

Local Scale Equivariance with Latent Deep Equilibrium Canonicalizer

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Abstract

Scale variation is a fundamental challenge in computer vision. Objects of the same class can have different sizes, and their perceived size is further affected by the distance from the camera. These variations are local to the objects, i.e., different object sizes may change differently within the same image. To effectively handle scale variations, we present a deep equilibrium canonicalizer (DEC) to improve the local scale equivariance of a model. DEC can be easily incorporated into existing network architectures and can be adapted to a pre-trained model. Notably, we show that on the competitive ImageNet benchmark, DEC improves both model performance and local scale consistency across four popular pre-trained deep-nets, e.g., ViT, DeiT, Swin, and BEiT.

1. Introduction

The perceived scale of an object in images varies due to multiple factors. First, intrinsic factors such as the object’s physical dimensions cause variations in the apparent size, e.g., individuals may differ in height. Second, extrinsic factors such as camera zoom and the object’s distance to the camera further affect the observed scale, even for the same object. How to effectively handle and represent this scale variation has been a fundamental question in computer vision. Inspired by human vision, which uses a hierarchy of scales for perception [42], earlier works adopted image pyramids [1, 12] and variants [34, 55, 65] to extract features at different image resolutions [21, 31, 39, 40].

In deep learning, ideas similar to image pyramids have been introduced [14, 20, 24, 76] and studied under the broad area of scale invariance/equivariance [48, 56, 57, 67]. These works aim to design specialized deep-nets such that the network’s output is transformed in a predefined way when the input image undergoes scale changes (image resizing). Mathematically, equivariance is formalized as equality statements over groups; see Sec. 3 for a review. A typical approach is to design a group equivariant architecture [15] by modifying the filter operations. Note, we use the term “equivariance” in the strict mathematical sense of the *exact equality statement*. We

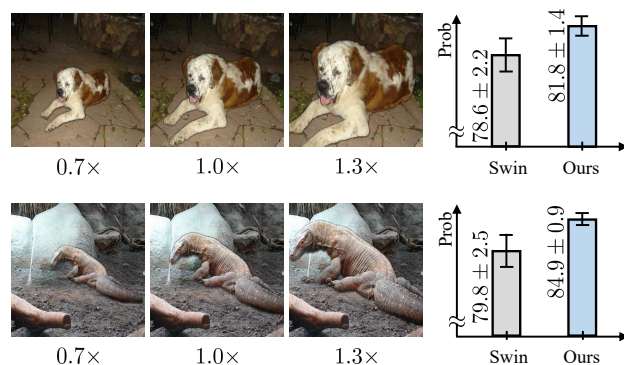


Figure 1. Local scaling resizes different parts of an image independently, e.g., the dog is scaled while the background remains unchanged. For the Swin model [38], we show the distribution of predicted probabilities for the object class across a set of locally scaled images. Observe that the predicted probabilities are not stable, indicating that the Swin model is not consistent across local scaling. We introduce Deep Equilibrium Canonicalizer (Ours) to address this issue.

will use the term “consistency” to mean an approximation of equivariance, i.e., without the mathematical guarantees.

While scale equivariance has been well studied, these works all focused on *global scaling*, i.e., changes in image resolution. Differently, we are interested in *local scaling*, e.g., the resizing of each part of an input image differently (at a fixed resolution), which more closely matches the scaling of objects in the real world. See illustration in Fig. 1. Ideally, we aim to design deep-nets that are local scale equivariant. However, real-world local scaling does not form a group as the operation is not invertible, e.g., when two objects change size and occlude each other. To address this, we propose an approximation of the real-world local scaling operation, namely monotone scaling, which we will show to be a group.

Our approach to making a deep-net monotone scale equivariant is based on the framework of canonicalization [44, 46]. Briefly, a canonicalizer transforms an input into a “standard” (canonical) form so that the deep network can extract invariant features. A canonicalizer can be modeled as finding a stationary point of a learned energy function with respect to the transformation (group) and then using that point as the “standard”. Instead of this optimization approach, we propose

to utilize a deep equilibrium model [4] for directly predicting this point, *i.e.*, amortized optimization [2]. We name our approach the Deep Equilibrium Canonicalizer (DEC). We then incorporate DEC into the latent space of existing deep-net architectures to improve the local scale consistency.

To evaluate local scale consistency, we first conduct experiments on semantic segmentation using synthetic data where we can control the scale variation. We create a dataset of realistic renderings using physical simulation based on the Google Scan Objects [19]. Next, we consider the task of image classification. Inspired by the MNIST-scale dataset used in prior works [29, 48, 57], we create a dataset of locally scaled MNIST images where each image contains multiple MNIST digits, resized to a different scale. Finally, we evaluate a locally scaled version of ImageNet [17]. Overall, our proposed DEC, when incorporated into existing models, achieves a higher performance metric and further improves the local scale consistency over the existing base model.

Our main contributions are as follows:

- Motivated by local scaling in the real world, we propose to study how to design deep-nets that are equivariant to monotone scaling.
- We propose DEC, a novel latent canonicalization approach based on deep equilibrium models, which can be easily incorporated into existing deep-net architectures.
- We demonstrate that the proposed DEC improves model performance and local scale consistency on two vision tasks, notably, the competitive benchmark of ImageNet across four different ViT architectures.

2. Related Works

Scale consistency has a long history in computer vision. Early works focus on developing methods that are invariant to scaling [40, 66] with techniques such as Laplacian image pyramids [11, 55] or wavelet analysis [62]. Later works aim to improve scale consistency by learning transformations applied either to the input images or directly to convolutional filters [16, 27, 52, 59] or by directly learning local/global scale estimation [8, 33]. More recently, variants of vision transformer models have been proposed to tackle scale consistency, *e.g.*, incorporating the structure of image pyramids [22, 60]. While some of these works use the term invariance/equivariance, these approaches generally do not achieve the equality condition on a group, *i.e.*, they are not guaranteed to be scale equivariant in the strict sense. In the equivariant literature, there is a line of work studying global scaling [48, 56, 57, 67, 69] through designing specialized architecture based on group theory.

