesco Caggioni, Frank Kanayet, Frank Seide, Gabriela Medina Florez, Gabriella Schwarz, Gada Badeer, Georgia Swee, Gil Halpern, Grant Herman, Grigory Sizov, Guangyi, Zhang, Guna Lakshminarayanan, Hakan Inan, Hamid Shojanazeri, Han Zou, Hannah Wang, Hanwen Zha, Haroun Habeeb, Harrison Rudolph, Helen Suk, Henry As-

pegren, Hunter Goldman, Hongyuan Zhan, Ibrahim Damlaj, Igor Molybog, Igor Tufanov, Ilias Leontiadis, Irina-Elena Veliche, Itai Gat, Jake Weissman, James Geboski, James Kohli, Janice Lam, Japhet Asher, Jean-Baptiste Gaya, Jeff Marcus, Jeff Tang, Jennifer Chan, Jenny Zhen, Jeremy Reizenstein, Jeremy Teboul, Jessica Zhong, Jian Jin, Jingyi Yang, Joe Cummings, Jon Carvill, Jon Shepard, Jonathan McPhie, Jonathan Torres, Josh Ginsburg, Junjie Wang, Kai Wu, Kam Hou U, Karan Saxena, Kartikay Khandwal, Katayoun Zand, Kathy Matosich, Kaushik Veeraraghavan, Kelly Michelena, Keqian Li, Kiran Jagadeesh, Kun Huang, Kunal Chawla, Kyle Huang, Lailin Chen, Lakshya Garg, Lavender A, Leandro Silva, Lee Bell, Lei Zhang, Liangpeng Guo, Licheng Yu, Liron Moshkovich, Luca Wehrstedt, Madian Khabsa, Manav Avalani, Manish Bhatt, Martynas Mankus, Matan Hasson, Matthew Lennie, Matthias Reso, Maxim Groshev, Maxim Naumov, Maya Lathi, Meghan Keneally, Miao Liu, Michael L. Seltzer, Michal Valko, Michelle Restrepo, Mihir Patel, Mik Vyatskov, Mikayel Samvelyan, Mike Clark, Mike Macey, Mike Wang, Miquel Jubert Hermoso, Mo Metanat, Mohammad Rastegari, Munish Bansal, Nandhini Santhanam, Natascha Parks, Natasha White, Navyata Bawa, Nayan Singhal, Nick Egebo, Nicolas Usunier, Nikhil Mehta, Nikolay Pavlovich Laptev, Ning Dong, Norman Cheng, Oleg Chernoguz, Olivia Hart, Omkar Salpekar, Ozlem Kalinli, Parkin Kent, Parth Parekh, Paul Saab, Pavan Balaji, Pedro Rittner, Philip Bontrager, Pierre Roux, Piotr Dollar, Polina Zvyagina, Prashant Ratanchandani, Pritish Yuvraj, Qian Liang, Rachad Alao, Rachel Rodriguez, Rafi Ayub, Raghotham Murthy, Raghu Nayani, Rahul Mitra, Rangaprabhu Parthasarathy, Raymond Li, Rebekkah Hogan, Robin Battey, Rocky Wang, Russ Howes, Ruty Rinott, Sachin Mehta, Sachin Siby, Sai Jayesh Bondu, Samyak Datta, Sara Chugh, Sara Hunt, Sargun Dhillon, Sasha Sidorov, Satadru Pan, Saurabh Mahajan, Saurabh Verma, Seiji Yamamoto, Sharadh Ramaswamy, Shaun Lindsay, Shaun Lindsay, Sheng Feng, Shenghao Lin, Shengxin Cindy Zha, Shishir Patil, Shiva Shankar, Shuqiang Zhang, Shuqiang Zhang, Sinong Wang, Sneha Agarwal, Soji Sajuyigbe, Soumith Chintala, Stephanie Max, Stephen Chen, Steve Kehoe, Steve Satterfield, Sudarshan Govindaprasad, Sumit Gupta, Summer Deng, Sungmin Cho, Sunny Virk, Suraj Subramanian, Sy Choudhury, Sydney Goldman, Tal Remez, Tamar Glaser, Tamara Best, Thilo Koehler, Thomas Robinson, Tianhe Li, Tianjun Zhang, Tim Matthews, Timothy Chou, Tzook Shaked, Varun Vontimitta, Victoria Ajayi, Victoria Montanez, Vijai Mohan, Vinay Satish Kumar, Vishal Mangla, Vlad Ionescu, Vlad Poenaru, Vlad Tiberiu Mihailescu, Vladimir Ivanov, Wei Li, Wenchen Wang, Wenwen Jiang, Wes Bouaziz, Will Constable, Xiaocheng Tang, Xiaojian Wu, Xiaolan Wang, Xilun Wu, Xinbo Gao, Yaniv Kleinman, Yanjun Chen, Ye Hu, Ye Jia, Ye Qi, Yenda Li, Yilin Zhang, Ying Zhang, Yossi Adi, Youngjin Nam, Yu, Wang, Yu Zhao, Yuchen Hao, Yundi Qian, Yunlu Li, Yuzi He, Zach Rait, Zachary DeVito, Zef Rosnbrick, Zhaoduo Wen, Zhenyu Yang, Zhiwei Zhao, and Zhiyu Ma. 2024. [The llama 3 herd](#)

