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Autofocus Method for SAR Based on Dynamically Randomized Block Sampling in Wavenumber Domain

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Abstract

Synthetic Aperture Radar (SAR) systems on small unmanned aerial vehicles (UAVs) are challenged by complex, spatially-variant motion errors, which conventional autofocus methods often fail to adequately correct. Specifically, approaches using fixed sub-block partitioning struggle to adapt to the non-uniform spatial distribution of phase errors, while static sample selection mechanisms lack the flexibility to optimize estimation throughout the iterative process. To overcome these limitations, this paper proposes a novel autofocus algorithm that integrates two key innovations. The first, adaptive slant-range wavenumber sub-block partitioning, dynamically adjusts the division granularity based on the local severity of the phase error, ensuring an optimal trade-off between estimation accuracy and robustness. The second, a sample selection strategy guided by a Dynamic Probability Density Function (DPDF), adaptively modifies sample selection probability during iterations, balancing broad exploration in early stages with focused exploitation of high-quality samples in later stages. Integrated within a fast factorized back-projection (FFBP) framework, the method leverages the coherence between azimuth phase error (APE) and non-systematic range cell migration (NsRCM) for joint estimation and correction. Validation using both simulated and measured UAV SAR data demonstrates that the proposed method significantly enhances focusing quality.

Keywords: synthetic aperture radar, motion error, autofocus, adaptive partitioning, dynamic probability density function

1 Introduction

SAR is a crucial remote sensing tool due to its all-weather, all-day imaging capability. Recent advancements have enabled miniaturized SAR systems on UAV platforms, facilitating flexible, high-resolution imaging and making UAV-SAR a significant research focus. However, small UAVs are highly susceptible to airflow disturbances, causing deviations from the intended trajectory. Lacking high-precision navigation systems, these platforms suffer from significant motion errors, which manifest as APE and NsRCM [1]. These errors severely degrade image quality, often rendering images unrecognizable. Consequently, effective motion error compensation through data-driven autofocus algorithms is a critical challenge.

To address these spatially-variant errors, data-driven autofocus methods, such as Phase Gradient Autofocus (PGA) and its derivatives, have been integrated into sub-block processing frameworks [2–4]. This strategy involves dividing the data into range sub-blocks and estimating the phase error within each block. However, despite their progress, these approaches face two persistent and critical limitations that hinder their performance in complex scenarios: First, the prevalent use of a fixed sub-block division strategy lacks adaptability. Methods like the range-dependent Map-Drift (RDMD) [5] or standard sub-block PGA often employ uniform, predefined block sizes. This rigid approach fails to adapt to the non-uniform spatial distribution of phase errors, leading to over-segmentation in regions of gradual error change (increasing computational load) or under-segmentation where errors fluctuate drastically (degrading estimation accuracy).

Second, sample selection within these sub-blocks often relies on inflexible, static mechanisms. For instance, Local Maximum Likelihood Weighted PGA (LML-WPGA) [6] uses a fixed threshold, which can either neglect useful information from weaker scatterers or be biased by a few dominant ones. While an improvement, Local Maximum Likelihood Weighted Stochastic PGA (LML-WSPGA) [7] employs a static Probability Density Function (PDF) based on the Signal-to-Clutter Ratio (SCR). This inflexibility limits the algorithm’s ability to adapt its sampling strategy throughout the iterative process; it cannot perform a broad exploration for samples in the initial, high-uncertainty stages, nor can it efficiently exploit high-quality samples for refinement in later stages.

To overcome these specific challenges, this paper proposes a novel autofocus algorithm, termed ASDP-WSPGA, which introduces two synergistic innovations. To solve the problem of rigid partitioning, we introduce Adaptive Slant-Range Partitioning (ASRP). This strategy dynamically adjusts the sub-block boundaries based on the estimated local severity of the phase error, ensuring that the partitioning granularity optimally matches the actual error characteristics. To address the inflexibility of sample selection, we develop a DPDF. The DPDF adaptively modifies sample selection probabilities throughout the iterative process, enabling a broad exploration of the sample space in early stages and a focused exploitation of high-quality samples for precise estimation in later stages. Integrated within a Quasi-Polar Grid (QPG) FFBP framework [8], the algorithm effectively estimates and corrects both APE and NsRCM.

This paper is structured as follows. Section 2 derives the spatially-varying characteristics of phase errors in the wavenumber domain. Section 3 details the proposed

ASRP strategy and the DPDF construction. Finally, Section 4 and Section 5 present simulation experiments and measured data results, respectively, to validate the algorithm's efficacy.

2 SAR Imaging Model

2.1 SAR Platform Motion Model

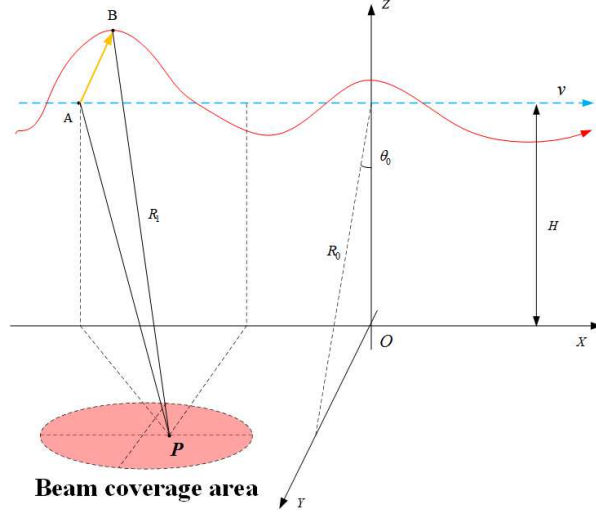


Figure 1 Airborne SAR geometric relationship

The platform motion model for UAV-SAR is illustrated in Figure 1. The radar operates at a flight altitude H , with a velocity v . The shortest slant distance to the scene is R_0 , corresponding to a downward-looking angle θ_0 . In Figure 1, the blue dashed line indicates the preset UAV trajectory, while the red solid line represents its actual flight path. Consider an arbitrary point $P(x_0, y_0, 0)$ within the beam coverage area. At the moment of t , the preset Antenna Phase Center (APC) position of the radar is $A(vt, 0, H)$, whereas its actual position is $B(vt + \Delta x(t), \Delta y(t), H + \Delta z(t))$. Here, $\Delta x(t)$, $\Delta y(t)$, and $\Delta z(t)$ are the respective three-axis motion errors of the platform. R_1 denote the actual slant distances from the UAV to point P . Consequently, the slant distance error for point P is

$$\begin{aligned} \Delta R \approx & \Delta x(t) \cos \theta_s(t) - \Delta y(t) \sin \theta_0 \sin \theta_s(t) \\ & + \Delta z(t) \sin \theta_0 \cos \theta_s(t) \end{aligned} \quad (1)$$

Here, $\theta_s(t)$ represents the real-time squint angle of the radar APC with respect to point P , which satisfies

$$\cos \theta_s(t) \approx \frac{vt - x_0}{\sqrt{R_1}}, \quad \cos \theta_0 = \frac{H}{R_0} \quad (2)$$

Due to the existence of $\cos \theta_s(t)$ in the three-dimensional trajectory deviation, the slant distance error ΔR of the target points at different positions exhibits an anisotropic characteristic, and the traditional self-focusing method is no longer applicable.