Group equivariant models. Designing group equivariant architectures has been explored due to their strong theoretical guarantees and robustness [13, 15, 41, 47, 72]. Their development has been extended to incorporate equivariance in rotation [63], shift [54, 75], permutation [23, 35, 36, 51,

70, 71, 74], and sampling [49, 53, 68]. Global scale equivariant architectures are primarily constructed by integrating over the scale space of images [67], through the use of steerable filter [56] or applying spectral-domain operations for anti-aliasing [48]. However, the current literature lacks equivariant methods for local scaling (as it does not form a group) and only handles the global scaling operations.

Equivariant via canonicalization. With the rise of pre-trained models, there is growing interest in adapting existing models to be equivariant rather than developing entirely new equivariant architectures. This has led to the area of group equivariant adaptation, which primarily aims to “standardize” the input through the use of a canonicalization module [44, 46] or by averaging over the group elements to achieve more stable output [9, 10]. However, these methods target discrete symmetry groups with a few parameters, *e.g.*, the 2D rotation group ($SO(2)$) characterized by a single parameter. These approaches are limited when the size and dimensionality of the transformation group increases.

Deep Equilibrium Models (DEQs) are a class of implicit neural networks that compute their outputs by solving for the fixed point of a parameterized transformation [4, 5]. Unlike traditional deep networks with a predefined number of layers, DEQs effectively model an infinitely deep, weight-tied recurrent network, making them highly parameter-efficient. These models have been applied successfully in various applications, including optical flow estimation [6] and video landmark detection [43]. In this work, we found DEQs to be a suitable architecture for modeling a canonicalizer for adapting deep-nets to be equivariant.

3. Preliminaries

We provide a review of the definition of equivariance and invariance. For readability, we describe these concepts using one-dimensional functions. Next, we review the canonicalization framework for achieving equivariant models.

Global scaling operation. The global scaling operation $\mathbf{R}_a$ on a 1D function $f \in \mathcal{U}$, where $\mathcal{U} \triangleq \{f : [0, 1] \rightarrow \mathbb{R}\}$, is defined as

$$\mathbf{R}_a[f](x) = f(a^{-1}x), \quad (1)$$

where $a \in \mathbb{R}^+$ is the scaling factor. Specifically, $\mathbf{R}_a$ with $a > 1$ is an upscaling operation, and $a < 1$ is a downscaling operation.

Group and equivariance. A group is a mathematical structure consisting of a set G , which captures the structure of symmetry transformations. The action of a group element $g \in G$ on any function $f \in \mathcal{U}$ is represented by a transformation $\mathbf{T}(\cdot; g)$ as

$$\mathbf{T}(f; g) = f(g^{-1} \cdot x), \quad (2)$$

where $g^{-1} \cdot x$ denotes the transformation of x by the inverse element g^{-1} . In other words, the function is evaluated at the transformed input $g^{-1} \cdot x$. Here, g acts as the parameter of the transformation $\mathbf{T}$. By definition, the transformation needs to satisfy the invertibility condition, $\mathbf{T}^{-1}(f; g) = \mathbf{T}(f; g^{-1})$, for any group element g .

Next, a neural network $\mathcal{M}$ is said to be *equivariant* with respect to the group G if $\mathcal{M}$ commutes with the action of G , *i.e.*, satisfying the following equality condition

$$\mathcal{M}(\mathbf{T}(f; g)) = \mathbf{T}(\mathcal{M}(f); g). \quad (3)$$

That is, transforming the input similarly transforms the output. The neural network $\mathcal{M}$ is *invariant* if

$$\mathcal{M}(\mathbf{T}(f; g)) = \mathcal{M}(f), \quad (4)$$

which means transforming the input by a group element $g \in G$ does not change the output.

Learned canonicalization function. A canonicalization function can modify $\mathcal{M}$, ensuring the resulting $\tilde{\mathcal{M}}$ is equivariant. The canonicalization function $h : \mathcal{U} \rightarrow G$ is designed to be equivariant with respect to the group G , *i.e.*,

$$h(\mathbf{T}(f; g)) = g \cdot h(f), \forall g \in G, \quad (5)$$

where $g \cdot h(f)$ denotes a group product. Using this canonicalization function, the adapted model $\tilde{\mathcal{M}}$ is defined as:

$$\tilde{\mathcal{M}}(f) \triangleq \mathbf{T}(\mathcal{M}(\mathbf{T}^{-1}(f; \tilde{g})); \tilde{g}), \quad (6)$$

where $\tilde{g} \triangleq h(f)$. In particular, the canonicalization module maps all possible transformed versions of a function f to a canonical input represented as $\mathbf{T}^{-1}(f; \tilde{g})$.

The canonicalization function h can be modeled directly by group equivariant networks [28, 44]. In the absence of an equivariant model, an alternative approach is to formulate the canonicalizer as an optimization problem [41, 46]

$$h(f) \triangleq \arg \min_{g \in G} E(\mathbf{T}^{-1}(f; g)). \quad (7)$$

Here, $E : \mathcal{U} \rightarrow \mathbb{R}$ is a learnable *energy function*, *e.g.*, modeled with a neural network.

4. Latent Deep Equilibrium Canonicalizer

In Sec. 4.1, we formulate the monotone scaling group to approximate the local scaling transformation. Next, we introduce our DEC module in Sec. 4.2 for effective canonicalization on a given module. Lastly, we propose latent canonicalization for adapting pretrained models in Sec. 4.3.