- of models. *Preprint*, arXiv:2407.21783.
- Yoav Gur-Arieh, Roy Mayan, Chen Agassy, Atticus Geiger, and Mor Geva. 2025. Enhancing automated interpretability with output-centric feature descriptions. *arXiv preprint arXiv:2501.08319*.
- Wes Gurnee, Neel Nanda, Matthew Pauly, Katherine Harvey, Dmitrii Troitskii, and Dimitris Bertsimas. 2023. [Finding neurons in a haystack: Case studies with sparse probing](#). *Transactions on Machine Learning Research*.
- Zhengfu He, Wentao Shu, Xuyang Ge, Lingjie Chen, Junxuan Wang, Yunhua Zhou, Frances Liu, Qipeng Guo, Xuanjing Huang, Zuxuan Wu, et al. 2024. Llama scope: Extracting millions of features from llama-3.1-8b with sparse autoencoders. *arXiv preprint arXiv:2410.20526*.
- Evan Hernandez, Sarah Schwettmann, David Bau, Teona Bagashvili, Antonio Torralba, and Jacob Andreas. 2021. Natural language descriptions of deep visual features. In *International Conference on Learning Representations*.
- Jing Huang, Atticus Geiger, Karel D’Oosterlinck, Zhengxuan Wu, and Christopher Potts. 2023. [Rigorously assessing natural language explanations of neurons](#). In *Proceedings of the 6th BlackboxNLP Workshop: Analyzing and Interpreting Neural Networks for NLP*, pages 317–331, Singapore. Association for Computational Linguistics.
- Jing Huang, Zhengxuan Wu, Christopher Potts, Mor Geva, and Atticus Geiger. 2024. [RAVEL: Evaluating interpretability methods on disentangling language model representations](#). In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 8669–8687, Bangkok, Thailand. Association for Computational Linguistics.
- Andrej Karpathy, Justin Johnson, and Li Fei-Fei. 2015. Visualizing and understanding recurrent networks. *arXiv preprint arXiv:1506.02078*.
- Daniel Lee and H. Seung. 1999. [Learning the parts of objects by non-negative matrix factorization](#). *Nature*, 401:788–91.
- Amit Arnold Levy and Mor Geva. 2025. [Language models encode numbers using digit representations in base 10](#). In *Proceedings of the 2025 Conference of the Nations of the Americas Chapter of the Association for Computational Linguistics: Human Language Technologies (Volume 2: Short Papers)*, pages 385–395, Albuquerque, New Mexico. Association for Computational Linguistics.
- Tom Lieberum, Senthooan Rajamanoharan, Arthur Conmy, Lewis Smith, Nicolas Sonnerat, Vikrant Varma, Janos Kramar, Anca Dragan, Rohin Shah, and Neel Nanda. 2024. [Gemma scope: Open sparse autoencoders everywhere all at once on gemma 2](#). In *Proceedings of the 7th BlackboxNLP Workshop: Analyzing and Interpreting Neural Networks for NLP*, pages 278–300, Miami, Florida, US. Association for Computational Linguistics.
- Johnny Lin. 2023. [Neuronpedia: Interactive reference and tooling for analyzing neural networks](#). Software available from neuronpedia.org.
- Hanxiao Liu, Zihang Dai, David So, and Quoc V Le. 2021. [Pay attention to MLPs](#). In *Advances in Neural Information Processing Systems*.
- Samuel Marks, Can Rager, Eric J Michaud, Yonatan Belinkov, David Bau, and Aaron Mueller. 2025. [Sparse feature circuits: Discovering and editing interpretable causal graphs in language models](#). In *The Thirteenth International Conference on Learning Representations*.
- Samuel Marks and Max Tegmark. 2024. [The geometry of truth: Emergent linear structure in large language model representations of true/false datasets](#). In *First Conference on Language Modeling*.
- Luke Melas-Kyriazi, Christian Rupprecht, Iro Laina, and Andrea Vedaldi. 2022. Deep spectral methods: A surprisingly strong baseline for unsupervised semantic segmentation and localization. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 8364–8375.
- Kevin Meng, David Bau, Alex J Andonian, and Yonatan Belinkov. 2022. [Locating and editing factual associations in GPT](#). In *Advances in Neural Information Processing Systems*.
- Abhinav Menon, Manish Shrivastava, David Krueger, and Ekdeep Singh Lubana. 2025. [Analyzing \(in\)abilities of SAEs via formal languages](#). In *Proceedings of the 2025 Conference of the Nations of the Americas Chapter of the Association for Computational Linguistics: Human Language Technologies (Volume 1: Long Papers)*, pages 4837–4862, Albuquerque, New Mexico. Association for Computational Linguistics.
- Aaron Mueller, Jannik Brinkmann, Millicent Li, Samuel Marks, Koyena Pal, Nikhil Prakash, Can Rager, Aruna Sankaranarayanan, Arnab Sen Sharma, Jiuding Sun, Eric Todd, David Bau, and Yonatan Belinkov. 2024. [The quest for the right mediator: A history, survey, and theoretical grounding of causal interpretability](#). *Preprint*, arXiv:2408.01416.
- Aaron Mueller, Atticus Geiger, Sarah Wiegrefe, Dana Arad, Iván Arcuschin, Adam Belfki, Yik Siu Chan, Jaden Fiotto-Kaufman, Tal Haklay, Michael Hanna, Jing Huang, Rohan Gupta, Yaniv Nikankin, Hadas Orgad, Nikhil Prakash, Anja Reusch, Aruna Sankaranarayanan, Shun Shao, Alessandro Stolfo, Martin Tutek, Amir Zur, David Bau, and Yonatan Belinkov. 2025. [Mib: A mechanistic interpretability benchmark](#). *Preprint*, arXiv:2504.13151.
- Neel Nanda, Andrew Lee, and Martin Wattenberg. 2023. [Emergent linear representations in world models](#)

of self-supervised sequence models. In *Proceedings of the 6th BlackboxNLP Workshop: Analyzing and Interpreting Neural Networks for NLP, BlackboxNLP@EMNLP 2023, Singapore, December 7, 2023*, pages 16–30. Association for Computational Linguistics.

Chris Olah, Nick Cammarata, Ludwig Schubert, Gabriel Goh, Michael Petrov, and Shan Carter. 2020. [Zoom in: An introduction to circuits](https://distill.pub/2020/circuits/zoom-in). *Distill*. <https://distill.pub/2020/circuits/zoom-in>.

James Oldfield, Christos Tzelepis, Yannis Panagakis, Mihalis Nicolaou, and Ioannis Patras. 2023. [Panda: Unsupervised learning of parts and appearances in the feature maps of GANs](#). In *The Eleventh International Conference on Learning Representations*.