2.2 Wavenumber Domain Imaging Model in a Quasi-polar Coordinate System

References [9–13] establish a linear mapping between the phase error in the wavenumber domain and the azimuth time domain, approximating this error as spatially invariant. Fundamentally, this constitutes a simplified processing approach reliant on linearization and specific coordinate transformations. However, this approximation becomes invalid when actual conditions—such as the magnitude of motion errors, scene size, resolution requirements, or imaging geometry—deviate beyond the tolerance limits imposed by these simplifying assumptions. The geometrical relationship of a point

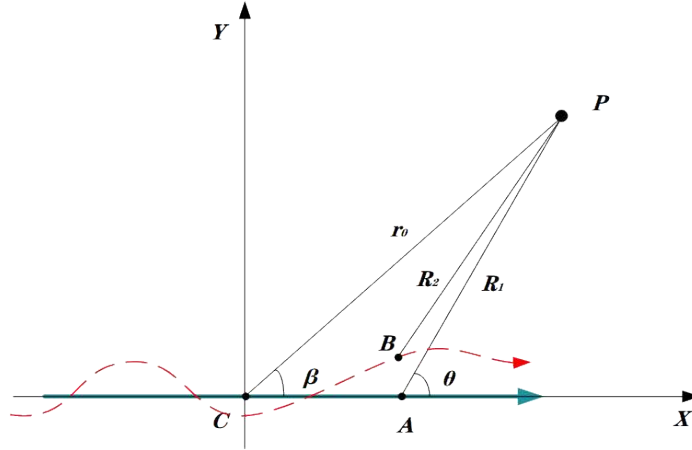


Figure 2 Motion model in the quasi-polar coordinate system

target P within both the Cartesian coordinate system and the quasi-polar coordinate system is illustrated in Figure 2. In this setup, the X axis defines the azimuth direction, and the Y axis represents the range direction. Assuming that the radar platform moves along the X -axis within a sub-aperture, the blue solid line depicts the ideal motion trajectory, while the red dashed line indicates the actual motion trajectory. The Y coordinate of this trajectory is constant at y_c . Point C , with coordinate

(x_c, y_c) , represents the APC at the center of the sub-aperture and serves as its reference point. Point A denotes the ideal instantaneous position of the radar at azimuth slow time t , $(x(t), y_c)$, whereas B is its actual position. P is an arbitrary target point in the illuminated area with coordinates (x, y) in Cartesian coordinate system and (r_0, β) in quasi-polar coordinate system. Here, r_0 is the reference slant-range from the sub-aperture center C to target P , and β is the squint angle of P relative to C . R_1 signifies the instantaneous slant-range from the ideal radar position A to P , and θ is the instantaneous squint angle of P with respect to A . The relationships are given by $r_0 = \sqrt{(x - x_c)^2 + (y - y_c)^2}$ and $\cos \beta = \frac{x - x_c}{r_0}$.

Let $\Delta X(t) = x(t) - x_c$. The ideal instantaneous slant-range to P , denoted as $R_1(t; r_0, \cos \beta)$, is then given by

$$R_1(t; r_0, \cos \beta) = \sqrt{r_0^2 + \Delta X(t)^2 - 2r_0 \Delta X(t) \cos \beta} \quad (3)$$

The actual instantaneous slant-range $R_2(t; r_0, \cos \beta)$ is

$$R_2(t; r_0, \cos \beta) = R_1(t; r_0, \cos \beta) + \Delta R(t; r_0, \cos \beta) \quad (4)$$

The expression for FFBP imaging of P in the quasi-polar coordinate system is

$$i(r, \cos \theta; r_0, \cos \beta) = \iint s(f_r, t; r_0, \cos \beta) \cdot \exp \left[j \frac{4\pi(f_c + f_r)}{c} R_1(t; r, \cos \theta) \right] df_r dt \quad (5)$$

where $s(f_r, t; r_0, \cos \beta) = A \cdot \exp \left[-j \frac{4\pi(f_c + f_r)}{c} R_2(t; r_0, \cos \beta) \right]$ represents the linear frequency modulated signal after range compression. Here, f_r is the range frequency, f_c represents the carrier frequency, and A is the amplitude coefficient. Based on Eq. (4), the phase error in this signal can be expressed as

$$\Delta \Phi(r_0, \beta) = - \frac{4\pi(f_c + f_r)}{c} \cdot \Delta R(t; r_0, \cos \beta) \quad (6)$$

Performing a first-order Taylor expansion for $R_1(t; r, \theta)$ around the reference point (r_0, β) yields

$$R_1(t; r, \cos \theta) \approx R_1(t; r_0, \cos \beta) + \frac{\partial R_1}{\partial r_0}(r - r_0) + \frac{\partial R_1}{\partial \cos \beta}(\cos \theta - \cos \beta) \quad (7)$$

At the reference point, the partial derivatives are

$$\begin{cases} \frac{\partial R_1}{\partial r_0} = \frac{r_0 - \Delta X(t) \cos \beta}{R_1(t; r_0, \cos \beta)} \\ \frac{\partial R_1}{\partial \cos \beta} = \frac{-r_0 \Delta X(t)}{R_1(t; r_0, \cos \beta)} \end{cases} \quad (8)$$

The wavenumbers K_1 (slant range) and K_2 (azimuthal) are defined as

$$\begin{cases} K_1 = \frac{4\pi(f_c + f_r)}{c} \frac{\partial R_1}{\partial r_0} \\ \quad = \frac{4\pi(f_c + f_r)}{c} \cdot \frac{r_0 - \Delta X(t) \cos \beta}{R_1(t; r_0, \cos \beta)} \\ K_2 = \frac{4\pi(f_c + f_r)}{c} \frac{\partial R_1}{\partial \cos \beta} \\ \quad = -\frac{4\pi(f_c + f_r)}{c} \cdot \frac{r_0 \Delta X(t)}{R_1(t; r_0, \cos \beta)} \end{cases} \quad (9)$$

Substituting Eqs. (7), (8) and (9) into Eq. (5) results in

$$i(K_1, K_2; r_0, \beta) = \iint_{Q_1 Q_2} \exp \left[\begin{aligned} &-j (K_1 r_0 + K_2 \cos \beta) \\ &+ j \Delta \varphi(K_1, K_2; r_0, \cos \beta) \end{aligned} \right] \cdot \exp [j (K_1 r + K_2 \cos \theta)] dK_1 dK_2 \quad (10)$$

where $\Delta \varphi(K_1, K_2; r_0, \cos \beta)$ is the phase error after variable substitution, and Q_1 and Q_2 represent the support domains of K_1 and K_2 , respectively. From Eq. (9), we obtain

$$\frac{K_1}{K_2} = \frac{\Delta X(t) \cos \beta - r_0}{r_0 \Delta X(t)} = \frac{x_0}{r_0^2} - \frac{1}{\Delta X(t)} \quad (11)$$

Eq. (11) indicates that both the range wavenumber and the slant-range wavenumber are continuously influenced by r_0 , thereby exhibiting two-dimensional spatial variance. Reference [8] states that during the synthetic aperture time, the term x_0/r_0^2 is approximately constant and can be considered negligible, leading to the simplification

$$\frac{K_1}{K_2} \approx -\frac{1}{\Delta X(t)} \quad (12)$$

2.2.1 Analysis of the Coupling Effect between APE and NsRCM

Decomposing the phase error $\Delta\varphi(K_1, K_2; r_0, \cos\beta)$ along the azimuthal and slant-range directions yields

$$\begin{aligned}\Delta\varphi &= -\frac{4\pi(f_c + f_r)}{c}\Delta\gamma[x(t)] \\ &= K_1\Delta\gamma_r[x(t)] + K_2\Delta\gamma_{\cos\theta}[x(t)]\end{aligned}\quad (13)$$

where $\Delta\gamma[x(t)]$ represents the system motion error, which is a function of the azimuthal position $x(t)$. The terms $\Delta\gamma_r[x(t)]$ and $\Delta\gamma_{\cos\theta}[x(t)]$ are the components of this motion error along the slant-range wavenumber K_1 and the azimuth wavenumber K_2 , respectively.

