

Connecting randomized benchmarking results to physical error rates

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What Randomized Benchmarking Actually Measures

In [1], the authors discuss what randomized benchmarking (RB) experiments measure. In particular, a common analysis considers contexts where the RB decay rate quantifies a depolarizing error rate. The authors emphasize that in general the decay rate cannot be used to produce an estimator for the fidelity — the fidelity is gauge dependent, while the RB decay rate is an observable quantity.

Markovian Error Generators

In [2], the authors characterize all $d^2(d^2 - 1)$ Markovian error generators that can be exponentiated to produce all Markovian error channels. The characterization is given in Fig. 1. These errors represent a low-level error model that can occur in systems without memory, and represent a first step towards the analysis of error correction circuits.

Sector	Dimension	Action	Example effect (Bloch sphere)	ϵ_I (Error Probability)	θ_I (Error Amplitude)
Hamiltonian	$d^2 - 1$	$H_P[\rho] = -i[P, \rho]$		0	1
Stochastic (Pauli)	$d^2 - 1$	$S_P[\rho] = P\rho P - \mathbb{1}\rho\mathbb{1}$		1	0
Stochastic (Pauli-correlation)	$\binom{d^2 - 1}{2}$	$C_{P,Q}[\rho] = P\rho Q + Q\rho P - \frac{1}{2}\{P, Q\}\rho$		0	0
					1
Active	$\binom{d^2 - 1}{2}$	$A_{P,Q}[\rho] = i\left(P\rho Q - Q\rho P + \frac{1}{2}\{P, Q\}\rho\right)$		0	1
					0

Figure 1 Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Error Model

We consider a simple gate set consisting of both X and $Y - \pi/2$ rotation. These two gates generate the single qubit Clifford group, and so can be used for both Direct randomized benchmarking, and as a compilation target for Clifford randomized benchmarking. We introduce a *relational error*, so that the angle between the axes of rotation is $\pi/2 + \theta$. This can be represented as

$$X_{\pi/2} \rightarrow X_{\pi/2}$$

$$Y_{\pi/2} \rightarrow Z_{\theta} Y_{\pi/2} Z_{-\theta}.$$

Direct Randomized Benchmarking with Physical Errors

In [3] the authors introduce direct randomized benchmarking (DRB). DRB takes as input a sampling distribution over the primitive gates, and uses it to produce a decay rate. By tuning the sampling distribution and the relational error rate, we can tune the DRB error rate. We would like to find an analytic expression for this error rate. Fig. 2 shows the theoretical DRB error rate for different values of the sampling ratio, along with sampled points from numerical experiments.

Direct Randomized Benchmarking with Physical Errors (Continued)

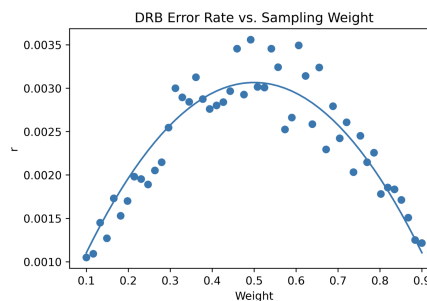
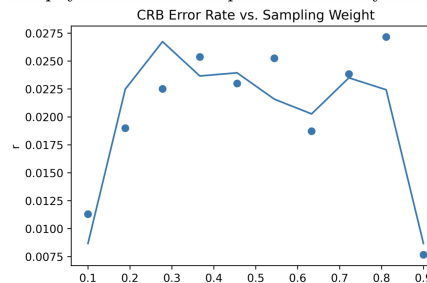


Figure 2

Clifford Randomized Benchmarking with Physical Errors

Clifford randomized benchmarking (CRB) is described in [4]. Random elements are drawn from the Clifford group to form variable length sequences, and followed by the inverse element to the composed sequence. We can view CRB as DRB with a uniform sampling distribution over the Clifford group, together with a compiler. Figure 3 shows the theoretical CRB error rate for different values of the sampling ratio, along with sampled points from numerical experiments. In order to have particular weights, the sequences may be longer, hence the physical error rate per Clifford may increase.



Relationship to QISENET Proposal

The characterization of quantum errors in error correcting circuits is crucial to the development of error corrected quantum computers. Modeling these errors as physical processes is often difficult, due to the variety of quantum computing architectures in addition to mathematical difficulties such as choices of gauges, and the exponential size of the quantum computer's Hilbert space. Here we provide a concrete first step towards understanding the impact of physical error rates, assumed to be Markovian, on a total error rate. Moreover, by specifying on Clifford circuits as in randomized benchmarking, we are naturally posed to consider stabilizer error correction circuits which also consist of Clifford circuits.

Acknowledgements

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- [2] Robin Blume-Kohout, Marcus P da Silva, Erik Nielsen, Timothy Proctor, Kenneth Rudinger, Mohan Sarovar, and Kevin Young. A taxonomy of small markovian errors. *arXiv preprint arXiv:2103.01928*, 2021.
- [3] Timothy J Proctor, Arnaud Carignan-Dugas, Kenneth Rudinger, Erik Nielsen, Robin Blume-Kohout, and Kevin Young. Direct randomized benchmarking for multiqubit devices. *Physical review letters*, 123(3):030503, 2019.
- [4] Easwar Magesan, Jay M Gambetta, and Joseph Emerson. Scalable and robust randomized benchmarking of quantum processes. *Physical review letters*, 106(18):180504, 2011.