OpenAI, :, Aaron Hurst, Adam Lerer, Adam P. Goucher, Adam Perelman, Aditya Ramesh, Aidan Clark, AJ Ostrow, Akila Welihinda, Alan Hayes, Alec Radford, Aleksander Mądry, Alex Baker-Whitcomb, Alex Beutel, Alex Borzunov, Alex Carney, Alex Chow, Alex Kirillov, Alex Nichol, Alex Paino, Alex Renzin, Alex Tachard Passos, Alexander Kirillov, Alexi Christakis, Alexis Conneau, Ali Kamali, Allan Jabri, Allison Moyer, Allison Tam, Amadou Crookes, Amin Tootoochian, Amin Tootoonchian, Ananya Kumar, Andrea Vallone, Andrej Karpathy, Andrew Braunstein, Andrew Cann, Andrew Codisoti, Andrew Galu, Andrew Kondrich, Andrew Tulloch, Andrew Mishchenko, Angela Baek, Angela Jiang, Antoine Pelisse, Antonia Woodford, Anuj Gosalia, Arka Dhar, Ashley Pantuliano, Avi Nayak, Avital Oliver, Barret Zoph, Behrooz Ghorbani, Ben Leimberger, Ben Rossen, Ben Sokolowsky, Ben Wang, Benjamin Zweig, Beth Hoover, Blake Samic, Bob McGrew, Bobby Spero, Bogo Gierlter, Bowen Cheng, Brad Lightcap, Brandon Walkin, Brendan Quinn, Brian Guarraci, Brian Hsu, Bright Kellogg, Brydon Eastman, Camillo Lugaresi, Carroll Wainwright, Cary Bassin, Cary Hudson, Casey Chu, Chad Nelson, Chak Li, Chan Jun Shern, Channing Conger, Charlotte Barette, Chelsea Voss, Chen Ding, Cheng Lu, Chong Zhang, Chris Beaumont, Chris Hallacy, Chris Koch, Christian Gibson, Christina Kim, Christine Choi, Christine McLeavey, Christopher Hesse, Claudia Fischer, Clemens Winter, Coley Czarnecki, Colin Jarvis, Colin Wei, Constantin Koumouzelis, Dane Sherburn, Daniel Kappler, Daniel Levin, Daniel Levy, David Carr, David Farhi, David Mely, David Robinson, David Sasaki, Denny Jin, Dev Valladares, Dimitris Tsipras, Doug Li, Duc Phong Nguyen, Duncan Findlay, Edede Oiwoh, Edmund Wong, Ehsan Asdar, Elizabeth Proehl, Elizabeth Yang, Eric Antonow, Eric Kramer, Eric Peterson, Eric Sigler, Eric Wallace, Eugene Brevdo, Evan Mays, Farzad Khorasani, Felipe Petroski Such, Filippo Raso, Francis Zhang, Fred von Lohmann, Freddie Sulit, Gabriel Goh, Gene Oden, Geoff Salmon, Giulio Starace, Greg Brockman, Hadi Salman, Haiming Bao, Haitang Hu, Hannah Wong, Haoyu Wang, Heather Schmidt, Heather Whitney, Heewoo Jun, Hendrik Kirchner, Henrique Ponde de Oliveira Pinto, Hongyu Ren,

Huiwen Chang, Hyung Won Chung, Ian Kivlichen, Ian O’Connell, Ian O’Connell, Ian Osband, Ian Silber, Ian Sohl, Ibrahim Okuyucu, Ikai Lan, Ilya Kostrikov, Ilya Sutskever, Ingmar Kanitscheider, Ishaan Gulrajani, Jacob Coxon, Jacob Menick, Jakub Pachocki, James Aung, James Betker, James Crooks, James Lennon, Jamie Kiros, Jan Leike, Jane Park, Jason Kwon, Jason Phang, Jason Teplitz, Jason Wei, Jason Wolfe, Jay Chen, Jeff Harris, Jenia Varavva, Jessica Gan Lee, Jessica Shieh, Ji Lin, Jiahui Yu, Jiayi Weng, Jie Tang, Jieqi Yu, Joanne Jang, Joaquin Quinonero Candela, Joe Beutler, Joe Landers, Joel Parish, Johannes Heidecke, John Schulman, Jonathan Lachman, Jonathan McKay, Jonathan Uesato, Jonathan Ward, Jong Wook Kim, Joost Huizinga, Jordan Sitkin, Jos Kraaijeveld, Josh Gross, Josh Kaplan, Josh Snyder, Joshua Achiam, Joy Jiao, Joyce Lee, Juntang Zhuang, Justyn Harriman, Kai Fricke, Kai Hayashi, Karan Singhal, Katy Shi, Kevin Karthik, Kayla Wood, Kendra Rimbach, Kenny Hsu, Kenny Nguyen, Keren Gu-Lemberg, Kevin Button, Kevin Liu, Kiel Howe, Krithika Muthukumar, Kyle Luther, Lama Ahmad, Larry Kai, Lauren Itow, Lauren Workman, Leher Pathak, Leo Chen, Li Jing, Lia Guy, Liam Fedus, Liang Zhou, Lien Mamitsuka, Lilian Weng, Lindsay McCallum, Lindsey Held, Long Ouyang, Louis Feuvrier, Lu Zhang, Lukas Kondraciuk, Lukasz Kaiser, Luke Hewitt, Luke Metz, Lyric Doshi, Mada Aflak, Maddie Simens, Madelaine Boyd, Madeleine Thompson, Marat Dukhan, Mark Chen, Mark Gray, Mark Hudnall, Marvin Zhang, Marwan Aljube, Mateusz Litwin, Matthew Zeng, Max Johnson, Maya Shetty, Mayank Gupta, Meghan Shah, Mehmet Yatbaz, Meng Jia Yang, Mengchao Zhong, Mia Glaese, Mianna Chen, Michael Janer, Michael Lampe, Michael Petrov, Michael Wu, Michele Wang, Michelle Fradin, Michelle Pokrass, Miguel Castro, Miguel Oom Temudo de Castro, Mikhail Pavlov, Miles Brundage, Miles Wang, Minal Khan, Mira Murati, Mo Bavarian, Molly Lin, Murat Yesildal, Nacho Soto, Natalia Gimelshein, Natalie Cone, Natalie Staudacher, Natalie Summers, Natan LaFontaine, Neil Chowdhury, Nick Ryder, Nick Stathas, Nick Turley, Nik Tezak, Niko Felix, Nithanth Kudige, Nitish Keskar, Noah Deutsch, Noel Bundick, Nora Puckett, Ofir Nachum, Ola Okelola, Oleg Boiko, Oleg Murk, Oliver Jaffe, Olivia Watkins, Olivier Godement, Owen Campbell-Moore, Patrick Chao, Paul McMillan, Pavel Belov, Peng Su, Peter Bak, Peter Bakkum, Peter Deng, Peter Dolan, Peter Hoeschele, Peter Welinder, Phil Tillet, Philip Pronin, Philippe Tillet, Prafulla Dhariwal, Qiming Yuan, Rachel Dias, Rachel Lim, Rahul Arora, Rajan Troll, Randall Lin, Rapha Gontijo Lopes, Raul Puri, Reah Miyara, Reimar Leike, Renaud Gaubert, Reza Zamani, Ricky Wang, Rob Donnelly, Rob Honsby, Rocky Smith, Rohan Sahai, Rohit Ramchandani, Romain Huet, Rory Carmichael, Rowan Zellers, Roy Chen, Ruby Chen, Ruslan Nigmatullin, Ryan Cheu, Saachi Jain, Sam Altman, Sam Schoenholz, Sam Toizer, Samuel Miserendino, Sandhini Agarwal, Sara Culver, Scott Ethersmith, Scott Gray, Sean Grove, Sean Metzger, Shamez Hermani, Shantanu Jain, Shengjia Zhao, Sherwin Wu, Shino Jomoto, Shi-