4.1. Local Scaling

In contrast to the global scaling (Eq. (1)), scalings in a real-world image are often local. As shown in Fig. 1, different

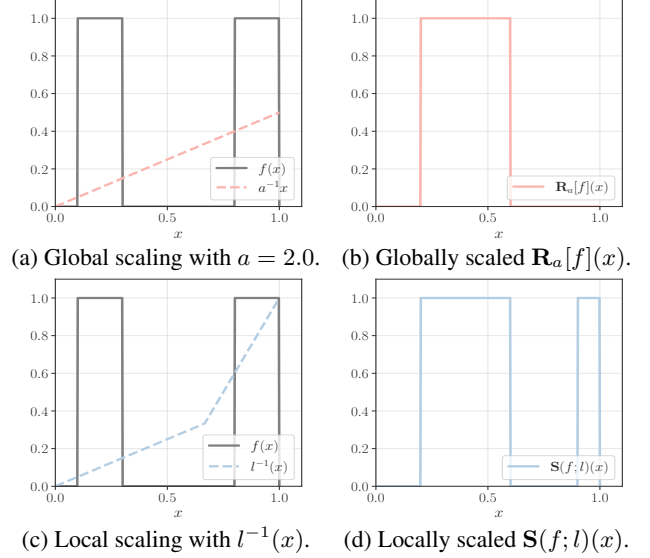


Figure 2. **Comparison of global and local scaling.** (a-b) In global scaling, all locations of $f(x)$ are scaled by a constant factor. (c-d) In local scaling, each segment of $f(x)$ is scaled differently.

regions in an image have different scaling factors. To approximate such transformations, we propose monotone scaling operations. For readability, the idea is first discussed in 1D and then extended to 2D for images.

Monotone scaling operation. To apply different scaling factors at different parts of the domain, we propose to perform the scaling by a function $l : [0, 1] \rightarrow [0, 1]$. A monotone scaling is defined as follows

$$\mathbf{S}(f; l)(x) \triangleq f(l^{-1}(x)), \quad (8)$$

where l^{-1} is the inverse function of l and the local scaling factor at t is $\frac{dl}{dt}$. To preserve the smoothness and invertibility of this transformation, we restrict the functions l to be strictly monotonic, increasing, and continuous. In Fig. 2, we provide a comparison between the global and our local scaling operation.

Monotone scaling group. Denote the set of all continuous and strictly monotonic increasing functions l as L . The set L forms a group structure, where the group product “.” is represented as the function composition as

$$(l_1 \cdot l_2)(x) = (l_1 \circ l_2)(x) = l_1(l_2(x)). \quad (9)$$

We formally state this in Lemma 1.

Lemma 1. *The set of all continuous strictly monotonic increasing functions L is a group under the binary operation of function composition.*

Proof. We provide the proof of this lemma in Sec. A3.1. $\square$

Lemma 1 states that the proposed monotone scaling operation forms a group. Hence, we can formally define equiv-

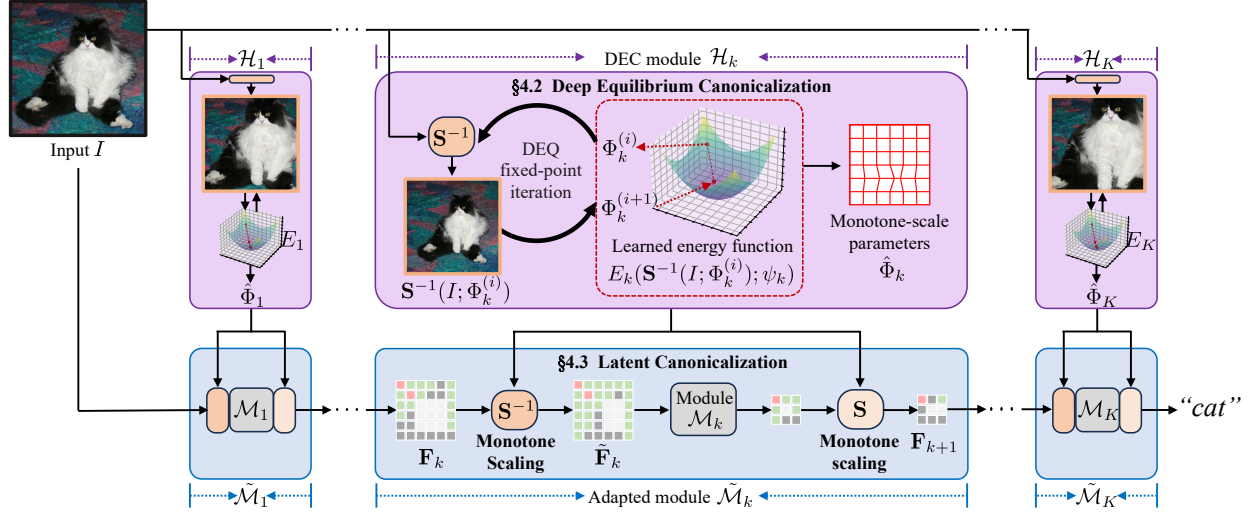


Figure 3. **Illustration of the proposed method.** We formulate a neural network as a sequence of modules $\{\tilde{\mathcal{M}}_k\}_{k=1:K}$. We perform latent canonicalization (see Sec. 4.3) on the latent feature $\mathbf{F}_k$ of each intermediate module. The monotone scaling in the latent canonicalization, *i.e.*, $\mathbf{S}^{-1}$ and $\mathbf{S}$, are controlled by the parameters Φ_k found by our DEC module $\mathcal{H}_k$ (see 4.2). Each $\mathcal{H}_k$ finds the optimal $\hat{\Phi}_k$ through an iterative fixed-point solver on the energy function E_k until convergence.

