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A new educational grading system based on fuzzy techniques

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Abstract

In this article, the authors propose a new educational grading system based on fuzzy techniques including five interrelated modules. Through an example used to illustrate the process of the proposed fuzzy educational grading system, we find that the proposed system satisfy the essential needs for educational grading systems as well as can overcome the limitations of some existing fuzzy educational grading systems. After making a discussion between the marks obtained from the proposed fuzzy educational grading system and the marks by the traditional method, we find that the proposed evaluation system can evaluate students' answer sheets in a more objective and intelligent manner. The aim of the proposed method is not to replace existing fuzzy educational grading systems but to enrich the present systems for students' learning achievement evaluation.

Keywords: Evaluation methodologies; Data science applications in education; Educational grading system; Learning achievement; Fuzzy techniques

1. Introduction

The evaluation of students' learning achievement is the process of gathering information from a test that can reflect how well individual students are achieving the curriculum expectations in a course and determining students'

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performance levels in relation to educational objectives. To evaluate a student, one of the important factors to be considered is the teacher's view on the knowledge level of the student, including the presentation skill, the time taken to present the correct answer, the effectiveness of the methods used to derive a particular result, and the accuracy of the results. These factors are subjective by nature. It often occurs that one teacher deems some work to be of high quality, the other may view it as poor. Sometimes, although two teachers agree that some work is of satisfactory quality, they may grade it significantly different, based on their interpretation of what is meant by satisfactory. In order to solve the subjective judging problems of teachers, we can use fuzzy set as a tool, which includes the potential of modeling linguistic variables and hence can provide a mechanism for capturing the uncertainty introduced by subjectivity.

Since its introduction by Zadeh [15], the fuzzy set theory has been widely used in educational grading systems. Biswas [3] presented a fuzzy evaluation method for applying fuzzy sets in students' answer sheets evaluation. Chen and Lee [4] extended the work of Biswas to propose two methods for evaluating students' answer sheets using fuzzy sets, which could overcome the drawbacks of Biswas's work [3] due to the fact that they do not need to perform the complicated matching operations and they can evaluate students' answer sheets in a more fair manner. Echauz and Vachtsevanos [6] presented a fuzzy educational grading system for translating the marks by the traditional method into letter-grades. Law [9] introduced a method for using fuzzy numbers in education grading systems and building the membership functions of several linguistic values with different weights. Ma and Zhou [10] proposed a fuzzy set approach to assessing the outcomes of student-centered learning through encouraging students and teachers to express their views and to agree on the evaluation criteria and evaluation marks in fuzzy terms. Weon and Kim [14] pointed out that the educational grading system should consider three important factors of the questions given to students: the difficulty, the importance, and the complexity. They developed an evaluation strategy based on fuzzy membership functions. Bai and Chen [1] proposed a method for applying fuzzy membership functions and fuzzy inference rules for the same purpose. Saleh and Kim [11] proposed a method for evaluating students' answer sheets through a fuzzy system which considers the difficulty, the importance and the complexity of questions. Wang and Chen [13] presented a method for evaluating students' answer sheets using fuzzy numbers associated with degrees of confidence of teachers. Chen and

Li [5] presented a method for students' answer sheets evaluation based on interval-valued intuitionistic fuzzy sets, where the fuzzy marks awarded to the answers of students' answer sheets are represented by interval-valued intuitionistic fuzzy sets. Salunkhe et al. [12] revisited the Zadeh-Deshpande formalism for evaluating students' answer sheets using the concept of the degree of confidence via the "degree of match" and the fuzzy inference system in students' performance evaluation.

From the previous studies, it can be found that fuzzy numbers, fuzzy sets, fuzzy rules, and fuzzy systems have been used for various existing educational grading systems, which are based on two main kinds of methods, i.e., methods applying fuzzy inference and methods applying only fuzzy arithmetic. On the one hand, fuzzy educational grading systems have some advantages compared with the traditional grading system. First, it includes the potential of modeling linguistic variables and hence can provide a mechanism for capturing the uncertainty introduced by subjectivity. Second, it can reduce the impact of subjectivity in the process of grading. Third, the fuzzy logic and the technologies are developed in view of human intelligence and behavior, and thus have the ability to consider teachers' views and experiences. Fourth, the differences of views and personal mistakes of teachers can be eliminated in a decision process. On the other hand, several limitations of the existing fuzzy educational grading systems should be noted. First, although the optimism index of each teacher to each question has been considered in some studies [5, 13], it was determined subjectively by teachers. Second, the teachers participated in an evaluation were classified into three types based on the given mark matrix. Then the marks were transformed based on the classification of teachers [2]. However, only teachers' influences on marks have been considered. Third, to consider the effect of the quality of answer sheets on marks, four factors from teachers and answer sheets were applied to adjust the original marks, i.e., the accuracy rate, the time rate, the complexity and the importance in some studies [1, 7, 11]. However, there was no formalized way to determine the complexity, the importance and the time rate of each question. Fourth, the effects of teachers and answer sheets to the final marks were considered separately in different studies.