- rong Wu, Shuaiqi, Xia, Sonia Phene, Spencer Papay, Srinivas Narayanan, Steve Coffey, Steve Lee, Stewart Hall, Suchir Balaji, Tal Broda, Tal Stramer, Tao Xu, Tarun Gogineni, Taya Christianson, Ted Sanders, Tejal Patwardhan, Thomas Cunningham, Thomas Degry, Thomas Dimson, Thomas Raoux, Thomas Shadwell, Tianhao Zheng, Todd Underwood, Todor Markov, Toki Sherbakov, Tom Rubin, Tom Stasi, Tomer Kaftan, Tristan Heywood, Troy Peterson, Tyce Walters, Tyna Eloundou, Valerie Qi, Veit Moeller, Vinnie Monaco, Vishal Kuo, Vlad Fomenko, Wayne Chang, Weiye Zheng, Wenda Zhou, Wesam Manassra, Will Sheu, Wojciech Zaremba, Yash Patil, Yilei Qian, Yongjik Kim, Youlong Cheng, Yu Zhang, Yuchen He, Yuchen Zhang, Yujia Jin, Yunxing Dai, and Yury Malkov. 2024. [Gpt-4o system card](#). *Preprint*, arXiv:2410.21276.
- Kiho Park, Yo Joong Choe, Yibo Jiang, and Victor Veitch. 2025. [The geometry of categorical and hierarchical concepts in large language models](#). In *The Thirteenth International Conference on Learning Representations*.
- Kiho Park, Yo Joong Choe, and Victor Veitch. 2024. The linear representation hypothesis and the geometry of large language models. In *Proceedings of the 41st International Conference on Machine Learning*, ICML’24. JMLR.org.
- Gonalo Paulo, Alex Mallen, Caden Juang, and Nora Belrose. 2024. Automatically interpreting millions of features in large language models. *arXiv preprint arXiv:2410.13928*.
- Gonalo Paulo and Nora Belrose. 2025. [Sparse autoencoders trained on the same data learn different features](#). *Preprint*, arXiv:2501.16615.
- Robert Peharz and Franz Pernkopf. 2012. [Sparse nonnegative matrix factorization with \$\ell_0\$ -constraints](#). *Neurocomputing*, 80:38–46. Special Issue on Machine Learning for Signal Processing 2010.
- Alec Radford, Jeffrey Wu, Rewon Child, David Luan, Dario Amodei, and Ilya Sutskever. 2019. [Language models are unsupervised multitask learners](#). *OpenAI*. Accessed: 2024-11-15.
- Nina Rimskey, Nick Gabrieli, Julian Schulz, Meg Tong, Evan Hubinger, and Alexander Turner. 2024. [Steering llama 2 via contrastive activation addition](#). In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 15504–15522, Bangkok, Thailand. Association for Computational Linguistics.
- D. E. Rumelhart, J. L. McClelland, and PDP Research Group, editors. 1986. *Parallel Distributed Processing. Volume 1: Foundations*. MIT Press, Cambridge, MA.
- Hassan Sajjad, Nadir Durrani, Fahim Dalvi, Firoj Alam, Abdul Khan, and Jia Xu. 2022. [Analyzing encoded concepts in transformer language models](#). In *Proceedings of the 2022 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pages 3082–3101, Seattle, United States. Association for Computational Linguistics.
- Lee Sharkey, Bilal Chughtai, Joshua Batson, Jack Lindsey, Jeff Wu, Lucius Bushnaq, Nicholas Goldowsky-Dill, Stefan Heimersheim, Alejandro Ortega, Joseph Bloom, Stella Biderman, Adria Garriga-Alonso, Arthur Conmy, Neel Nanda, Jessica Rumbelow, Martin Wattenberg, Nandi Schoots, Joseph Miller, Eric J. Michaud, Stephen Casper, Max Tegmark, William Saunders, David Bau, Eric Todd, Atticus Geiger, Mor Geva, Jesse Hoogland, Daniel Murfet, and Tom McGrath. 2025. [Open problems in mechanistic interpretability](#). *Preprint*, arXiv:2501.16496.
- Paul Smolensky. 1986. Neural and conceptual interpretation of PDP models. In James L. McClelland, David E. Rumelhart, and the PDP Research Group, editors, *Parallel Distributed Processing: Explorations in the Microstructure of Cognition: Psychological and Biological Models*, volume 2, pages 390–431. MIT Press.
- Hyun Ah Song and Soo-Young Lee. 2013. Hierarchical representation using nmf. In *Neural Information Processing*, pages 466–473, Berlin, Heidelberg. Springer Berlin Heidelberg.
- Gemma Team, Morgane Riviere, Shreya Pathak, Pier Giuseppe Sessa, Cassidy Hardin, Surya Bhupatiraju, L  onard Hussenot, Thomas Mesnard, Bobak Shahriari, Alexandre Ram  , Johan Ferret, Peter Liu, Pouya Tafti, Abe Friesen, Michelle Casbon, Sabela Ramos, Ravin Kumar, Charline Le Lan, Sammy Jerome, Anton Tsitsulin, Nino Vieillard, Piotr Stanczyk, Sertan Girgin, Nikola Momchev, Matt Hoffman, Shantanu Thakoor, Jean-Bastien Grill, Behnam Neyshabur, Olivier Bachem, Alanna Walton, Aliaksei Severyn, Alicia Parrish, Aliya Ahmad, Allen Hutchison, Alvin Abdagic, Amanda Carl, Amy Shen, Andy Brock, Andy Coenen, Anthony Laforge, Antonia Paterson, Ben Bastian, Bilal Piot, Bo Wu, Brandon Royal, Charlie Chen, Chintu Kumar, Chris Perry, Chris Welty, Christopher A. Choquette-Choo, Danila Sinopalnikov, David Weinberger, Dimple Vijaykumar, Dominika Rogozi  ska, Dustin Herbison, Elisa Bandy, Emma Wang, Eric Noland, Erica Moreira, Evan Senter, Evgenii Eltyshov, Francesco Visin, Gabriel Rasskin, Gary Wei, Glenn Cameron, Gus Martins, Hadi Hashemi, Hanna Klimczak-Pluci  ska, Harleen Batra, Harsh Dhand, Ivan Nardini, Jacinda Mein, Jack Zhou, James Svensson, Jeff Stanway, Jetha Chan, Jin Peng Zhou, Joana Carrasqueira, Joana Iljazi, Jocelyn Becker, Joe Fernandez, Joost van Amersfoort, Josh Gordon, Josh Lipschultz, Josh Newlan, Ju yeong Ji, Kareem Mohamed, Kartikeya Badola, Kat Black, Katie Millican, Keelin McDonnell, Kelvin Nguyen, Kiranbir Sodhia, Kish Greene, Lars Lowe Sjoesund, Lauren Usui, Laurent Sifre, Lena Heuermann, Leticia Lago, Lilly McNealus, Livio Baldini Soares,