ariance of w.r.t. the monotone scaling group as

$$\text{Equivariance} : \mathcal{M}(\mathbf{S}(I; l)) = \mathbf{S}(\mathcal{M}(I); l) \quad (10)$$

and invariance as

$$\text{Invariance} : \mathcal{M}(\mathbf{S}(I; l)) = \mathcal{M}(I). \quad (11)$$

Parameterization of l . The family of possible functions in l is very flexible. For simplicity, we consider a parametric l in the form of a piecewise linear function where the domain $[0, 1]$ is discretized uniformly into N discrete points as $\{x_0 = 0, x_1, \dots, x_N = 1\}$. In this case, l has the form

$$l_\Phi(x) = \phi_{n-1} + \frac{\phi_n - \phi_{n-1}}{x_n - x_{n-1}} \times (x - x_{n-1}), \quad (12)$$

when $\Phi = \{\phi_0, \dots, \phi_N\}$ are the parameters. We impose the restriction $\phi_{n-1} < \phi_n$ to make the l monotonic increase and further we impose $\phi_0 = 0$ and $\phi_N = 1$ to make the function bijective. For the ease of notation, we denote the monotone scaling by l_Φ as $\mathbf{S}(f; \Phi)$, by dropping the l , *i.e.*, $\mathbf{S}(f; \Phi) \triangleq \mathbf{S}(f; l_\Phi)$.

A similar piecewise linear l can be constructed for monotone scaling operation to images $I : [0, 1]^2 \rightarrow \mathbb{R}$ by extending the definition of l to the 2D domain, *i.e.*,

$$\mathbf{S}(I; l)(x, y) = I(l^{-1}(x, y)), \quad (13)$$

where $l : [0, 1]^2 \rightarrow [0, 1]^2$. We illustrate this monotone scaling operation in Fig. 4. More details are provided in Appendix A2.2. With the parameterization and monotone scaling group defined, we now discuss how to design an effective canonicalizer for this group.

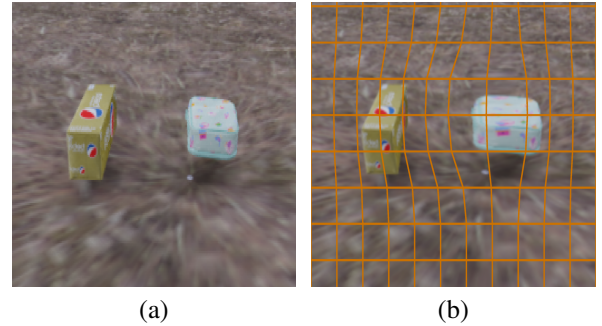


Figure 4. Scaling by monotone function $l : [0, 1]^2 \rightarrow [0, 1]^2$. (a) The original image. (b) Scaled image. The warped grid visualizes the effect of the monotone scaling operation.

4.2. Deep Equilibrium Canonicalization (DEC)

As reviewed in Sec. 3, one approach to formulate a canonicalizer is by solving for a stationary point, *e.g.*, a minimum of an energy function with respect to the group elements in Eq. (7). However, solving Eq. (7) per example and training it end-to-end is slow and memory intensive. Instead, we propose to leverage the idea of amortized optimization [2], *i.e.*, we learn a model to directly predict the solution. In our case, we choose this prediction model to be a Deep equilibrium model to be used as the canonicalizer.

Deep equilibrium model (DEQ) is a type of implicit neural network where the output is expressed as the fixed/stationary point of a learnable non-linear transformation $\mathcal{H}$, *i.e.*,

$$\hat{\mathbf{z}} = \mathcal{H}(\hat{\mathbf{z}}; I, \psi), \quad (14)$$

where $\mathcal{H}$ is the non-linear transformation with parameter ψ and $\hat{\mathbf{z}}$ is the equilibrium hidden state. The equilibrium can

be calculated implicitly by solving the equilibrium condition

$$\mathbf{z} - \mathcal{H}(\mathbf{z}; I, \psi) = \mathbf{0} \quad (15)$$

DEQ can be trained from end to end like any other differentiable module, where the backpropagation can be efficiently computed using implicit differentiation.

DEC module. We propose to formulate the canonicalization module using a DEQ. That is, we model the optimization in Eq. (7) as a fixed point of a non-linear system $\mathcal{H}$. Specifically, we treat Φ as the hidden state $\mathbf{z}$ described in Eq. (14) and define the update as

$$\mathcal{H}(\Phi^{(i+1)}; I, \psi) \triangleq \Phi^{(i)} - \nabla_{\Phi^{(i)}} E(\mathbf{S}^{-1}(I; \Phi^{(i)}); \psi). \quad (16)$$

As shown in Fig. 3, our DEC finds the optimal monotone-scale parameters $\hat{\Phi}$ via DEQ fixed-point iteration to canonicalize the input I .