Note that a strategic fuzzy educational grading system not only provides guidance for making important evaluation decisions for teachers, but also ensures that all students receive fair grading so as not to limit students' present and future opportunities. Thus the system should regularly be reviewed and improved to ensure that it is precise, fair, and beneficial to all students. In

the process of designing a fuzzy educational grading system, two aspects are crucial. First, it should be guaranteed that evaluation resources are being used efficiently and effectively. Second, the evaluation system needs the transparency, objectivity, logical inference, and easy computer implementation.

In order to overcome the limitations of the existing fuzzy evaluation methods and satisfy the essential needs for educational grading systems, this study aims to build a new educational grading system based on fuzzy techniques including five interrelated modules. In the first module, the optimism index of each teacher to each question is determined. In the second module, a fuzzy evaluation method is proposed to calculate each student's mark based on the optimism index given in Module 1. In the third module, the fuzzy marks obtained in Module 2 are transformed into normal-type marks according to the classification of teachers. In the fourth module, the fuzzy marks obtained in Module 2 is adjusted through applying four factors from answer sheets in the adjustment. In the last module, the transformed marks and adjusted marks respectively obtained in Modules 3 and 4 are combined to final marks through a Mamdani type fuzzy inference system in fuzzy tool box of MATLAB. An example is used to illustrate the process of the proposed fuzzy educational grading system, through which we find that the proposed system satisfy the essential needs for educational grading systems as well as can overcome the limitations of some existing fuzzy educational grading systems. According a discussion between the marks obtained from the proposed fuzzy educational grading system and the marks by the traditional method, we find that the proposed evaluation system can evaluate students' answer sheets in a more objective and intelligent manner.

The article is organized into five sections. We begin with proposing a new educational grading system based on fuzzy techniques including five interrelated modules in Section 2. We then give an example to illustrate the process of the proposed evaluation system in Section 3. The fourth section makes a discussion of the marks by the proposed fuzzy educational grading system and the marks by the traditional method. We conclude the article with a discussion about the contributions and limitations of the study in the last section.

2. A new educational grading system based on fuzzy techniques

Suppose that there are n teachers $T_1, T_2, \dots, T_n$ to evaluate l answer sheets $A_1, A_2, \dots, A_l$ with m questions $Q_1, Q_2, \dots, Q_m$. Assume the total mark of each answer sheet is 100 with Q_i carrying q_i marks, where $q_i \in (0, 100]$ and $\sum_{i=1}^m q_i = 100$. If the answer sheets are evaluated by the traditional method, then the marks of l answer sheets evaluated by n teachers can be listed as shown in Table 1. Although this method is simple and direct, it ignores some important information from answer sheets. First, each student's mark is directly evaluated by subjective judgments of teachers without considering that some questions are neither correct nor incorrect, which can only be evaluated by the mark of linguistic data. Second, the mark of an answer sheet evaluated by some teacher is calculated by directly accumulating the marks of all questions without considering the optimism index of each teacher to each question in the process of grading. Third, the total mark of each answer sheet is obtained from taking the weighted mean of all marks given by all teachers without considering the types of teachers. Fourth, the number of students getting the same mark may be high, where the knowledge levels, skills sets of two students are really different. How to consider these factors and remove these differences is not considered in the existing literature. In view of there four points, we propose a fuzzy educational grading system including five interrelated modules as follows.

Module 1: *Determining the optimism index of each teacher to each question by the traditional method*

Consider that the traditional mark of each answer sheet evaluated by each teacher is calculated without considering the optimism index of the teacher to the question, we determine the optimism index of each teacher to each question in this module.

Suppose that there are n teachers $T_1, T_2, \dots, T_n$ to evaluate l answer sheets $A_1, A_2, \dots, A_l$ with m questions $Q_1, Q_2, \dots, Q_m$ by the traditional method as shown in Table 1, then we can get a mark matrix $\mathbf{G}$ expressed as

$$\mathbf{G} = \begin{matrix} & \begin{matrix} T_1 & T_2 & \dots & T_n \end{matrix} \\ \begin{matrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{matrix} & \begin{pmatrix} \bar{g}_{11} & \bar{g}_{12} & \dots & \bar{g}_{1n} \\ \bar{g}_{21} & \bar{g}_{22} & \dots & \bar{g}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{g}_{m1} & \bar{g}_{m2} & \dots & \bar{g}_{mn} \end{pmatrix} \end{matrix}$$

Table 1: The marks of l answer sheets with m questions evaluated by n teachers by the traditional method