Substituting Eq. (12) into $\Delta X(t) = x(t) - x_c$, we obtain $x(t) = \Delta X(t) + x_c = -K_2/K_1 + x_c$. Subsequently, substituting this expression for $x(t)$ into Eq. (13) gives

$$\begin{aligned}\Delta\varphi(K_1, K_2) &= K_1\Delta\gamma_r\left[x_c - \frac{K_2}{K_1}\right] \\ &\quad + K_2\Delta\gamma_{\cos\theta}\left[x_c - \frac{K_2}{K_1}\right]\end{aligned}\quad (14)$$

Expanding Eq. (14) as a Taylor series around the center of the slant-range wavenumber $K_{1,c}$ yields. For brevity in the subsequent equations, we will denote the argument of the functions as $(\cdot) = \left(x_c - \frac{K_2}{K_{1,c}}\right)$

$$\left\{\begin{aligned}K_1\Delta\gamma_r\left(x_c - \frac{K_2}{K_1}\right) &\approx K_{1,c}\Delta\gamma_r(\cdot) \\ &\quad + \left[\frac{K_2}{K_{1,c}}\Delta\gamma_r'(\cdot) + \Delta\gamma_r(\cdot)\right](K_1 - K_{1,c}) \\ K_2\Delta\gamma_{\cos\theta}\left(x_c - \frac{K_2}{K_1}\right) &\approx K_2\Delta\gamma_{\cos\theta}(\cdot) \\ &\quad + \frac{K_2^2}{K_{1,c}^2}\Delta\gamma_{\cos\theta}'(\cdot)(K_1 - K_{1,c})\end{aligned}\right.\quad (15)$$

Here, $\Delta\gamma_r'(\cdot)$ and $\Delta\gamma_{\cos\theta}'(\cdot)$ represent the derivatives of $\Delta\gamma_r(\cdot)$ and $\Delta\gamma_{\cos\theta}(\cdot)$ with respect to their arguments, respectively. Meanwhile, the terms $K_{1,c}\Delta\gamma_r\left(x_c - \frac{K_2}{K_{1,c}}\right)$ and $K_2\Delta\gamma_{\cos\theta}\left(x_c - \frac{K_2}{K_{1,c}}\right)$ are functions of the azimuthal wavenumber K_2 . Therefore, these terms are considered azimuthal phase error (APE) components and can be further denoted as

$$\Delta\varphi_a(K_{1,c}, K_2) = K_{1,c}\Delta\gamma_r(\cdot) + K_2\Delta\gamma_{\cos\theta}(\cdot)\quad (16)$$

The remaining terms in Eq. (15) are related to the slant-range wavenumber K_1 . These can be classified as the phase error due to NsRCM and are denoted as

$$\Delta\varphi_r(K_1, K_2).$$

$$\begin{aligned}\Delta\varphi_r(K_1, K_2) &= \left[\frac{K_2}{K_{1,c}} \Delta\gamma'_r(\cdot) + \Delta\gamma_r(\cdot) \right] (K_1 - K_{1,c}) \\ &\quad + \frac{K_2^2}{K_{1,c}^2} \Delta\gamma'_{\cos\theta}(\cdot) (K_1 - K_{1,c}) \\ &= \Delta R_{NsRCM}(K_1, K_2) \cdot (K_1 - K_{1,c})\end{aligned}\tag{17}$$

where $\Delta R_{NsRCM}(K_1, K_2)$ is defined as

$$\begin{aligned}\Delta R_{NsRCM}(K_1, K_2) &= \frac{K_2}{K_{1,c}} \Delta\gamma'_r(\cdot) + \Delta\gamma_r(\cdot) \\ &\quad + \frac{K_2^2}{K_{1,c}^2} \Delta\gamma'_{\cos\theta}(\cdot)\end{aligned}\tag{18}$$

Based on the phase consistency constraints inherent in SAR echo signals, a correlation model can be established between APE $\Delta\varphi_a(K_{1,c}, K_2)$ and NsRCM $\Delta R_{NsRCM}(K_1, K_2)$.

$$\begin{aligned}\Delta R_{NsRCM}(K_2) &= \\ &\quad \frac{\Delta\varphi_a(K_{1,c}, K_2) - K_2 \cdot \Delta\varphi'_a(K_{1,c}, K_2)}{K_{1,c}}\end{aligned}\tag{19}$$

Eq. (19) expresses the NsRCM error as a function of the APE and its derivative with respect to the azimuth wavenumber. This implies that by accurately estimating the APE, information regarding the NsRCM can be simultaneously obtained, thereby enabling the effective correction of both error types.

2.2.2 Mapping of Time-domain Echo Signals to the Wavenumber Domain

In an ideal scenario, assuming the absence of motion errors and that the UAV traverses an ideal orbit, Eq. (10) can be reformulated as

$$\begin{aligned}i(K_1, K_2; r_0, \beta) &= \\ &\quad \iint_{Q_1 Q_2} \exp[-j(K_1 r_0 + K_2 \cos\beta)] \cdot \\ &\quad \exp[j(K_1 r + K_2 \cos\theta)] dK_1 dK_2\end{aligned}\tag{20}$$

In Eq. (20), there exist two pairs of Fourier transform: the first is between the cosine of the squint angle at the sub-aperture center $\cos\theta$ and the azimuthal wavenumber K_2 ; the second is between the slant-range, r , and the slant-range wavenumber, K_1 . Within the framework of FT theory, the FFBP imaging results are initially transformed into the azimuthal wavenumber domain using a Fast Fourier Transform (FFT) to facilitate motion error compensation. Subsequently, an inverse FFT (IFFT) is applied to transform the data back to the image domain, thereby yielding a focused image.

However, motion errors are inevitable during actual flight operations. Consequently, to ensure the validity of Eq. (20) in the presence of such errors, stringent constraints must be imposed. These constraints are derived from the characteristics of edge pixels within the imaging grid, as detailed in [8, 12, 14], and are expressed as follows

$$8L_a \cdot \sin \frac{\theta_w}{2} + 2\lambda \leq r_{min} \quad (21)$$

In this expression, L_a denotes the synthetic aperture length of the system, λ represents the SAR system wavelength, r_{min} corresponds to the shortest slant-range of the QPG, and θ_w indicates the antenna's azimuth beamwidth. This inequality elucidates that only when this specific condition is met can the QPG establish an effective FT relationship between (K_1, K_2) and $(r, \cos \theta)$, thereby enabling motion error compensation.

3 ASDP-WSPGA Algorithm

3.1 ASRP Strategy

The derivations in the preceding section establish that, in two-dimensional (2-D) wavenumber domain SAR imaging, a spatially variant mapping relationship typically exists between the slant-range wavenumber K_1 and the azimuth wavenumber K_2 , which can be generally denoted $K_1 \cdot \Delta X(t) = K_2$. This spatial variance renders the phase characteristics of the target echo complex, making uniform correction across the entire wavenumber domain challenging. After partitioning the sub-blocks, the phase error within a sub-block image, $\Delta\varphi(K_1, K_2; r_0, \cos \beta)$, can be approximated as being solely dependent on the central slant-range wavenumber, $K_{1,C}$, of that sub-block

$$\Delta\varphi(K_1, K_2; r_0, \cos \beta) \approx \Delta\varphi(K_{1,c}, K_2; r_0, \cos \beta) \quad (22)$$

Here, $K_{1,c,M} = \frac{1}{2} \cdot (K_{1,\max,M} + K_{1,\min,M})$ represents the central wave number of the M -th slant-range wave number sub-block.

However, conventional fixed sub-block partitioning typically employs a fixed bandwidth, which presents limitations when encountering varying degrees of range-dependent phase errors. In regions where phase errors vary gently, excessively narrow fixed sub-blocks increase computational load. Conversely, in areas with abrupt error variations, overly wide fixed sub-blocks can lead to excessive non-stationarity of the phase error within the sub-block, thereby degrading the accuracy of subsequent WPGA estimation.

For simplification, $\Delta\varphi(K_1, K_2; r_0, \cos \beta)$ will be denoted as $\Delta\varphi(K_1, K_2)$ in the subsequent derivations. In the 2-D wavenumber domain, the phase error of the SAR echo data after preliminary processing is represented by $\Delta\varphi(K_1, K_2)$. Within a specific slant-range wavenumber sub-block, M , its range is defined by $[K_{1,\max,M}, K_{1,\min,M}]$, its central wavenumber is $K_{1,c,M}$, and its width is $\Delta K_1 = K_{1,\max,M} - K_{1,\min,M}$. The objective is to approximate $\Delta\varphi(K_1, K_2)$ as a constant within this sub-block respect to K_1 .