Claim 1. *The optimal monotone-scale parameter, $\hat{\Phi}$, of energy function E obtained via gradient descent is a fixed point of $\mathcal{H}$.*

Proof. We assume that both DEQ iteration and the gradient descent optimization (Eq. (7)) start from the same initialization $\Phi^{(0)}$. Let $\Phi^\dagger$ be a fixed point of $\mathcal{H}$ for the input I . Therefore it satisfies

$$\Phi^\dagger = \mathcal{H}(\Phi^\dagger; I, \psi) \quad (17)$$

$$\Rightarrow \Phi^\dagger = \hat{\Phi}^\dagger - \nabla_{\hat{\Phi}^\dagger} E(\mathbf{S}^{-1}(I; \Phi^\dagger); \psi) \quad (18)$$

$$\Rightarrow \nabla_{\Phi^\dagger} E(\mathbf{S}^{-1}(I; \Phi^\dagger); \psi) = \mathbf{0} \quad (19)$$

This implies that the gradient of the energy function E , $\nabla_{\Phi} E$, evaluated at point $\Phi^\dagger$ is $\mathbf{0}$. Therefore, $\Phi^\dagger$ corresponds to some stationary point. Consequently, $\Phi^\dagger$ corresponds to a local minimum and thus represents the optimal parameter returned by the gradient descent optimization in Eq. (7). $\square$

The main insight of our canonicalizer design is that, instead of modeling $\mathcal{H}$ in terms of the gradient of E , we directly model $\mathcal{H}$ using a neural network with learnable parameters ψ and use it as the canonicalization module. Given an image I and monotone-scale parameter $\Phi^{(i)}$ at step i , $\mathcal{H}$ predict the next-step parameter $\Phi^{(i+1)}$ based on inversely scaled image $\mathbf{S}^{-1}(I, \Phi^{(i)})$. This process is repeated iteratively until convergence to the equilibrium point. The equilibrium point $\hat{\Phi}$ can be determined by using Anderson acceleration [3].

With our modeled canonicalizer defined, the remaining question is how to incorporate it into an existing deep-net.

4.3. Latent Canonicalization

Typically, canonicalization [41, 44, 46] methods apply the canonicalizer at the input of the deep-net. This approach is effective when the transformation of interest is fully represented by the group actions, such as 2D rotation. However, recall that monotone scaling is only an approximation of the

real-world local scaling. Therefore, we propose to canonicalize the latent features of an existing network. We demonstrate the process of latent canonicalization in Fig. 3. This is motivated by the hypothesis that the inductive bias of monotone scale equivariant features could better approximate an overall local scale equivariant deep-net.

Canonicalizer on the latent features. A deep-net $\mathcal{M}$ can be viewed as a composition of multiple layers, *i.e.*,

$$\mathcal{M}(I) = \mathcal{M}_K \circ \mathcal{M}_{K-1} \circ \dots \circ \mathcal{M}_1(I). \quad (20)$$

Using this notation, we denote the latent feature maps $\mathbf{F}_{k+1} \triangleq \mathcal{M}_k(\mathbf{F}_k)$ where the input feature is the image, *i.e.*, $\mathbf{F}_1 = I$. For each layer $\mathcal{M}_k$, we use canonicalization to adapt it to $\tilde{\mathcal{M}}_k$ following Eq. (6). Our overall adapted model with equivariance guarantee is a composition of all adapted modules, *i.e.* $\tilde{\mathcal{M}} = \tilde{\mathcal{M}}_K \circ \dots \circ \tilde{\mathcal{M}}_0$.

For monotone scale equivariance, we define each adapted module $\tilde{\mathcal{M}}_k$ as follows:

$$\tilde{\mathcal{M}}_k^{\text{Eq}}(\mathbf{F}_k) = \mathbf{S}(\mathcal{M}_k(\tilde{\mathbf{F}}_k); \Phi_k), \quad (21)$$

where $\tilde{\mathbf{F}}_k = \mathbf{S}^{-1}(\mathbf{F}_k; \Phi_k)$. That is, we first apply monotone scaling $\mathbf{S}^{-1}$ to each input feature of each layer, *i.e.*, $\mathbf{F}_k$, and subsequently apply its inverse $\mathbf{S}$ to the output of $\mathcal{M}_k$. This ensures that the latent feature is equivariant to monotone scaling. Similarly, for monotone scale invariance, we define the adapted modules as

$$\tilde{\mathcal{M}}_k^{\text{Inv}}(\mathbf{F}_k) = \mathcal{M}_k(\tilde{\mathbf{F}}_k). \quad (22)$$

5. Experiments

For evaluation, we aim to benchmark the local scale consistency of existing architectures and verify that the proposed DEC can improve consistency while maintaining model performance. To accurately measure the local scale consistency, we create datasets of locally scaled images where we know the underlying local scale transformation applied to each image. Specifically, we considered semantic segmentation on a dataset based on the Google Scan Objects [19] and image classification using the MNIST [32] dataset and the ImageNet [17] dataset. The implementation details are in the Appendix A1. More results are provided in the Appendix A5.

5.1. Semantic Segmentation

Dataset setup. For evaluating equivariance, we construct a locally scaled segmentation dataset by simulating real-world objects' scale variations. To have precise control over the variations, we render objects from Google Scanned Objects [19] on a randomly selected background in HDRI Haven [73]. For each image, we place two 3D objects and extract the ground-truth mask.

To introduce the local scaling effects, we vary the relative distance of each object from the camera and enable

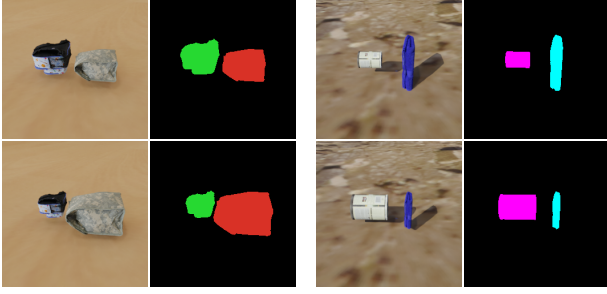


Figure 5. **Samples from locally scaled object segmentation dataset.** Each object in an image is scaled independently.