Answer sheets	T_1		T_2		$\dots$	T_n		Total marks S_h
	Qu.	Marks	Qu.	Marks		Qu.	Marks	
A_1	Q_1	g_{111}	Q_1	g_{112}		Q_1	g_{11n}	$\frac{1}{n} \sum_{j=1}^n \sum_{i=1}^m g_{1ij}$
	Q_2	g_{121}	Q_2	g_{122}		Q_2	g_{12n}	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	Q_m	g_{1m1}	Q_m	g_{1m2}		Q_m	g_{1mn}	
	g_{11}	$\sum_{i=1}^m g_{1i1}$	g_{12}	$\sum_{i=1}^m g_{1i2}$		g_{1n}	$\sum_{i=1}^m g_{1in}$	
A_2	Qu.	Marks	Qu.	Marks		Qu.	Marks	
	Q_1	g_{211}	Q_1	g_{212}		Q_1	g_{21n}	$\frac{1}{n} \sum_{j=1}^n \sum_{i=1}^m g_{2ij}$
	Q_2	g_{221}	Q_2	g_{222}		Q_2	g_{22n}	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	Q_m	g_{2m1}	Q_m	g_{2m2}		Q_m	g_{2mn}	
	g_{21}	$\sum_{i=1}^m g_{2i1}$	g_{22}	$\sum_{i=1}^m g_{2i2}$		g_{2n}	$\sum_{i=1}^m g_{2in}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_l	Qu.	Marks	Qu.	Marks		Qu.	Marks	
	Q_1	g_{l11}	Q_1	g_{l12}		Q_1	g_{l1n}	$\frac{1}{n} \sum_{j=1}^n \sum_{i=1}^m g_{lij}$
	Q_2	g_{l21}	Q_2	g_{l22}		Q_2	g_{l2n}	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
	Q_m	g_{lm1}	Q_m	g_{lm2}		Q_m	g_{lmn}	
	g_{l1}	$\sum_{i=1}^m g_{li1}$	g_{l2}	$\sum_{i=1}^m g_{li2}$		g_{ln}	$\sum_{i=1}^m g_{lin}$	

Note: g_{hj} stands for the mark of the h th answer sheet A_h ($1 \leq h \leq l$) evaluated by the j th teacher T_j ($1 \leq j \leq n$).

where $\bar{g}_{ij} \in [0, 100]$ stands for the mean of the i th question Q_i ($1 \leq i \leq m$) evaluated by the j th teacher T_j ($1 \leq j \leq n$). According to Table 1, we have $\bar{g}_{ij} = \frac{1}{h} \sum_{h=1}^l g_{hij}$.

Based on the mark matrix G , we define the optimism index as

$$\lambda_{ij} = \begin{cases} \frac{\bar{g}_{ij} - \min_j \bar{g}_{ij}}{\max_j \bar{g}_{ij} - \min_j \bar{g}_{ij}}, & \max_j \bar{g}_{ij} \neq \min_j \bar{g}_{ij} \\ 1, & \max_j \bar{g}_{ij} = \min_j \bar{g}_{ij} \end{cases}$$

where λ_{ij} is the optimism index of each teacher T_j to each question Q_i . The optimism index matrix is denoted as $\lambda = (\lambda_{ij})_{m \times n}$.

Module 2: Calculating each student's fuzzy mark based on the optimism index

In the process of grading by the traditional method, each student's mark is directly evaluated by the subjective judgment of the teacher. It is not easy to remove the existing subjectivity from evaluation, especially when the answer sheet is highly subjective by nature. While one teacher deems some work to be of high quality, the other may view it as poor. Although two teachers may agree that the work is of satisfactory quality, they may grade it significantly different, based on their interpretation of what is meant by satisfactory.

Note that fuzzy logic includes the potential of modeling linguistic variables and hence can provide a mechanism for capturing the uncertainty introduced by subjectivity. In order to reduce the impact of subjectivity in the process of grading, we propose a fuzzy evaluation method in this module to calculate each student's mark based on the optimism index of each teacher to each question given in Module 1.

Given nine satisfaction levels to evaluate each question in the answer sheet. The set of satisfaction levels is expressed as $X = \{\text{Extremely Good (EG), Very Good (VG), Good (GD), More or Less Good (MG), Fair (FR), More or Less Bad (MB), Bad (BD), Very Bad (VB), Extremely Bad (EB)}\}$. The degree of satisfaction of each satisfaction level is valued in an interval as shown in Table 2.

Suppose that the h th answer sheet is evaluated by the j th teacher. Assume that the satisfaction degree of each question Q_i of the h th answer sheet evaluated by the teacher T_j is valued in an interval and shown in Table 3.

The steps of calculating fuzzy marks of answer sheets are shown as follows.