Logan Kilpatrick, Lucas Dixon, Luciano Martins, Machel Reid, Manvinder Singh, Mark Iverson, Martin Görner, Mat Velloso, Mateo Wirth, Matt Davidson, Matt Miller, Matthew Rahtz, Matthew Watson, Meg Risdal, Mehran Kazemi, Michael Moynihan, Ming Zhang, Minsuk Kahng, Minwoo Park, Mofi Rahman, Mohit Khatwani, Natalie Dao, Nenshad Bardoliwalla, Nesh Devanathan, Neta Dumai, Nilay Chauhan, Oscar Wahltinez, Pankil Botarda, Parker Barnes, Paul Barham, Paul Michel, Pengchong Jin, Petko Georgiev, Phil Culliton, Pradeep Kupala, Ramona Comanescu, Ramona Merhej, Reena Jana, Reza Ardeshtir Rokni, Rishabh Agarwal, Ryan Mullins, Samaneh Saadat, Sara Mc Carthy, Sarah Cogan, Sarah Perrin, Sébastien M. R. Arnold, Sebastian Krause, Shengyang Dai, Shruti Garg, Shruti Sheth, Sue Ronstrom, Susan Chan, Timothy Jordan, Ting Yu, Tom Eccles, Tom Hennigan, Tomas Kocisky, Tulsee Doshi, Vihan Jain, Vikas Yadav, Vilobh Meshram, Vishal Dharmadhikari, Warren Barkley, Wei Wei, Wenming Ye, Woohyun Han, Woosuk Kwon, Xiang Xu, Zhe Shen, Zhitao Gong, Zichuan Wei, Victor Cotruta, Phoebe Kirk, Anand Rao, Minh Giang, Ludovic Peran, Tris Warkentin, Eli Collins, Joelle Barral, Zoubin Ghahramani, Raia Hadsell, D. Sculley, Jeanine Banks, Anca Dragan, Slav Petrov, Oriol Vinyals, Jeff Dean, Demis Hassabis, Koray Kavukcuoglu, Clement Farabet, Elena Buchatskaya, Sebastian Borgeaud, Noah Fiedel, Armand Joulin, Kathleen Kenealy, Robert Dadashi, and Alek Andreev. 2024. [Gemma 2: Improving open language models at a practical size](#). *Preprint*, arXiv:2408.00118.

Alexander Matt Turner, Lisa Thiergart, Gavin Leech, David Udell, Juan J. Vazquez, Ulisse Mini, and Monte MacDiarmid. 2024. [Steering language models with activation engineering](#). *Preprint*, arXiv:2308.10248.

Zhengxuan Wu, Aryaman Arora, Atticus Geiger, Zheng Wang, Jing Huang, Dan Jurafsky, Christopher D. Manning, and Christopher Potts. 2025. [Axbench: Steering llms? even simple baselines outperform sparse autoencoders](#). *Preprint*, arXiv:2501.17148.

Zhengxuan Wu, Aryaman Arora, Zheng Wang, Atticus Geiger, Dan Jurafsky, Christopher D Manning, and Christopher Potts. 2024. [ReFT: Representation fine-tuning for language models](#). In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*.

Zhengxuan Wu, Atticus Geiger, Thomas Icard, Christopher Potts, and Noah Goodman. 2023. [Interpretability at scale: Identifying causal mechanisms in alpaca](#). In *Thirty-seventh Conference on Neural Information Processing Systems*.

Zeyu Yun, Yubei Chen, Bruno Olshausen, and Yann LeCun. 2021. [Transformer visualization via dictionary learning: contextualized embedding as a linear superposition of transformer factors](#). In *Proceedings of Deep Learning Inside Out (DeeLIO): The 2nd Workshop on Knowledge Extraction and Integration for*

Deep Learning Architectures, pages 1–10, Online. Association for Computational Linguistics.

A Activation Decomposition with SNMF

In this section we describe in detail the process used to decompose MLP activations with SNMF.

Initialization The non-negative activation matrix $Y \in \mathbb{R}_{\geq 0}^{k \times n}$ is sampled element-wise from a uniform distribution on $[0, 1]$, and the feature matrix $Z \in \mathbb{R}^{d \times k}$ is sampled from a standard normal distribution. We chose this initialization for simplicity and left optimizations for future work.

Feature update With Y fixed we obtain Z :

$$Z \leftarrow AY^T(YY^T + \lambda I)^{-1},$$

where $\lambda > 0$ is a small regularisation constant that prevents ill-conditioning.