Method	ViT [18]		Swin [38]		DINOv2 [45]	
	mIoU $\uparrow$	EquE $\downarrow$	mIoU $\uparrow$	EquE $\downarrow$	mIoU $\uparrow$	EquE $\downarrow$
<i>Aug</i>	82.18	0.054	86.65	0.036	76.63	0.061
<i>Canon</i>	79.37	0.062	83.94	0.050	76.94	0.064
<i>AugL</i>	82.35	0.054	87.63	0.037	79.05	0.058
Ours	83.12	0.052	88.56	0.035	80.86	0.055

Table 1. **Results on local scale equivariance.** For each method, we report the mIoU(%) and EquE across 3 architectures.

independent scale changes for each object while keeping the background fixed. The dataset includes 100 distinct objects; see Fig. 5 for examples. We use 6,000 images for training and 2,000 for testing with an image resolution of 256×256 .

Evaluation metric. To measure performance, we report the mean intersection over union (mIoU). To measure consistency, we propose monotone scale equivariance error (EquE) on the predicted masked, defined as

$$\frac{1}{|\mathcal{D}|} \sum_{I \in \mathcal{D}} \mathbb{E}_{\Phi \sim \Omega} \|\mathcal{S}(\mathcal{M}(I); \Phi) - \mathcal{M}(\mathcal{S}(I; \Phi))\|_2^2 \quad (23)$$

over a dataset $\mathcal{D}$ and Ω denotes a uniform distribution over all possible monotone scale parameters.

Intuitively, EquE can be thought of as a *soft* definition, in the ℓ_2 sense, of the monotone scale equivariance in Eq. (10). That is, an equivariant model will achieve an EquE of zero.

Baselines. We consider the following alternative methods to encourage local scale consistency into a model:

- *Aug*: We train the architecture with random monotone scaling data augmentation on the training set. *Aug* also serves as the starting weights for DEC and the other baselines.
- *Canon*: We discretize the local scaling parameter and consider a set of 64 different parameters, including the identity transformation. We use a 3-layer CNN to model the energy function in Eq. (7).
- *AugL*: As one of the metrics is the equivariance error, we further fine-tune *Aug* by using EquE in Eq. (23) as part of the loss to encourage local scale equivariance.

The baselines and our DEC are applied to three deep-net architectures: ViT [18], Swin [38], and DINOv2 [45].

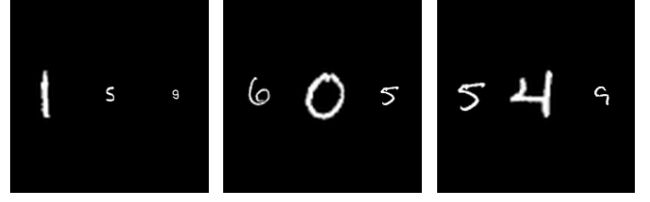


Figure 6. **Samples from locally scaled MNIST.** The task is to regress the number, *i.e.*, “159”, “605”, and “549” are the ground-truth for each of the images. In other words, an image classification task of 1000 classes; from 0 to 999.

Results. In Tab. 1, we compare our DEC with the equivariant adaptation baselines in semantic segmentation. We observe that ours achieves the highest mIoU and lowest EquE in all cases. Although *Aug* is effective in pursuing local scale equivariance, they are far from optimal. Comparison of *Aug* and ours shows that mere data augmentation with local scaling operations is not reliable enough to guarantee equivariance, nor does further guiding *Aug* with the Eq. (23) in *AugL*.

In contrast, *Canon* degrades the performance of *Aug* in most cases. That is, complete canonicalization on the input image only induces deterioration to the model. This shows the importance of canonicalization on latent features and replacing gradient optimization with our DEC, *i.e.*, solving a fixed-point equation iteratively.

Lastly, consistent improvements of ours on 3 distinct architectures demonstrate the generalizability of our model to different deep-nets. On average, ours outperforms the best baseline, *i.e.*, *AugL*, in mIoU by 1.17%.

5.2. Image Classification — MNIST

Dataset setup. Inspired by the MNIST-scale dataset from scale equivariance [29, 48, 57], we construct a locally scaled MNIST by creating 3-digit images from “000” to “999”. Each digit is randomly resized by a factor within $[0.4, 2.0]$, allowing for a scale variation of up to 5 times between different digits (see Fig. 6). The images have a resolution of 224×224 . This provides a suitable setup to test the local scale invariance, where the task is to predict the 1,000 number classes. As this task is relatively easy, we consider the more difficult setting with limited training data. The training set consists of 6,000 samples, while the test set contains 50,000 samples. The train/test images of the locally scaled dataset are sampled from the train/test split of the MNIST, respectively.

Evaluation Metric. We report the top-1 classification accuracy and the local scale invariance error

$$\text{InvE} \triangleq \frac{1}{|\mathcal{D}| |\mathcal{S}_I|} \sum_{I \in \mathcal{D}} \sum_{I_s \in \mathcal{S}_I} \|\mathcal{M}(I) - \mathcal{M}(I_s)\|_2^2. \quad (24)$$

Method	ResNet [25]		ViT [18]		DeiT [61]		Swin [38]		BEiT [7]		DINOv2 [45]	
	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$
<i>Aug</i>	93.09 $\pm$ 0.41	13.71 $\pm$ 1.65	91.23 $\pm$ 1.80	13.33 $\pm$ 1.72	90.91 $\pm$ 1.18	16.17 $\pm$ 4.94	94.26 $\pm$ 0.25	5.44 $\pm$ 1.65	94.70 $\pm$ 0.37	9.69 $\pm$ 1.94	95.80 $\pm$ 0.29	7.47 $\pm$ 1.33
<i>Canon</i>	94.22 $\pm$ 0.33	13.84 $\pm$ 1.05	93.73 $\pm$ 0.61	12.32 $\pm$ 2.35	95.01 $\pm$ 0.20	7.04 $\pm$ 0.70	95.67 $\pm$ 0.19	4.27 $\pm$ 1.86	95.92 $\pm$ 0.29	5.07 $\pm$ 1.65	96.63 $\pm$ 0.43	6.00 $\pm$ 3.12
<i>InvL</i>	93.67 $\pm$ 0.73	13.47 $\pm$ 2.01	94.79 $\pm$ 0.42	8.17 $\pm$ 2.60	94.06 $\pm$ 0.38	8.04 $\pm$ 2.88	94.78 $\pm$ 0.25	3.89 $\pm$ 0.62	95.26 $\pm$ 0.41	6.49 $\pm$ 0.78	96.66 $\pm$ 0.10	5.21 $\pm$ 1.34
Ours	96.09 $\pm$ 0.16	6.13 $\pm$ 2.01	95.64 $\pm$ 0.15	5.04 $\pm$ 3.37	96.46 $\pm$ 0.14	6.02 $\pm$ 2.71	96.91 $\pm$ 0.12	2.08 $\pm$ 1.09	97.05 $\pm$ 0.26	4.13 $\pm$ 1.70	97.90 $\pm$ 0.21	2.62 $\pm$ 1.69