Step1 : Based on the optimism index λ_{ij} of the teacher T_j to the question Q_i obtained in Module 1, calculate the expected fuzzy value $(E(x_k))_{ij}$

Table 2: Nine satisfaction levels with the corresponding degrees of satisfaction

Nine satisfaction levels	The corresponding satisfaction degrees
EG	$[0.91, 1]$
VG	$[0.81, 0.9]$
GD	$[0.71, 0.8]$
MG	$[0.61, 0.7]$
FR	$[0.51, 0.6]$
MB	$[0.41, 0.5]$
BD	$[0.25, 0.4]$
VB	$[0.1, 0.24]$
EB	$[0, 0.09]$

Note: We denote the set of satisfaction levels as $X = \{x_1, x_2, \dots, x_9\}$ with the corresponding satisfaction degrees being $[x_1^L, x_1^U], [x_2^L, x_2^U], \dots, [x_9^L, x_9^U]$.

Table 3: The satisfaction degree of each question evaluated by the j th teacher in the h th answer sheet

Question No.	Satisfaction levels								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	x_{h1j_1}	x_{h1j_2}	x_{h1j_3}	x_{h1j_4}	x_{h1j_5}	x_{h1j_6}	x_{h1j_7}	x_{h1j_8}	x_{h1j_9}
Q_2	x_{h2j_1}	x_{h2j_2}	x_{h2j_3}	x_{h2j_4}	x_{h2j_5}	x_{h2j_6}	x_{h2j_7}	x_{h2j_8}	x_{h2j_9}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Q_i	x_{hij_1}	x_{hij_2}	x_{hij_3}	x_{hij_4}	x_{hij_5}	x_{hij_6}	x_{hij_7}	x_{hij_8}	x_{hij_9}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Q_m	x_{hmj_1}	x_{hmj_2}	x_{hmj_3}	x_{hmj_4}	x_{hmj_5}	x_{hmj_6}	x_{hmj_7}	x_{hmj_8}	x_{hmj_9}

Note: $x_{hij_k} = [x_{hij_k}^L, x_{hij_k}^U] (k = 1, 2, \dots, 9)$.

of each satisfaction level x_k decided by the teacher T_j to the question Q_i , where $(E(x_k))_{ij} = (1 - \lambda_{ij})x_k^L + \lambda_{ij}x_k^U$, and then calculate the expected fuzzy value $E(x_{hij_k})$ of each interval value x_{hij_k} of the satisfac-

tion degree of question Q_i , where $E(x_{hij_k}) = (1 - \lambda_{ij})x_{hij_k}^L + \lambda_{ij}x_{hij_k}^U$.

Step2 : Based on the expected fuzzy values $(E(x_k))_{ij}$ and $E(x_{hij_k})$, the degree of satisfaction $D(x_{hij})$ of the question Q_i of the h th answer sheet evaluated by the teacher T_j is calculated as follows:

$$D(x_{hij}) = \frac{\sum_{k=1}^9 E(x_{hij_k})(E(x_k))_{ij}}{\sum_{k=1}^9 E(x_{hij_k})}. \quad (1)$$

Thus $0 \leq D(x_{hij}) \leq 1$ with $1 \leq i \leq m$.

Step3 : Calculate defuzzified crisp mark of each question Q_i in the h th answer sheet evaluated by the teacher T_j , where

$$p_{hij} = D(x_{hij}) \times q_i. \quad (2)$$

Thus the total mark of the student's h th answer sheet evaluated by the j th teacher can be expressed as

$$p_{hj} = \sum_{i=1}^m p_{hij}.$$

Then we obtain a fuzzy mark matrix $\mathbf{P}$ for each teacher on each student

$$\mathbf{P} = \begin{matrix} & \begin{matrix} T_1 & T_2 & \dots & T_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_l \end{matrix} & \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{l1} & p_{l2} & \dots & p_{ln} \end{pmatrix} \end{matrix}$$

The fuzzy total marks vector of dimension $l \times 1$ is

$$\mathbf{S}_F = \left(\frac{1}{n} \sum_{j=1}^n p_{1j}, \frac{1}{n} \sum_{j=1}^n p_{2j}, \dots, \frac{1}{n} \sum_{j=1}^n p_{lj} \right),$$

which is denoted as $\mathbf{S}_F = (\bar{S}_1, \bar{S}_2, \dots, \bar{S}_l)$.

Module 3: Transforming students' fuzzy marks to normal-type marks

The teachers participated in an evaluation can always be classified into three types, i.e., strict-type, lenient-type and normal-type. The marks given

by them are respectively strict-type marks, lenient-type marks and normal-type marks. Since the fuzzy marks obtained in Module 2 are calculated based on the optimism index of each teacher to each question, it will inevitably make some marks strict or lenient. Thus we aim to transform those strict-type or lenient-type marks to normal-type marks in this module.

The steps of transforming students' fuzzy marks to normal-type marks are shown as follows.