Activation update Keeping Z fixed, the non-negative activation matrix Y is updated using the multiplicative rule derived from the SNMF objective:

$$Y \leftarrow Y \odot \sqrt{\frac{[Z^T A]_+ + [Z^T Z]_- Y}{[Z^T A]_- + [Z^T Z]_+ Y}},$$

where $[X]_+$ ($[X]_-$) denotes the element-wise positive (negative) part of X , and $\odot$ is the Hadamard product. This update preserves the non-negativity of Y .

Sparsity and Normalization We impose an ℓ_0 constraint during training by applying a hard winner-take-all (WTA) operator to every column of Z , keeping only the largest $p\%$ of entries (by absolute value) and zeroing the remainder. In our experiments we used $p = 1$. To stabilize training we normalize every column of Y to unit ℓ_2 norm and rescale the corresponding column of Z so that ZY is unchanged. The sparsity method is adapted from [Peharz and Pernkopf \(2012\)](#), who employed a similar approach for enforcing sparsity in Non-negative Matrix Factorization.

B Cosine Similarity versus Projection

If we consider the formula for projection we get,

$$\begin{aligned} m_{\text{proj}}(S) &= \frac{1}{|S|} \sum_{\mathbf{x} \in S} \mathbf{f}^T \mathbf{x} \\ &= \|\mathbf{f}\| \cdot \frac{1}{|S|} \sum_{\mathbf{x} \in S} \|\mathbf{x}\| \cos \theta_x \end{aligned} \quad (7)$$

so for activating S_{act} and neutral S_{neu} ,

$$\frac{m_{\text{proj}}(S_{\text{act}})}{m_{\text{proj}}(S_{\text{neu}})} = \frac{\sum_{x \in S_{\text{act}}} \|\mathbf{x}\| \cos \theta_x}{\sum_{y \in S_{\text{neu}}} \|\mathbf{y}\| \cos \phi_y} \quad (8)$$

depends on the norms $\|\mathbf{x}\|$, $\|\mathbf{y}\|$. By contrast,

$$m_{\cos}(S) = \frac{1}{|S|} \sum_{\mathbf{x} \in S} \frac{\mathbf{f}^\top \mathbf{x}}{\|\mathbf{f}\| \|\mathbf{x}\|} = \frac{1}{|S|} \sum_{\mathbf{x} \in S} \cos \theta_{\mathbf{x}} \quad (9)$$

cancels $\|\mathbf{f}\|$ and $\|\mathbf{x}\|$. Since both unsupervised methods compare against different activating and neutral sets, using cosine similarity makes the activating/neutral ratio directly comparable across methods.

C SAE Training on MLP Activations

For causal evaluation we trained sparse autoencoders (SAEs) on MLP activations from LLaMA-3.1–8B at layers 0, 8, 16, 23, and 31 and Gemma-2–2B at layers 0, 6, 12, 18, and 25. For concept detection we additionally trained on layers 6, 12, 18, and 25 for LLaMA-3.1–8B. For training, we followed best practices as outlined in Gao et al. (2025). We trained both TopK and Gated architectures across a grid of hyperparameters: three learning rates (1e−4, 5e−4, 1e−5) and three sparsity levels per architecture. For TopK we used $k \in \{3, 5, 10\}$; for Gated, we used L_1 penalties from (5, 10, 20). Each SAE had 100 latents and was trained for 500 epochs. We observed that low regularization tended to produce overly dense features, while increased regularization led to large amounts of dead features. These challenges were particularly difficult to balance with the limited settings, where the smaller number of features (100) and smaller dataset made training less stable and led to training that was more sensitive to hyperparameter choice. To address this, we manually identified a reasonable lower and upper bound for the hyperparameters that didn’t create an obvious collapse of training.

We saved 10 checkpoints per run and evaluated them on a held-out validation set constructed identically to the training set, with four sentences per concept. To select the best checkpoint, we combined two metrics: (1) the average number of nonzero activations (to capture sparsity) and (2) downstream loss, measured by the KL divergence between the original model logits and the logits after injecting SAE reconstructions (Gao et al., 2025). We normalized both metrics and combined them to form

the selection objective. For Gemma, giving weight to sparsity led to a large amount of dead features, so only downstream loss was used for checkpoint selection. Overall, we trained 252 SAEs and evaluated 2520 checkpoints to identify the SAEs used.

D Comparison of Interpretability Results Across Four K Values

Here we describe the experimental settings of the concept detection benchmarking (§4) and provide comparisons for varying values of k .

D.1 SNMF Dataset

To construct the SNMF training data we randomly sampled 20 concepts from Neuronpedia (Lin, 2023). For each concepts we prompted GPT-4o-mini (OpenAI et al., 2024) to generate 10 unique sentences exemplifying the concept. Overall, the resulting dataset contains 200 sentences exemplifying 20 labeled concepts and additional non-labeled concepts. For the prompt used to generate the dataset refer to §G.1.

D.2 SNMF Training

We applied SNMF as described in §A with varying values of $k = 100, 200, 300, 400$. For Llama-3.1–8B and Gemma-2–2B, we kept the top 1 percent of indices in each MLP feature during training as described in §A, whereas for GPT-2 we keep the top 5 percent due to its smaller dimensions.

D.3 SAE Concepts

For the SAEs, we randomly sampled 100 features per layer from those identified by Neuronpedia (Lin, 2023), using the SAEs from Llamascope, Gemmascope, and OpenAI’s GPT-2 Small (He et al., 2024; Lieberum et al., 2024; Gao et al., 2025), all trained on MLP-out.

D.4 Evaluation

The evaluation procedure is identical across all methods and layers. For each concept and layer, we follow the same methodology: generate 5 activating sentences that exemplify the concept and 5 neutral sentences that do not. Each sentence is passed through the model, and we extract the activations at the target layer. For each activation, we compute the cosine similarity with the feature and record the maximum similarity across all tokens in the sentence. We then average these maximum scores across the activating sentences to obtain $\bar{a}_{\text{activating}}$, and across the neutral sentences to

obtain $\bar{a}_{\text{neutral}}$. Finally, we compute the Concept Detection score (S_{CD}) as the log-ratio between the two averages:

$$S_{CD} := \log \frac{\bar{a}_{\text{activating}}}{\bar{a}_{\text{neutral}}} \quad (10)$$

D.5 Results

Figures 6, 7 and 8, present results for varying values of k in SNMF, while for SAE the number of concepts is fixed at 100. Across all models, we observe that increasing k generally improves cosine similarity in most layers, suggesting that larger SNMF decompositions yield features that more cleanly activate for specific concepts. However, there are exceptions. For instance, in Gemma-2-2B at layer 12, performance degrades as k increases. This may indicate that setting k higher than the number of meaningful concepts present at a given layer can hurt performance.