Table 2. **Results of local scale invariance on MNIST.** For each method, we report the Acc(%) and InvE(10^{-2}) across 6 architectures, with error bars computed over five runs.

Scale	[0.4, 1.0]	[1.0, 2.0]	[2.0, 3.0]	Overall
<i>Aug</i>	91.64	96.32	93.84	93.93 $\pm$ 1.91
<i>Canon</i>	90.16	95.39	90.54	92.03 $\pm$ 2.38
<i>InvL</i>	91.98	96.26	93.88	94.04 $\pm$ 1.75
Ours	95.14	97.81	96.64	96.53 $\pm$ 1.09

Table 3. **Per-scale results on MNIST.** We report the accuracy across various local scaling ranges using the Swin [38] architecture.

Here, $\mathcal{S}_I$ consists of all variants of locally scaled I . Intuitively, InvE can be thought as a *soft* definition, in the ℓ_2 sense, of the local scale invariance in Eq. (11).

Baselines. We follow the same baselines of *Aug* and *Canon* from Sec. 5.1. Differently, as image classification should be invariant to local scaling, we instead have the baseline of *InvL*, which contains an additional loss of

$$\mathcal{L}_{\text{Inv}} = \mathbb{E}_{I \sim \mathcal{D}, \Phi \sim \Omega} \|\mathcal{M}(I) - \mathcal{M}(\mathcal{S}(I; \Phi))\|_2^2, \quad (25)$$

to encourage monotone scale invariance. We evaluate these baselines on commonly used image classification architectures, including ResNet [25], ViT [18], DeiT [61], Swin [38], BEiT [7], and DINOv2 [45].

Results. In Tab. 2, we compare our DEC with the baselines in image classification on MNIST. We have observations similar to the results in Tab. 1. *Aug* and *InvL* are not optimal while *AugL* degrades the performance of some architectures, e.g. ResNet, ViT, and Swin. Ours achieves the highest Acc and lowest InvE, no matter the architectures.

In particular, ours shows impressive results on local scale invariance and achieves a significant improvement in InvE compared to *Aug* and other baselines. Specifically, ours reduces InvE from 18.08 to 9.85 on ResNet and from 6.30 to 1.31 on DINOv2. This substantiates DEC’s ability to achieve local scaling invariance. On average, ours improves *Aug* on Acc by 4.10% and InvE by 8.91×10^{-2} . To further demonstrate the broad applicability of our model, we provide an additional comparison against hierarchical models [22, 60] in Sec. A5.1.

Analysis and ablation studies. In Tab. 3, we present the baseline results on Swin [38] across different local scale ranges. Specifically, we restrict the local scale range of each number digit in various scale brackets. We note that our method achieved higher accuracy in every scale range. Furthermore, by accessing different scale ranges, our method



Figure 7. **Samples from locally scaled ImageNet.** For each image, we show the target object at two different scales. Note that the scale of the background is always unchanged.

demonstrates the most consistent accuracy, reducing the std to 1.09 from 1.91. Additional ablations on DEC-related hyperparameters are provided in Sec. A5.2.

We also compare the time and memory requirements of DEC modules with differentiable optimization-based canonicalization, i.e., *Optim*. We train both methods with a batch size of 10 and image sizes of 224×224 . *Optim* takes 43.30 GB of GPU memory and spends 0.41s per iteration, while our DEC takes 5.75 GB and spends 0.19s. In other words, *Optim* requires more than 8 times the memory and twice the time than the DEC module. As discussed in Sec. 4.2, this makes it extremely difficult to apply *Optim* to deeper networks. We provide further discussion on runtime in Sec. A4.

5.3. Image Classification — ImageNet

Data setup. This experiment aims to benchmark local scale invariance in a more realistic real-world setting, where we create locally scaled images using ImageNet [17]. As shown in Fig. 7, only the target object in each image is scaled, while the background is kept the same.

In more detail, these images are created using the following procedure (see Fig. 4 for visualization): (a) We extract the bounding box of the target object by Grounding DINO [37] using the image labels. (b) With the bounding box, we extract a fine object mask using SAM [30] and (c) extract the object. (d) We inpaint the object mask region using LAMA [58]. (e) We scale the extracted object and (f) place the scaled object back onto the inpainted background. The object of interest is scaled by a factor within the range of [0.7, 1.3]. We fine-tune the pretrained models on a training dataset containing 10,000 images. We use 50,000 testing