Step1 : Based on the fuzzy mark matrix $\mathbf{P}$, calculate the optimism index of each teacher to each answer sheet $\mu = (\mu_{hj})_{l \times n}$, where

$$\mu_{hj} = \begin{cases} \frac{p_{hj} - \min_j p_{pj}}{\max_j p_{hj} - \min_j p_{hj}}, & \max_j p_{hj} \neq \min_j p_{hj} \\ 1, & \max_j p_{hj} = \min_j p_{hj} \end{cases}$$

For each answer sheet A_h , where $1 \leq h \leq l$, classify each teacher into the above-mentioned types according to the value of μ_{hj} as follows:

- (1) If $\mu_{hj} < \frac{1}{3}$, then the teacher T_j is a strict-type teacher and the given mark is a strict-type mark with respect to the answer sheet A_h .
- (2) If $\frac{1}{3} \leq \mu_{hj} \leq \frac{2}{3}$, then the teacher T_j is a normal-type teacher and the given mark is a normal-type mark with respect to the answer sheet A_h .
- (3) If $\mu_{hj} > \frac{2}{3}$, then the teacher T_j is a lenient-type teacher and the given mark is a lenient-type mark with respect to the answer sheet A_h .

Step2 : For each answer sheet A_h , where $1 \leq h \leq l$, count the numbers of strict-type teachers and lenient-type teachers and denote their numbers as r and s . Thus the number of normal-type teachers is $n - r - s$. Let $\tilde{p}_{ht}(1 \leq t \leq r)$ denote the mark of the answer sheet A_h graded by the strict-type teacher and $\tilde{p}_{hu}(1 \leq u \leq s)$ denote the mark of the answer sheet A_h graded by the lenient-type teacher and $\tilde{p}_{hv}(1 \leq v \leq (n - r - s))$ denote the mark of the answer sheet A_h graded by the normal-type teacher. For each answer sheet A_h , calculate the mean $\bar{p}_{S_h}$ of strict-type teachers, the mean $\bar{p}_{L_h}$ of lenient-type teachers and the mean $\bar{p}_{N_h}$ of normal-type teachers,

respectively, where

$$\begin{aligned}\bar{p}_{S_h} &= \frac{\sum_{t=1}^r \tilde{p}_{ht}}{r}, \\ \bar{p}_{L_h} &= \frac{\sum_{u=1}^s \tilde{p}_{hu}}{s}, \\ \bar{p}_{N_h} &= \frac{\sum_{v=1}^{(n-r-s)} \tilde{p}_{hv}}{n-r-s}.\end{aligned}$$

Then the total average strict-type, lenient-type and normal-type marks are expressed as

$$\begin{aligned}\bar{p}_S &= \frac{\sum_{h=1}^l \bar{p}_{S_h}}{l} = \frac{\sum_{h=1}^l \sum_{t=1}^r \tilde{p}_{ht}}{rl}, \\ \bar{p}_L &= \frac{\sum_{h=1}^l \bar{p}_{L_h}}{l} = \frac{\sum_{h=1}^l \sum_{u=1}^s \tilde{p}_{hu}}{sl}, \\ \bar{p}_N &= \frac{\sum_{h=1}^l \bar{p}_{N_h}}{l} = \frac{\sum_{h=1}^l \sum_{v=1}^{(n-r-s)} \tilde{p}_{hv}}{nl - rl - sl}.\end{aligned}$$

Step3 : Use the Lagrange interpolation technique to get the most appropriate relational function between $\bar{p}_{S_h}$ and $\bar{p}_{N_h}$ and that between $\bar{p}_{L_h}$ and $\bar{p}_{N_h}$, respectively, where $1 \leq h \leq l$. Then getting the relational function between $\bar{p}_{S_h}$ and $\bar{p}_{N_h}$ is equal to solving an interpolation polynomial through the points $(0, 0)$, $(100, 100)$ and $(50 + \bar{p}_S - \bar{p}_N, 50)$, and getting the relational function between $\bar{p}_{L_h}$ and $\bar{p}_{N_h}$ is equal to solving an interpolation polynomial through the points $(0, 0)$, $(100, 100)$ and $(50 + \bar{p}_L - \bar{p}_N, 50)$.

Step4 : According to dividing the Y axis by 100 to make the values on the Y axis between 0 and 1, we get the mark membership functions of lenient-type marks and strict-type marks. Based on the constructed mark membership functions, we perform fuzzy inference to transform strict-type and lenient-type marks to normal-type marks. We denote the transformed fuzzy mark matrix $\mathbf{P}'$ for each teacher on each student as

$$\mathbf{P}' = \begin{matrix} & \begin{matrix} T_1 & T_2 & \dots & T_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_l \end{matrix} & \begin{pmatrix} \hat{p}_{11} & \hat{p}_{12} & \dots & \hat{p}_{1n} \\ \hat{p}_{21} & \hat{p}_{22} & \dots & \hat{p}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{p}_{l1} & \hat{p}_{l2} & \dots & \hat{p}_{ln} \end{pmatrix} \end{matrix}$$