To quantify the accuracy of this approximation, a Taylor series expansion of the function $\Delta\varphi(K_1, K_2)$ is performed around the sub-block center $K_{1,c,M}$.

$$\begin{aligned}\Delta\varphi(K_1, K_2) &= \Delta\varphi(K_{1,c,M}, K_2) + \left. \frac{\partial\Delta\varphi}{\partial K_1} \right|_{K_{1,c,M}, K_2} (K_1 - K_{1,c,M}) \\ &+ \frac{1}{2} \left. \frac{\partial^2\Delta\varphi}{\partial K_1^2} \right|_{K_{1,c,M}, K_2} (K_1 - K_{1,c,M})^2 \\ &+ \frac{1}{6} \left. \frac{\partial^3\Delta\varphi}{\partial K_1^3} \right|_{K_{1,c,M}, K_2} (K_1 - K_{1,c,M})^3 + \dots\end{aligned}\quad (23)$$

When $\Delta\varphi(K_1, K_2)$ is approximated by a constant term solely dependent on the wavenumber $K_{1,C,M}$ at the center of the sub-block, the residual error originates from the linear term, quadratic term, cubic term, and higher-order terms in the Taylor expansion. To ensure the validity of this approximation, it is necessary to constrain the maximum values of these residual error terms. The first-order phase error, which manifests as a linear phase in the wavenumber domain, translates to a spatial shift of the Impulse Response Function (IRF) in the image domain via Fourier transform. It only alters the target's position in the image without broadening its energy distribution, and thus does not affect image sharpness. Conversely, the second-order phase error (QPE) causes a symmetric broadening of the IRF, leading to resolution degradation, while the cubic phase error (CPE) introduces asymmetric smearing or side-lobes, which reduces image contrast and readability. While Range Cell Migration Correction (RCMC) handles the bulk of migration from system parameters and geometry, this algorithm focuses primarily on these residual quadratic and cubic terms.

Therefore, to ensure the phase invariance within the block and maintain focus quality, and in accordance with [6], the magnitudes of the QPE and CPE are limited to within $\pi/4$. This limitation allows for the derivation of constraints on the sub-block width, ΔK_1 . Given that the K_1 values within the sub-block range from $[K_{1,C,M} - \frac{\Delta K_1}{2}, K_{1,C,M} + \frac{\Delta K_1}{2}]$, the maximum value of the quadratic term occurs at the sub-block edges

$$QPE_{\max} = \frac{1}{2} \left. \frac{\partial^2\Delta\varphi}{\partial K_1^2} \right|_{K_{1,c,M}, K_2} \left(\frac{\Delta K_1}{2} \right)^2 = \frac{1}{8} \Delta K_1^2 \cdot \left. \frac{\partial^2\Delta\varphi}{\partial K_1^2} \right|_{K_{1,c,M}, K_2} \quad (24)$$

Similarly, the maximum value of the cubic term is also attained at the sub-block boundaries

$$CPE_{\max} = \frac{1}{6} \left. \frac{\partial^3\Delta\varphi}{\partial K_1^3} \right|_{K_{1,c,M}, K_2} \left(\frac{\Delta K_1}{2} \right)^3 = \frac{1}{48} \Delta K_1^3 \cdot \left. \frac{\partial^3\Delta\varphi}{\partial K_1^3} \right|_{K_{1,c,M}, K_2} \quad (25)$$

To ensure these residual errors remain within acceptable limits, the following thresholds are established

$$\begin{cases} QPE_{\max} \leq \frac{\pi}{4} \\ CPE_{\max} \leq \frac{\pi}{4} \end{cases} \quad (26)$$

From the above conditions, the maximum allowable width of the current sub-block, $\Delta K_{r,\max,M}$, can be derived

$$\begin{cases} \Delta K_{r,\max,M} \leq \sqrt{\frac{2\pi}{\left|\frac{\partial^2 \Delta\varphi}{\partial K_1^2}\right| K_{1,c,M}, K_2}} \\ \Delta K_{r,\max,M} \leq \sqrt[3]{\frac{12\pi}{\left|\frac{\partial^3 \Delta\varphi}{\partial K_2^3}\right| K_{1,c,M}, K_2}} \end{cases} \quad (27)$$

Therefore, for the M -th sub-block, the maximum slant-range wavenumber width satisfies

$$\Delta K_{r,\max,M} = \min \left(\sqrt{\frac{2\pi}{\left|\frac{\partial^2 \Delta\varphi}{\partial K_1^2}\right| K_{1,c,M}, K_2}}, \sqrt[3]{\frac{12\pi}{\left|\frac{\partial^3 \Delta\varphi}{\partial K_2^3}\right| K_{1,c,M}, K_2}} \right) \quad (28)$$

In the actual iterative process, the second derivative $\frac{\partial^2 \Delta\varphi}{\partial K_1^2}$ and the third derivative $\frac{\partial^3 \Delta\varphi}{\partial K_2^3}$ are not known a priori. These values are derived from the phase error estimated in the preceding iteration.

The ASRP strategy is an iterative process that dynamically refines the sub-block partitioning in the slant-range wavenumber domain. The process begins with an initial coarse division of the entire wavenumber domain into several broad sub-blocks. Then, within each iteration, for every sub-block B_M , the internal phase error $\Delta\varphi(K_1, K_2)$ is first estimated using the WPGA method. From this estimate, the local variation of the phase error is quantified by computing its second and third derivatives with respect to K_1 ($\frac{\partial^2 \Delta\varphi}{\partial K_1^2}$ and $\frac{\partial^3 \Delta\varphi}{\partial K_1^3}$). These derivatives are subsequently used to calculate the maximum allowable width, $\Delta K_{r,\max,M}$, for that specific sub-block according to Eq. (28).

The core of the adaptive mechanism lies in the adjustment step. The algorithm compares the current block's actual width, $\Delta K_{r,M}$, with its calculated maximum allowable width. If a block is found to be wider than its limit, indicating that the phase error varies too rapidly for a constant approximation, it is split into two smaller sub-blocks at its midpoint. Conversely, if a block and its neighbor are both significantly narrower than their respective limits, they are considered for merging. This merging reduces computational load and increases the number of samples within the new block, thereby enhancing estimation robustness. This process of splitting and merging generates a new, refined set of sub-blocks that serves as the input for the subsequent iteration.

To prevent potential discontinuities in the final focused image, an overlapping region of 15% to 20% is maintained between adjacent sub-blocks, ensuring a smooth transition during the final phase error fusion. Through this iterative refinement, the ASRP strategy dynamically tailors the sub-block granularity to the actual spatial characteristics of the phase error. Wider blocks are used in regions of gentle variation to improve estimation robustness and efficiency, while narrower blocks are employed

where errors change abruptly to enhance spatial resolution and accuracy. The entire procedure of the ASRP strategy is summarized in the flowchart shown in Figure 3.

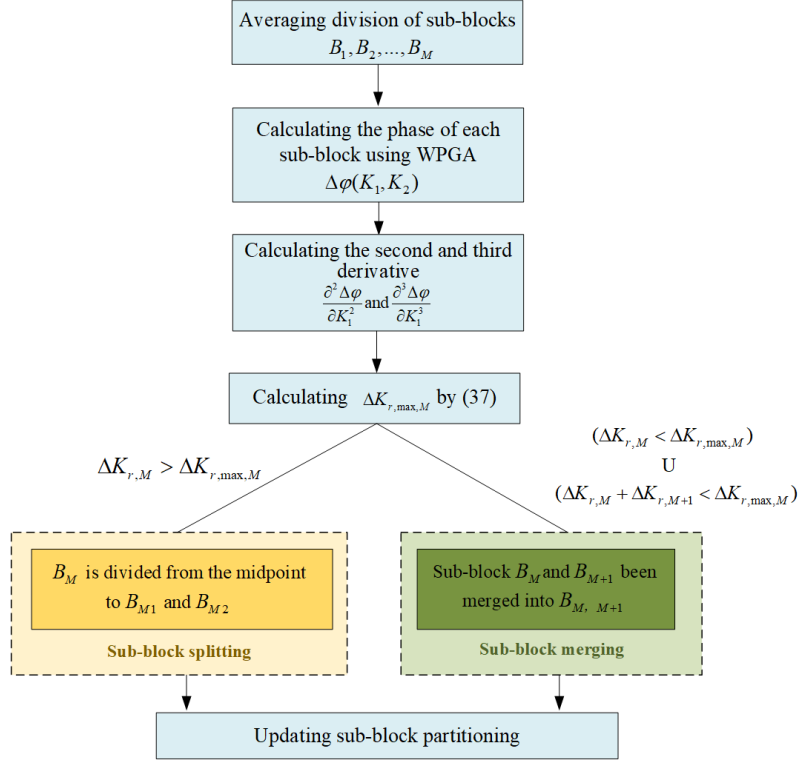


Figure 3 Flowchart of the adaptive slant-range wavenumber domain partitioning strategy

3.2 Construct DPDF

To address the challenge of sample selection, reference [7] proposes the LML-WSPGA algorithm. This algorithm, built upon the LML-WPGA framework, deviates from using a fixed threshold by constructing a Probability Density Function (PDF) based on the Signal-to-Clutter Ratio (SCR) of the samples. During each PGA iteration, samples within a sub-block are randomly selected according to this PDF and a predetermined number of required samples. Assuming that a range sub-block B_M contains n range cells, and $\delta_{M,n}$ represents the SCR of the n -th range cell within the M -th range sub-block, the PDF, $f(\delta_{M,n})$, parameterized by the SCRs of the range cells within that sub-block, is expressed as

$$f(\delta_{M,n}) = \frac{\delta_{M,n}}{\sum_n \delta_{M,n}} \quad (29)$$

Eq. (29) demonstrates that the SCR of a range cell determines the probability of its constituent samples being selected during the model parameter estimation process. In contrast to the fixed-threshold selection mechanism of LML-WPGA, this approach, after establishing the total number of samples to be selected, permits samples from each range cell to participate in the estimation process with a specific probability. This can be conceptualized as a soft-thresholding method, offering selection opportunities to samples from diverse range cells.