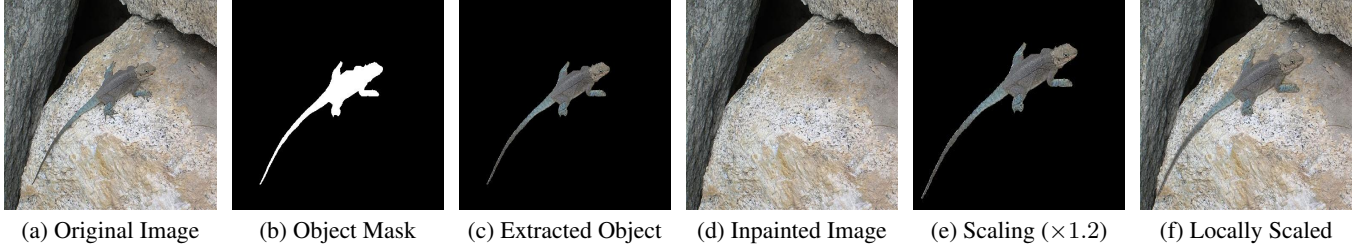


Table 4. **Generation procedure of locally scaled ImageNet.** Given (a) an image of a lizard, we first (b) obtain a mask of the lizard and (c) extract the lizard from the image. We then (d) inpaint the region without the lizard to get the background. Finally, (e) we scale the lizard by $1.2\times$ and (f) place it back onto the background to get the locally scaled image.

Method	ViT		DeiT		Swin		BEiT	
	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$	Acc $\uparrow$	InvE $\downarrow$
<i>Base</i>	80.15	8.18	69.06	11.13	77.94	8.93	84.70	6.29
<i>Aug</i>	80.15	8.16	69.06	11.12	77.96	8.92	84.66	6.27
<i>Canon</i>	77.72	10.20	64.79	12.74	74.67	11.40	83.34	8.03
<i>InvL</i>	80.16	8.16	69.06	11.12	77.97	8.92	84.66	6.27
Ours	80.36	8.10	69.27	11.08	78.32	8.82	85.08	6.24

Table 5. **Results of local scale invariance on ImageNet.** For each method, we report the Top-1 Acc(%) and InvE(10^{-2}) across 4 architectures.

Method	Local Scales								Overall
	0.7	0.8	0.9	1.0	1.1	1.2	1.3		
<i>Base</i>	74.66	76.44	78.07	79.55	79.48	78.88	78.53	77.94 ± 1.65	
<i>Aug</i>	74.71	76.47	78.07	79.56	79.48	78.87	78.57	77.96 ± 1.64	
<i>Canon</i>	71.37	73.28	74.87	76.04	76.04	75.79	75.32	74.67 ± 1.61	
<i>InvL</i>	74.73	76.46	78.07	79.58	79.48	78.87	78.57	77.97 ± 1.64	
Ours	75.31	77.04	78.65	79.86	79.60	79.05	78.73	78.32 ± 1.49	

Table 6. **Per-scale results on ImageNet.** We report the Top-1 Acc(%) across various local scaling factors. Swin [38].

images for evaluation.

Baselines. For ImageNet, we use open-source models from Pytorch Image Models [64]. We fine-tune the pretrained weights using their training script, which we denote as *Base*. All other baselines are initialized from *Base*. We consider four different architectures ViT [18], DeiT [61], Swin [38], and BEiT [7]. During fine-tuning, all baselines follow the best practices of data augmentation and fine-tuning strategy for vision transformers [26, 38], including techniques such as label-smoothing, cutmix, etc.

Results. We present the ImageNet results in Tab. 5. We observe that our proposed DEC achieves the highest Acc with low InvE. Furthermore, we notice that the *Canon* is once again degrading the performance of the *Base* by distorting the input image. In Tab. 6, we report the performance of the baselines on Swin on different local scales. Our approach achieves the highest Acc on all local scales. Lastly, DEC produces the most consistent performance across different architectures, lowering the std of the Top-1 Acc to 1.49 from 1.65 of the *Base* model.

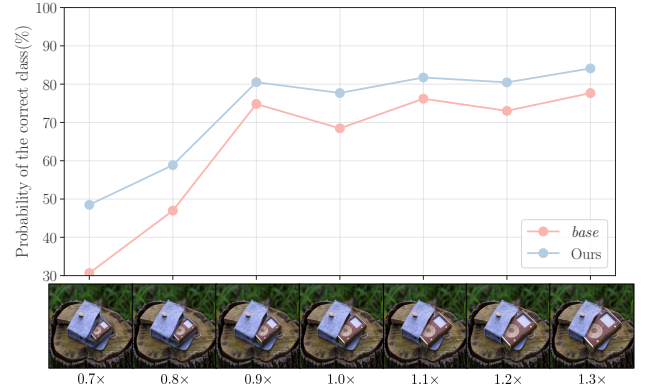


Figure 8. **Comparison on per-scale probability of correctness.** We locally scale the same input image within the range of $[0.7, 1.3]$ and report the probability of the correct class. Across all scales, our performance is 73.11 ± 0.13 while *Base*’s is 63.97 ± 0.17 .

We visualize DEC’s consistency across multiple scales in Fig. 8. We show the probability of the correct class with the same image but locally scaled differently. We apply both methods to Swin [38]. Compared to *Base*, ours performs better on all scales and is more robust and invariant to challenging scenarios when the scale is extreme. In particular, when the input image is locally scaled by $0.7\times$, our method outperforms *Base* by 17.8%.

6. Conclusion

In this work, we introduce the Deep Equilibrium Canonicalizer (DEC) to improve the local scale consistency of deep networks, tackling the fundamental challenge of scale variation in computer vision. We demonstrate that DEC can be adapted into existing deep-net architectures to improve both performance and scale consistency. Extensive experiments on three datasets and six architectures demonstrate its effectiveness. Notably, on the ImageNet dataset, DEC further improves the pre-trained model’s accuracy, even for the original unscaled images. Finally, we demonstrate the potential of DEQ as an effective canonicalizer for achieving equivariance and hope that this work inspires future DEC efforts toward other equivariances.

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