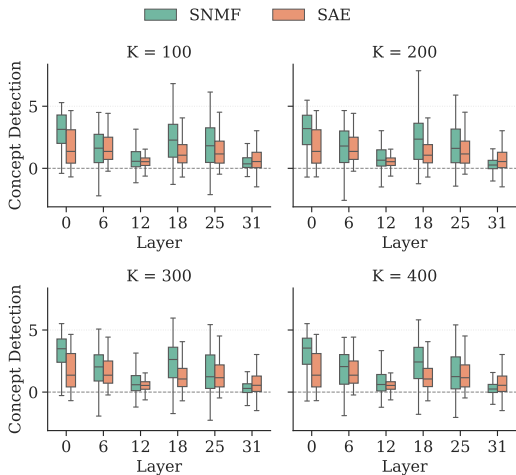


Figure 6: Interpretability scores for Llama-3.1-8B across $K = 100, 200, 300, 400$.

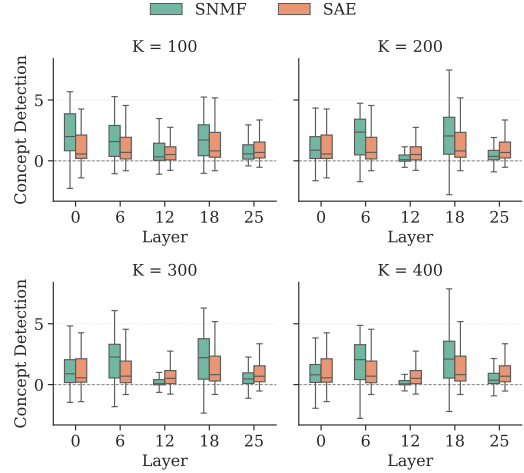


Figure 7: Interpretability scores for Gemma-2-2B across $K = 100, 200, 300, 400$.

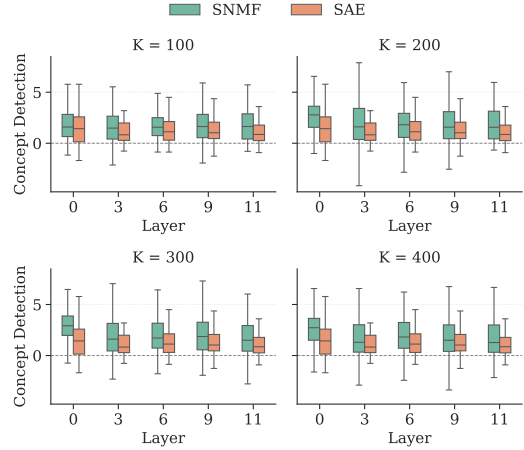


Figure 8: Interpretability scores for GPT-2 Small across $K = 100, 200, 300, 400$.

E Fluency Scores of Causal Benchmark

Figure 9 presents the Fluency score results from §5. Notably, we see a low score for Layer 0 showing the increased difficulty in maintaining model coherence at lower layer interventions.

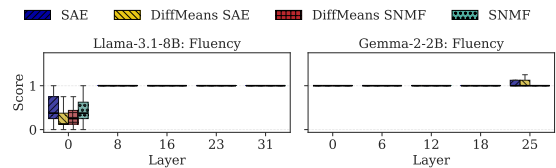


Figure 9: Fluency scores for Llama-3.1-8B and Gemma-2-2B across all methods: SAE, DiffMeans trained on SAE concepts, DiffMeans trained on SNMF concepts and SNMF.

F Experimental Details of Neuron Compositionality

F.1 Recursive SNMF Algorithm

We follow the method outlined in Song and Lee (2013). Initially, we decompose the activations as described in §A to obtain matrices Y_0 and Z_0 . Then we recursively decompose the matrix Z_i to produce matrices Y_{i+1} and Z_{i+1} . To improve the integration of the different MLP feature hierarchies, following Song and Lee (2013), we jointly fine-tune all layers using gradient descent to minimize the reconstruction loss between the original activations A and their multi-layer approximation:

$$\mathcal{L} = \frac{1}{2} \|A - Z_L Y_L \cdots Y_1 Y_0\|_F^2$$

The gradients are propagated backward through the factorization layers, and each Z_i and Y_i is updated using gradient descent. This yields a set of multi-level MLP features, where higher-layer features encode increasingly abstract combinations of co-activated neurons.

F.2 Dataset Generation: Time Units

We prompted GPT-o4-mini-high to generate a dataset focused on the concept of units of time: years, months, weekdays, minutes etc. We chose this concept because prior work (Engels et al., 2025) has shown that GPT-2 represents weekdays and months in a multi-dimensional manner, where each day of the week is positioned near the ones that follow it, forming a circular structure in a lower-dimensional subspace. We use the following prompt:

Dataset Generation Prompt

Generate a dataset of sentences that are all related to days of the week, months, years, dates etc. Make sure to include plenty of both broad and specific terms including all names of the days and months and for each term over 10 instances. Generate 200 sentences. Format the dataset with two columns: Prompt, Label. Format as a json in the format <label>: [<sentences>].

F.3 Dataset Generation: General

We constructed a more general dataset by prompting GPT-4o for 50 random nouns as base concepts. For each noun, we queried ConceptNet to retrieve related concepts that represent subcomponents or subcategories using relations like HasA, PartOf, or TypeOf. For each of the related concepts, we

prompted Gemini-2.0-flash (Google, 2025) to generate five distinct sentences exemplifying the relationship in natural language. This process resulted in a total of 873 sentences that reflect a diverse range of semantically related sentence groups.