However, the PDF employed in LML-WSPG possesses static characteristics. Specifically, once the SCR of a sample is determined, its selection probability remains fixed throughout the iterative autofocus process. This static nature imposes a fundamental limitation, as it fails to adapt to the dynamic evolution of the information state during the iterations. The iterative autofocus process is inherently a progression from high uncertainty to increasing certainty. In the initial stages, the image is severely defocused, and the calculated SCR values are unreliable. An apparently strong scatterer might simply be the result of constructive interference from several blurred targets, while a true strong scatterer's energy may be so smeared that its SCR is low. A static PDF based on these initial, unreliable SCRs will cause the algorithm to be overly biased towards a few "false-strong" points, risking premature convergence to a local optimum. This strategy restricts the exploration of the entire sample space and may neglect potentially higher-quality samples that would become apparent in later iterations. Conversely, in the later stages of iteration, the image is nearly focused, and the phase error estimation requires fine-tuning. At this point, the SCR becomes a highly reliable metric of sample quality. The algorithm should concentrate on using the most stable, high-SCR samples to achieve precise phase correction and ensure final focusing accuracy. A static PDF, if designed to be "flat" for initial exploration, would continue to introduce phase noise from medium-to-low quality samples, thereby hindering convergence speed and final accuracy. It fails to perform an efficient exploitation of the best available information.

To overcome the above limitations and endow the algorithm with enhanced adaptability and flexibility across diverse iterative stages and data characteristics, this paper, building upon the LML-WSPGA framework, introduces a sample selection methodology guided by a DPDF. The original PDF based on SCR, as given in Eq. (29), is refined into this DPDF. Specifically, in the k -th iteration, for the n -th sample within sub-block B_M , its selection probability, $f(\delta_{M,n}, k)$, is defined as

$$f(\delta_{M,n}, k) = \frac{\delta_{M,n}^{\frac{1}{T(k)}}}{\sum_n \delta_{M,n}^{\frac{1}{T(k)}}} \quad (30)$$

where $\delta_{M,n}$ still denotes the SCR of the sample. The parameter $T(k)$ varies with the iterations number, k , and controls the sharpness of the selection probability distribution. In this paper, $T(k)$ is defined as

$$T(k) = T_0 \cdot \mu^{k-1} \quad (31)$$

In Eq. (31), T_0 is the initial value of $T(k)$, and $\mu \in (0, 1)$ is a decay factor. At the beginning of the iteration, the distribution of $f(\delta_{M,n}, k)$ is relatively flat, granting even medium- and low-SCR samples a high probability of selection. As the iteration progresses, $f(\delta_{M,n}, k)$ becomes progressively sharper. Consequently, the algorithm becomes more inclined to select high-SCR samples, thereby accelerating convergence and improving the final estimation accuracy.

After determining the selection probability $f(\delta_{M,n}, k)$ for each sample, to select I samples within the current sub-block for phase error estimation, the inverse transformation sampling method based on the cumulative distribution function (CDF) is adopted:

(1) Construction of the CDF

For all samples within the current sub-block, indexed by their original serial number $n = 1, 2, \dots, n_{\max}$, the cumulative probability, $F_{M,j}$, is computed as follows:

$$\begin{cases} F_{M,1} = f(\delta_{M,1}, k) \\ F_{M,2} = f(\delta_{M,1}, k) + f(\delta_{M,2}, k) \\ \vdots \\ F_{M,j} = \sum_{l=1}^j f(\delta_{M,l}, k) \\ \vdots \\ F_{M,n_{\max}} = \sum_{l=1}^{n_{\max}} f(\delta_{M,l}, k) = 1 \end{cases} \quad (32)$$

This construction ensures that each range cell j corresponds to a specific interval $[F_{M,j-1}, F_{M,j}]$ on the cumulative probability scale, the length of which is equal to its individual selection probability.

(2) Independent Random Sampling

To select I samples, conduct I independent random sampling processes. In the i -th sampling (where $i = 1, 2, \dots, I$), a random number u_i is generated from the uniform distribution $U(0, 1)$, which is used to identify the selected sample. The specific sample chosen is the one corresponding to the smallest original serial index, j^* , such that

$$u_i \leq F_{M,j^*} \quad (33)$$

Essentially, the algorithm finds which cumulative probability interval $[F_{M,j-1}, F_{M,j}]$ the random number u_i falls into. The sample j^* associated with this interval is then selected.

After I such sampling iterations, I samples are selected. These, along with their corresponding SCR values and phase information, constitute the set of strong scattering points for the current sub-block. This set is subsequently used by the phase gradient autofocus algorithm to estimate the average phase error within that sub-block. The overall process of dynamically constructing and utilizing the PDF is illustrated in Figure 4.

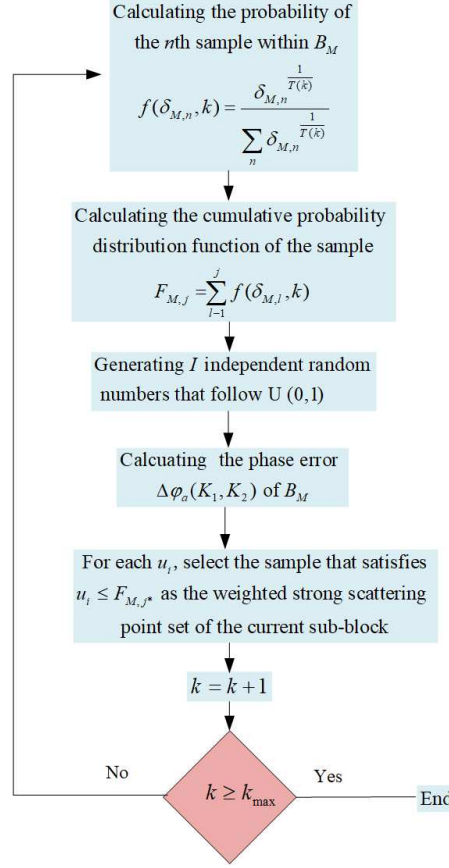


Figure 4 Constructing the flowchart of the DPDF

3.3 Overall Algorithm

In order to describe the proposed algorithm more clearly, Figure 5 shows the general flowchart of SAR imaging.

4 Simulation and Result

To validate the proposed method (ASDP-WSPGA), we conduct an ablation study comparing it against three other methods:

- **Method 1:** Baseline (Uniform Partitioning + Static PDF).
- **Method 2:** ASRP Only (Adaptive Partitioning + Static PDF).
- **Method 3:** DPDF Only (Uniform Partitioning + DPDF).