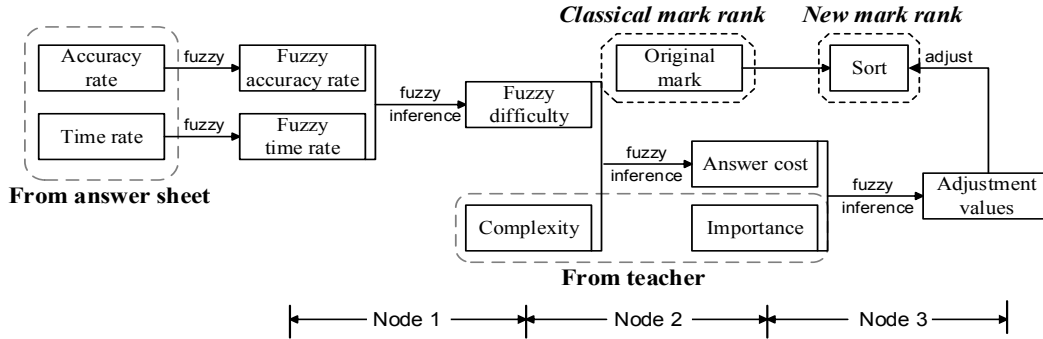


Figure 1: Block diagram of Bai and Chen's mark adjustment system.

Then the transformed total fuzzy marks vector of dimension $l \times 1$ is expressed as

$$\mathbf{S}_T = \left(\frac{1}{n} \sum_{j=1}^n \hat{p}_{1j}, \frac{1}{n} \sum_{j=1}^n \hat{p}_{2j}, \dots, \frac{1}{n} \sum_{j=1}^n \hat{p}_{lj} \right),$$

which is denoted as $\mathbf{S}_T = (\hat{S}_1, \hat{S}_2, \dots, \hat{S}_l)$.

Module 4: An adjustment of students' fuzzy marks

The marks given in the above-mentioned modules are obtained totally from teachers' perspective without considering the effect of the quality of answer sheets on students' marks. There might be a situation where there are several students getting the same mark from the same teacher but their knowledge levels and skills sets are really different. In order to remove these differences, some important factors to evaluate an answer sheet should be considered in the evaluation.

In the work of Weon and Kim [14], they pointed out that an educational grading system should consider three important factors of the questions given to students: the difficulty, the importance, and the complexity. These three factors were applied in Bai and Chen's fuzzy educational grading system [1], whose block diagram is expressed as Figure 1. In this evaluation system, four factors from teachers and answer sheets were applied to adjust the original marks, i.e., the accuracy rate, the time rate, the complexity, and the importance. This system was efficient to make a ranking among students achieving the same mark in the traditional grading system. However, it was

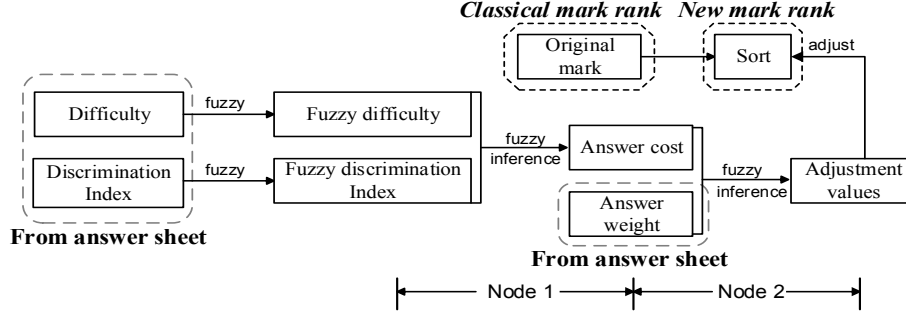


Figure 2: Block diagram of our mark adjustment system.

pointed out in [8] that there exist several problems in Bai and Chen's system. First, this system introduces redundancy in the evaluation process because the complexity and the importance presumably have been already taken into consideration in course of the definition of the original mark. Second, there is no formalized way for a teacher to determine the complexity and the importance of each question. Third, it is not clear how the base time for each question is defined. Meanwhile, it seems not too efficient to measure the answer time of each student to each question.

In this module, in order to avoid the problems in Bai and Chen's system, we construct a new fuzzy educational grading system to adjust the original marks based on some information about the test given in the above-mentioned three modules. The block diagram of the proposed fuzzy evaluation system is expressed as Figure 2.

It is shown that four factors from answer sheets are applied in the adjustment, i.e., the difficulty, the discrimination index, the answer cost, and the answer weight. These factors can be directly determined by the information given in Modules 1-3. In this sense, the proposed evaluation system can be seen as an improved and simplified alternative to Bai and Chen's system.

The steps of operating the system are presented as follows.