F.4 Tree Construction

To identify hierarchical MLP features, we apply recursive SNMF with $K = [400, 200, 100, 50]$. Each matrix Y_i maps features from level $i+1$ to those in Z_i , allowing us to trace how higher-level features activate lower-level ones. To construct the tree, we connect each feature at level $i+1$ to its top-activated features in Z_i according to Y_i , using a fixed threshold to prune weak edges. This forms a directed tree structure that reflects the compositional organization of features across levels. We label the nodes corresponding to the MLP features using the top activating contexts associated with the MLP features. We obtain the mapping for an MLP feature at layer i in the hierarchy to the contexts by multiplying the coefficient matrices 0 to i : $Y^{(0 \rightarrow i)} = Y_0 Y_1 \cdots Y_i$

F.5 Additional Hierarchical Decompositions

In Figures 10, 11, 12, 13 and 14 we provide additional examples of hierarchical MLP features found by applying the described methodology on the general dataset on MLP activations extracted from Layer 0. We manually select the examples to provide a variety of compositions. We note that these are a select few from a larger pool of identified hierarchical MLP features.

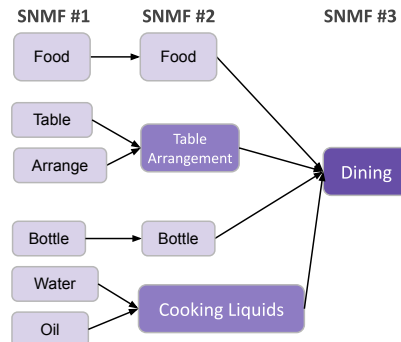


Figure 10: Merging of Dining related MLP features.

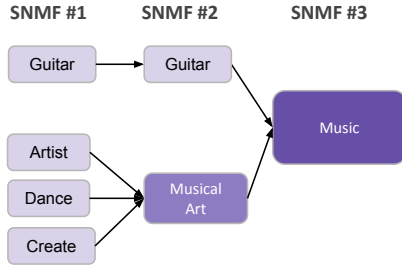


Figure 11: Merging of Music related MLP features.

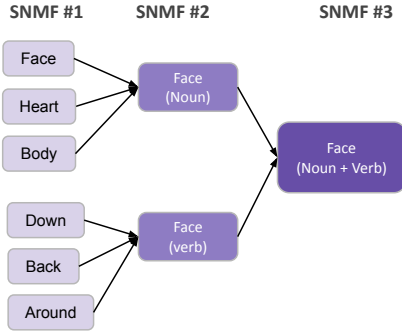


Figure 12: Hierarchy showing that semantically different, but syntactically related features exhibit merging as well.

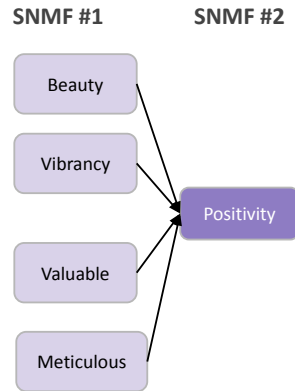


Figure 13: Hierarchy showing that semantically related features share neurons.

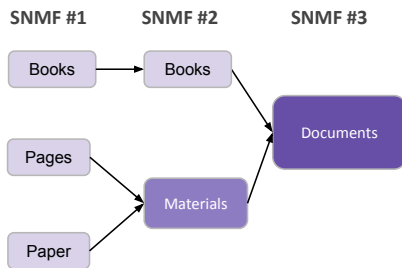


Figure 14: Hierarchy showing that compositional concepts exhibit neuron sharing.

G Concept Detection Prompts

In this section we present the prompts used for the concept detection experiments.

G.1 Generating Dataset From Concept Descriptions

The following prompt is used sequentially to generate the dataset employed in identifying MLP features with SNMF in Section 4 and Section 5. To generate the dataset, we provide the concept and prompt the model m times, where m is the desired number of sentences per concept.

Dataset Generation Prompt

Generate a sentence that exemplifies and strongly exhibits the concept of {concept}.
Make sure the sentence is different from the following list: {current_sentences}.
Output the sentence **ONLY** without additional text.

G.2 Generating Activating and Neutral Sentences

The following prompts are used to generate sentences that exemplify a concept (activating sentences) and those that do not (neutral sentences). To construct the list, we sequentially prompt the model to generate a new sentence while conditioning on the current list to encourage diversity.

Activating Prompt

You are given a description of an LLM concept.
Given the description, generate a sentence that contains tokens that would activate the feature. Make sure your generated sentence exemplifies all the key terms and structures specified in the description.
Ensure that the sentence is a full, grammatically correct English sentence.

Category description:

{description}

Make sure you generate a sentence that is distinct from: {previous_sentences}

Output only the sentence without any additional text.

Neutral Prompt

You are given a description of an LLM concept.

Your objective is to generate a neutral sentence that should not activate the feature.

This means that you must not include in the sentence any tokens that relate to the feature.

Have the generated sentence be on a completely unrelated topic.

Concept description:

{description}

Make sure you generate a sentence that is distinct from: {previous_sentences}

Output only the sentence without any additional text.

Make sure to exclude any tokens that may activate the concept.

G.3 Input Description Prompt

The following prompt is used to generate a natural language description for each concept, based on the top activated tokens in the matrix Y , which represents token-level activations for each MLP feature. Specifically, for a given MLP feature, we extract the tokens with the top m highest activations and use them as input to the prompt. The model is asked to interpret the shared semantic content of these tokens. We then parse the *Results* section of the model's response to retrieve the final concept description.

Input Description Prompt

You are given a set of tokens, their surrounding context (words before and after the token), and an importance score.

Your task is to determine what is the connection between all the tokens.

Instructions:

1. **Focus on High-Importance Samples:**

Examine only the token-context pairs with the highest importance scores. If a significant drop is observed beyond a threshold, ignore the lower-scoring pairs.

2. **Assess Token Consistency vs. Contextual Patterns:**

- **Token Consistency:** If the high-importance samples are mostly identical tokens or strongly related tokens, then consider the tokens only as the primary contributor.
- **Contextual Patterns:** If the tokens are not related to one another, then focus on common semantic, syntactic, or structural patterns in the surrounding contexts.

3. Always choose the simplest and most obvious underlying connection. If inspecting the tokens alone is enough to find a connection, do not mention or utilize the contexts.

Output Format:

- **Analysis:** <reason what the underlying connection is>
- **Results:** <Single sentence description of the single most obvious connection between the tokens>

Input:

Token-Context Pairs: {token_context_str}

Remember, find the most obvious connection prioritizing connecting the tokens alone without the contexts and only if you cannot find any connection between the tokens then you may inspect their contexts.