Simulations used the UAV SAR parameters in Table 1 and a scene with nine point targets (Table 2). Complex motion errors were introduced. BP imaging requires transforming the data into a processing grid, as shown for the Quasi-Polar Grid (QPG)

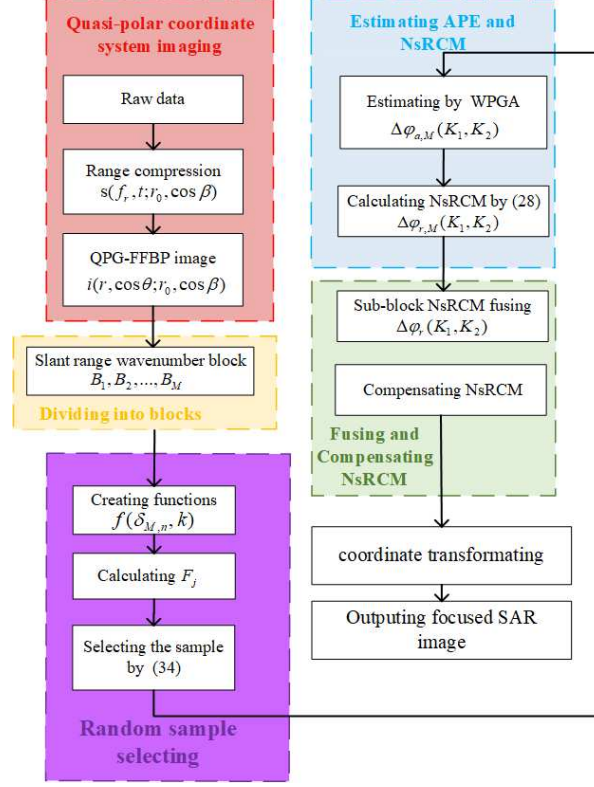


Figure 5 The algorithm flowchart

Table 1 System Parameters

parameters	value
Band	X
Bandwidth/GHz	0.75
PRF/Hz	333
Distance/m	633–2884
Height/m	300
Speed/ m·s ⁻¹	8

Table 2 Target Point Coordinates

Target Points	Azimuth /m	Slant Range /m
1, 2, 3	200	950, 1400, 1850
4, 5, 6	450	950, 1400, 1850
7, 8, 9	700	950, 1400, 1850

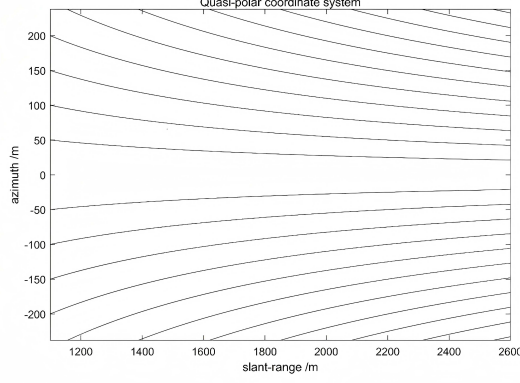


Figure 6 Quasi-polar coordinate system

in Figure 6. The QPG's nonlinear sampling causes azimuth contours to become curved, meaning targets at the same azimuth angle but different ranges are not collinear.

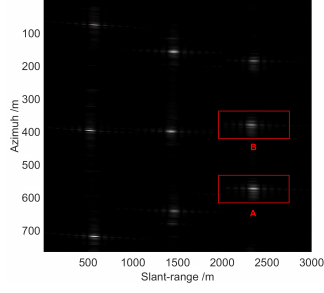
The results of the ablation study are in Figure 7. The baseline (Method 1) is severely defocused. Methods 2 and 3 show partial improvements but suffer from residual artifacts. The proposed ASDP-WSPGA algorithm (Figure 7j), integrating both strategies, achieves superior focusing quality, suggesting a synergistic effect. The point targets are sharp and compact with well-suppressed sidelobes.

Table 3 Performance parameters of point A

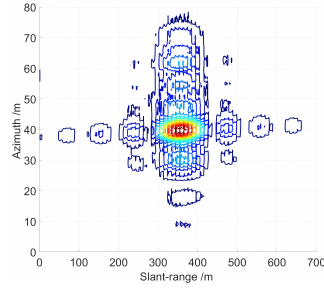
Method	Slant-range		Azimuth	
	PSLR /dB	ISLR /dB	PSLR /dB	ISLR /dB
Method 1	-13.6083	-9.5419	-12.2184	-4.5983
Method 2	-14.4924	-11.2592	-13.5799	-14.8245
Method 3	-14.2393	-7.7490	-12.8983	-11.8127
The proposed method	-13.7815	-11.1374	-13.5013	-17.4687

Table 4 Performance parameters of point B

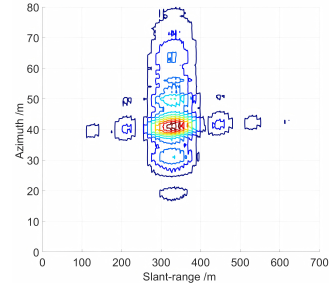
Method	Slant-range		Azimuth	
	PSLR /dB	ISLR /dB	PSLR /dB	ISLR /dB
Method 1	-14.2635	-6.5736	-12.6430	-2.9613
Method 2	-13.7628	-11.3039	-13.6493	-16.8249
Method 3	-13.8159	-7.6570	-13.1831	-11.1244
The proposed method	-14.8880	-11.5578	-13.8483	-15.9545



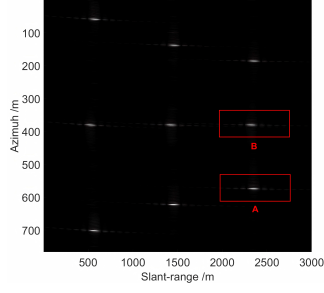
(a)



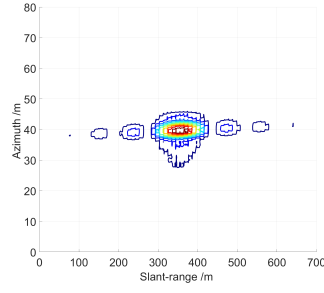
(b)



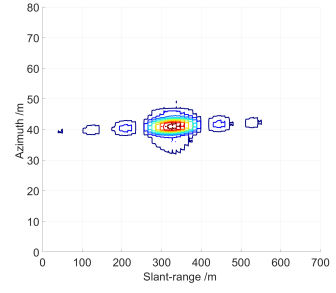
(c)



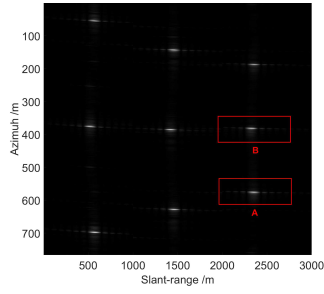
(d)



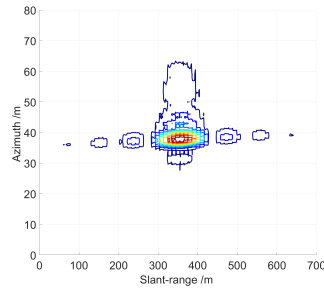
(e)



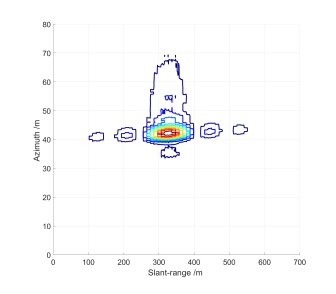
(f)



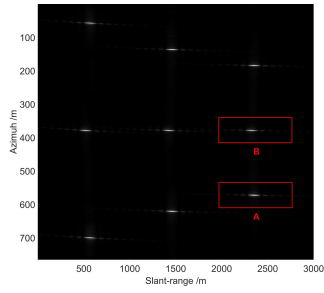
(g)



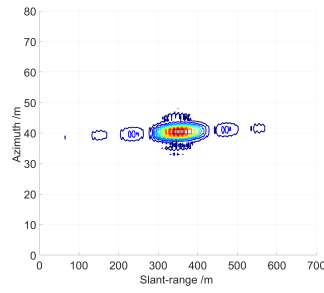
(h)



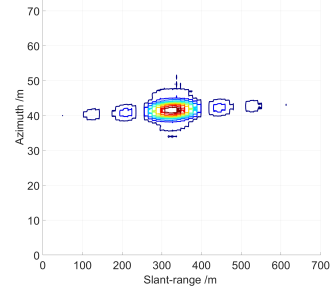
(i)



(j)



(k)



(l)

Figure 7 Simulation results of point target imaging. (a) Results from Method 1. (b) Magnified view of region A in (a). (c) Magnified view of region B in (a). (d)-(f) Corresponding results for Method 2. (g)-(i) Corresponding results for Method 3. (j)-(l) Corresponding results for the proposed method