Step1 : In Step 3 of Module 2, we obtain the mark of each question Q_i in each answer sheet A_h given by the teacher T_j is p_{hij} . Take the weighted average of the marks given by all teachers as students' marks in each question, we obtain the mark matrix $\mathbf{M}$ for each question on each

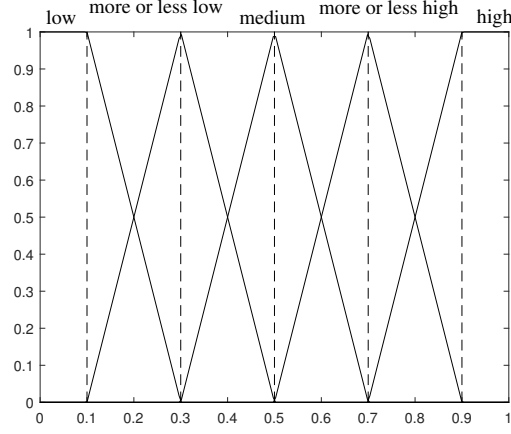


Figure 3: Membership functions of the five fuzzy sets.

answer sheet as follows:

$$\mathbf{M} = \begin{matrix} & \begin{matrix} Q_1 & Q_2 & \dots & Q_m \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_l \end{matrix} & \begin{pmatrix} \frac{1}{n} \sum_{j=1}^n p_{11j} & \frac{1}{n} \sum_{j=1}^n p_{12j} & \dots & \frac{1}{n} \sum_{j=1}^n p_{1mj} \\ \frac{1}{n} \sum_{j=1}^n p_{21j} & \frac{1}{n} \sum_{j=1}^n p_{22j} & \dots & \frac{1}{n} \sum_{j=1}^n p_{2mj} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} \sum_{j=1}^n p_{l1j} & \frac{1}{n} \sum_{j=1}^n p_{l2j} & \dots & \frac{1}{n} \sum_{j=1}^n p_{lmj} \end{pmatrix} \end{matrix}$$

For each question Q_i , where $1 \leq i \leq m$, choose the top 20% of the students who have higher mark to be the “high group”, choose the bottom 20% of the students who have lower mark to be the “low group”. Calculate the means of high group and low group and denote them as $\bar{H}_i$ and $\bar{L}_i$, respectively. Then the difficulty of question Q_i is calculated by

$$d_i = 1 - \frac{\bar{H}_i + \bar{L}_i}{2q_i}, \quad (3)$$

where $0 \leq d_i \leq 1, 1 \leq i \leq m$. Fuzzify each d_i based on five fuzzy sets “low”, “more or less low”, “medium”, “more or less high” and “high” as shown in Figure 3 and calculate its membership value of the difficulty belonging to each fuzzy set, respectively. Then we

obtain the fuzzy difficulty matrix $\mathbf{D}$ as follows:

$$\mathbf{D} = \begin{matrix} & \text{LW} & \text{ML} & \text{ME} & \text{MH} & \text{HI} \\ \begin{matrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{matrix} & \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{m1} & d_{m2} & d_{m3} & d_{m4} & d_{m5} \end{pmatrix} \end{matrix}$$

where $d_{iw} \in [0, 1] (1 \leq i \leq m, 1 \leq w \leq 5)$ denotes the membership value of the difficulty of the i th question Q_i belonging to the five fuzzy sets.

Step2 : Based on the mark matrix $\mathbf{M}$ for each question on each answer sheet, calculate the discrimination index of question Q_i as follows:

$$f_i = \frac{\bar{H}_i - \bar{L}_i}{q_i}, \quad (4)$$

where $0 \leq f_i \leq 1, 1 \leq i \leq m$. Fuzzify each f_i based on the five fuzzy sets shown in Figure 3 and calculate its membership value of the discrimination index belonging to each fuzzy set, respectively. Then we obtain the fuzzy discrimination index matrix $\mathbf{F}$ as follows:

$$\mathbf{F} = \begin{matrix} & \text{LW} & \text{ML} & \text{ME} & \text{MH} & \text{HI} \\ \begin{matrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{matrix} & \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} & f_{15} \\ f_{21} & f_{22} & f_{23} & f_{24} & f_{25} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{m1} & f_{m2} & f_{m3} & f_{m4} & f_{m5} \end{pmatrix} \end{matrix}$$

where $f_{iw} \in [0, 1] (1 \leq i \leq m, 1 \leq w \leq 5)$ denotes the membership value of the discrimination index of the i th question Q_i belonging to the five fuzzy sets.

Step3 : List the fuzzy inference rules $\mathfrak{R}_A$ used for fuzzy inference in Table 4. Based on the fuzzy difficulty matrix $\mathbf{D}$, the fuzzy discrimination index matrix $\mathbf{F}$ and the fuzzy inference rules, perform the fuzzy inference to evaluate the answer cost of each question.