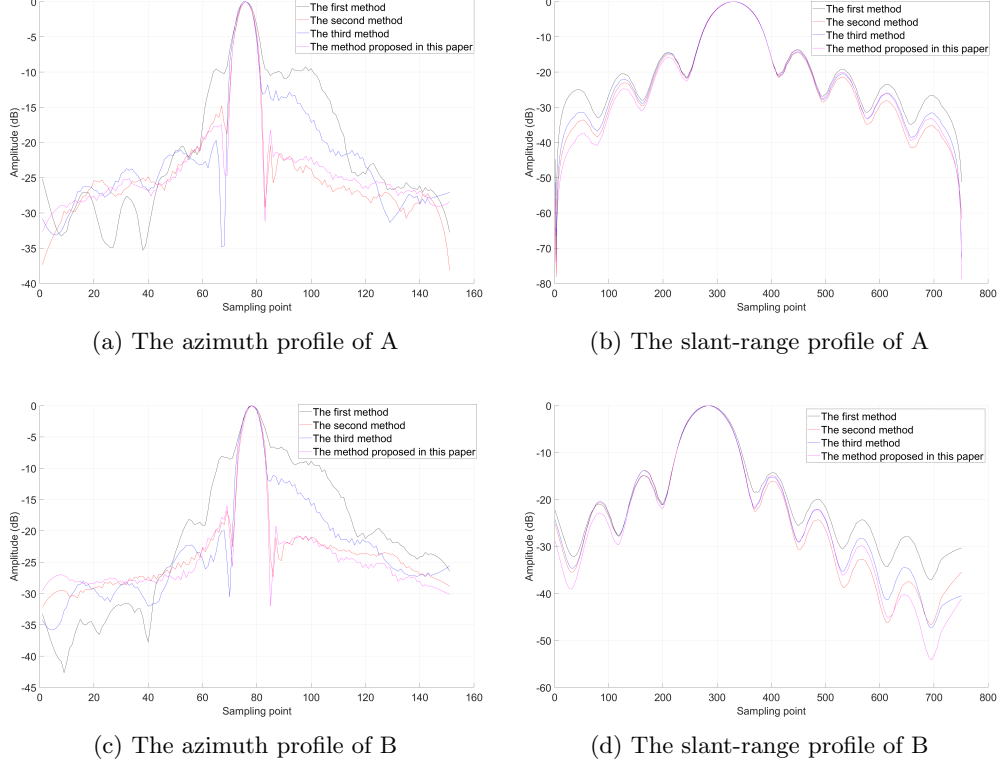


Figure 8 The performance of target A and B

This synergy is quantitatively confirmed by the metrics in Table 3 and Table 4, and visually by the profiles in Figure 8. For Point A's azimuth ISLR, Method 2 improves upon the baseline by 10.22 dB and Method 3 by 7.21 dB. The proposed method achieves a 12.87 dB improvement, demonstrating a super-additive gain. This positive feedback loop—where ASRP provides better input for DPDP, which in turn yields a more accurate phase estimate to guide the next ASRP iteration—is key to the algorithm's performance.

5 Measured Data and Result

Real SAR data were acquired by a UAV over Shijiazhuang, China (see optical image in Figure 10a). The low flight altitude and significant trajectory deviations (Figure 9),

caused by a low-precision IMU, resulted in severe range-dependent phase errors, making this dataset ideal for validation.

The data were processed using all four methods, with results shown in Figure 10. The baseline (b) is severely defocused. Method 2 (c) and Method 3 (d) show partial improvements. The proposed method (e) achieves the best visual quality, with sharp, well-defined contours and textures. For quantitative analysis, strong scatterers, Point A and Point B, were selected from the magnified Regions (1, 2).

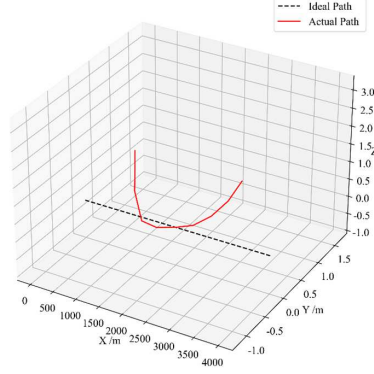


Figure 9 Flight trajectory of the carrier aircraft

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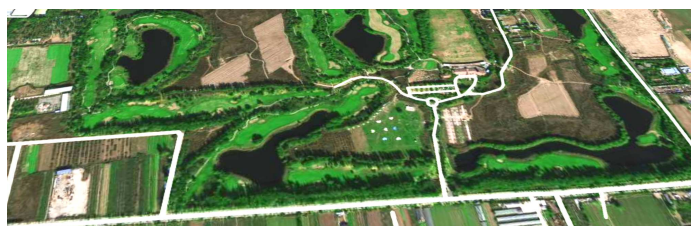
A detailed analysis of the contour maps for Point A and Point B (Figure 11) confirms the superiority of the proposed algorithm. While the other methods show significant energy dispersion and asymmetric sidelobes, the proposed method consistently produces highly compact and symmetric contour patterns, indicating more accurate phase correction.

Image entropy is used for quantitative assessment, where lower values indicate better focus:

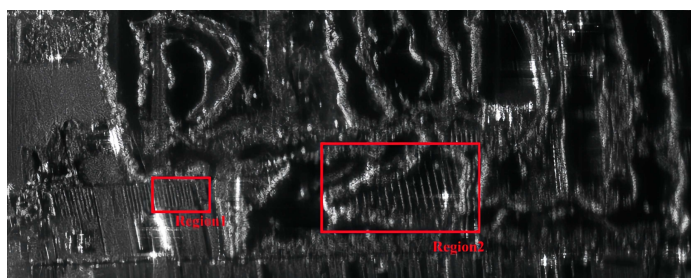
$$\begin{cases} E = -\sum_{i=1}^m \sum_{j=1}^n \rho_{ij} \log \rho_{ij} \\ \rho_{ij} = \frac{|x_{ij}|^2}{C} \\ C = \sum_{i=1}^m \sum_{j=1}^n |x_{ij}|^2 \end{cases} \quad (34)$$

As shown in Table 5, the proposed method achieves the lowest entropy, reducing it by 27.86% for the full scene compared to the baseline, which confirms its superior focusing capability.

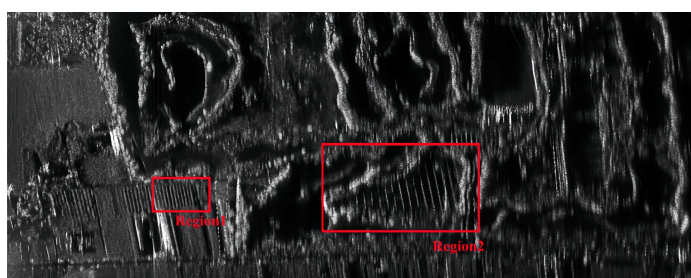
The improved performance comes at the cost of increased computational complexity due to the iterative ASRP and DPDF sampling. This trade-off prioritizes



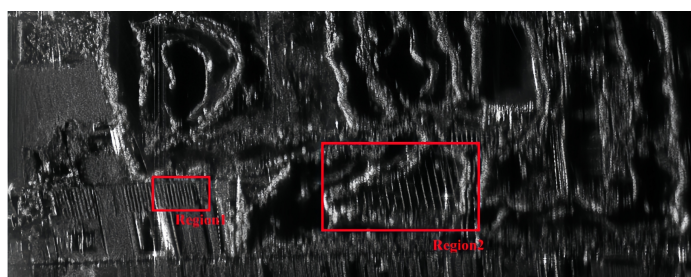
(a) Scene optical image



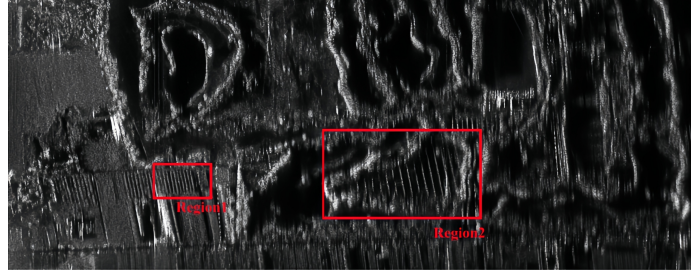
(b) Method 1 processes the result



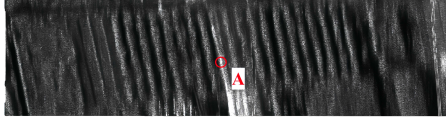
(c) Method 2 processes the result



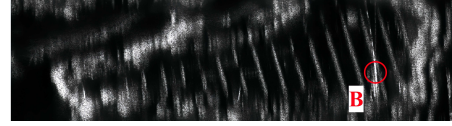
(d) Method 3 processes the result



(e) The proposed method processes the result

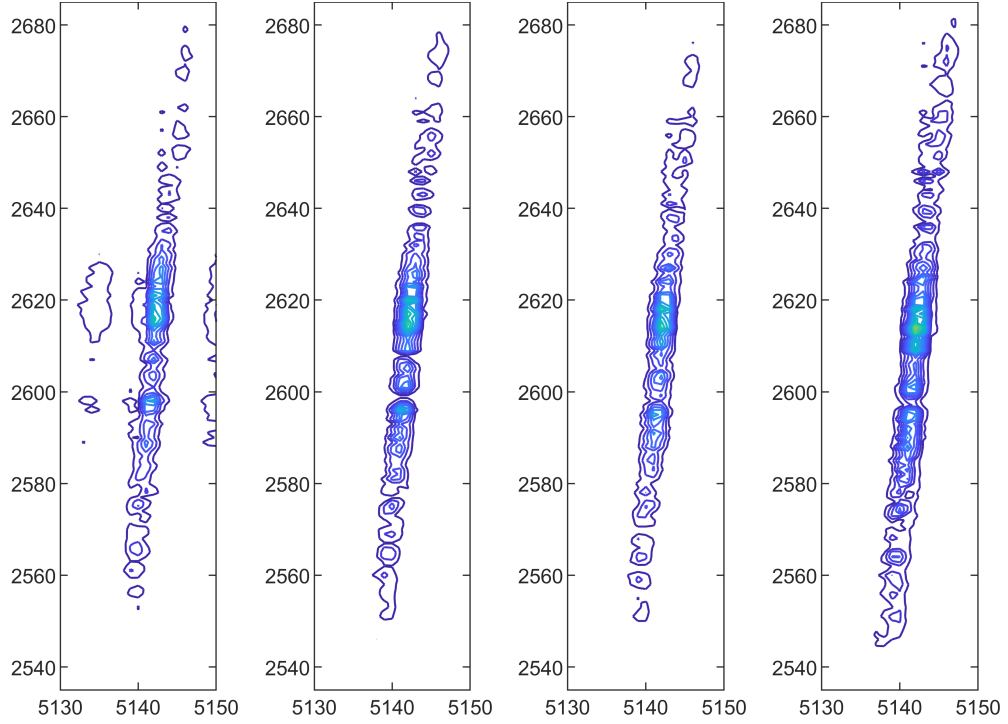


(f) Magnified view of Region 1

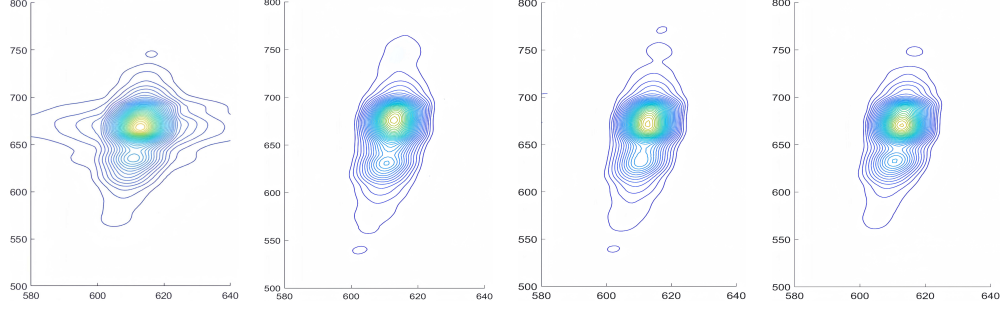


(g) Magnified view of Region 2

Figure 10 Scene image results. (a) Optical image. (b)-(e) SAR images processed by Method 1, Method 2, Method 3, and the proposed method, respectively. (f, g) Magnified views of Region 1 and Region 2.



(a) Imaging results for Point A.



(b) Imaging results for Point B.

Figure 11 Comparison of magnified contour maps for two selected point targets. Columns from left to right correspond to Method 1, Method 2, Method 3, and the proposed Method. (a) Results for Point A. (b) Results for Point B.

focusing accuracy. To further demonstrate robustness, Figure 12 shows that the proposed algorithm achieves a better fit to the true phase error compared to the other methods.

Table 5 Image entropy and relative change statistics

Scene	Method 1	Method 2		Method 3		The proposed method
	E_1	E_2	$\frac{E_1 - E_2}{E_1}$	E_3	$\frac{E_1 - E_3}{E_1}$	E_4
Region1	6.7039	6.3990	4.55%	6.3535	5.23%	6.3169
Region 2	6.7011	6.2476	6.77%	6.2157	7.24%	5.8787
Full scene	0.8726	0.6545	25.00%	0.6975	20.07%	0.6295

6 Conclusions

This paper has addressed the challenging problem of correcting range-dependent phase errors and compensating for NsRCM in SAR systems aboard small UAVs. A novel autofocus algorithm, ASDP-WSPGA, was proposed, which adapts to the 2-D spatially variant characteristics of motion errors through innovative mechanisms for adaptive sub-block adjustment and dynamic sample selection. By leveraging the coherence between APE and NsRCM, the algorithm jointly estimates and corrects these errors. Results from both simulated and measured SAR data demonstrate that the proposed algorithm can achieve efficient and high-precision autofocus for severe, spatially-variant motion errors without navigation assistance.

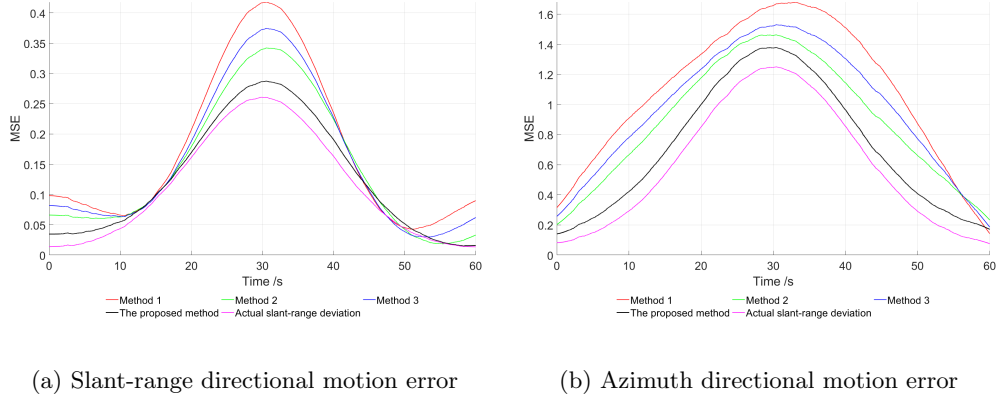


Figure 12 Comparison of Motion Error Results

Declarations

- **Funding:** This work is supported by the Research of Military Internal Scientific Project under grant number KYSJZQZL2023.
- **Conflict of interest:** The authors declare no conflicts of interest.
- **Ethics approval:** Not applicable.
- **Consent to participate:** Not applicable.
- **Consent for publication:** Not applicable.
- **Data availability:** The data that support the findings of this study are available on request from the corresponding author, Yanheng Ma, upon reasonable request.
- **Code availability:** Not applicable.
- **Author contribution:** Conceptualization, X.Z. and Y.M.; methodology, X.Z.; software, X.Z.; validation, X.Z., Y.M. and L.C.; formal analysis, X.Z.; investigation, W.L.; resources, H.H.; data curation, X.Z.; writing—original draft preparation, X.Z.; writing—review and editing, Y.M.; visualization, X.Z.; supervision, Y.M.; project administration, Y.M. All authors have read and agreed to the published version of the manuscript.

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