Step4 : Based on the values of d_i and f_i , where $1 \leq i \leq m$, calculate the weight θ_d of the difficulty and the weight θ_f of the discrimination

Table 4: Fuzzy inference rules $\mathfrak{R}_A(\mathfrak{R}_E)$ using for infer the answer cost (adjustment value)

Fuzzy discrimination index	Fuzzy difficulty				
	LW	ML	ME	MH	HI
LW	LW	LW	ML	ML	ME
ML	LW	ML	ML	ME	MH
ME	ML	ML	ME	MH	MH
MH	ML	ME	MH	MH	HI
HI	ME	MH	MH	HI	HI

Note: Fuzzy discrimination index and fuzzy difficulty are respectively fuzzy answer cost and fuzzy answer weight for fuzzy rules $\mathfrak{R}_E$.

index, respectively, shown as follows:

$$\theta_d = \frac{1}{2} \times \left(\frac{1}{2} + \frac{\sum_{i=1}^m \sum_{k=1}^m |d_i - d_k|}{\sum_{i=1}^m \sum_{k=1}^m |d_i - d_k| + \sum_{i=1}^m \sum_{k=1}^m |f_i - f_k|} \right), \quad (5)$$

$$\theta_f = \frac{1}{2} \times \left(\frac{1}{2} + \frac{\sum_{i=1}^m \sum_{k=1}^m |f_i - f_k|}{\sum_{i=1}^m \sum_{k=1}^m |d_i - d_k| + \sum_{i=1}^m \sum_{k=1}^m |f_i - f_k|} \right). \quad (6)$$

Step5 : Let us denote the fuzzy answer cost matrix $\mathbf{A}$ as

$$\mathbf{A} = \begin{matrix} & \begin{matrix} \text{LW} & \text{ML} & \text{ME} & \text{MH} & \text{HI} \end{matrix} \\ \begin{matrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{matrix} & \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & a_{m5} \end{pmatrix} \end{matrix}$$

where $a_{iw} \in [0, 1] (1 \leq i \leq m, 1 \leq w \leq 5)$ denotes the membership value of the answer cost of the i th question Q_i belonging to the five fuzzy sets. For convenience, the five fuzzy sets can be seen as five level: 1 for “low”, 2 for “more or less low”, 3 for “medium”, 4 for “more or less high”, and 5 for “high”. When the level of difficulty, w_d , and the level of discrimination, w_f , are given, the level of answer cost, w , can be determined by the fuzzy inference rules $\mathfrak{R}_A$ given in Table 4. The value of a_{iw} is obtained by

$$a_{iw} = \max_{\{(w_d, w_f) | \mathfrak{R}_A(w_d, w_f) = w\}} \{\theta_d d_{i, w_d} + \theta_f f_{i, w_f}\}, \quad (7)$$

where $1 \leq i \leq m, 0 \leq w \leq 5$. Calculate the degree of answer cost a_i of each question Q_i according to the COG defuzzification method as follows:

$$a_i = \frac{0.1 \times a_{i1} + 0.3 \times a_{i2} + 0.5 \times a_{i3} + 0.7 \times a_{i4} + 0.9 \times a_{i5}}{a_{i1} + a_{i2} + a_{i3} + a_{i4} + a_{i5}}, \quad (8)$$

where $1 \leq i \leq m$.

Step6 : The answer weight of the questions is an important factor to be considered, which is used to measure the impact of each question on the evaluation of each answer sheet. If the difference between the marks of all the answer sheets on a certain question is greater, then the greater the role of the question in the evaluation of the answer sheet, that is, the greater the weight of the question. Based on the mark matrix $\mathbf{M}$ for each question on each answer sheet, the answer weight of question Q_i is calculated by

$$c_i = \frac{1}{2} \times \left(\frac{1}{m} + \frac{\sum_{h=1}^l \sum_{t=1}^l \left| \frac{1}{n} \sum_{j=1}^n p_{hij} - \frac{1}{n} \sum_{j=1}^n p_{tij} \right|}{\sum_{i=1}^m \sum_{h=1}^l \sum_{t=1}^l \left| \frac{1}{n} \sum_{j=1}^n p_{hij} - \frac{1}{n} \sum_{j=1}^n p_{tij} \right|} \right), \quad (9)$$

where $0 \leq c_i \leq 1, 1 \leq i \leq m$. Fuzzify each c_i based on the five fuzzy sets shown in Figure 3 and calculate its membership value of the answer weight belonging to each fuzzy set, respectively. Then we obtain the fuzzy answer weight matrix $\mathbf{C}$ as follows:

$$\mathbf{C} = \begin{matrix} & \begin{matrix} \text{LW} & \text{ML} & \text{ME} & \text{MH} & \text{HI} \end{matrix} \\ \begin{matrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{matrix} & \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{m1} & c_{m2} & c_{m3} & c_{m4} & c_{m5} \end{pmatrix} \end{matrix}$$

where $c_{iw} \in [0, 1] (1 \leq i \leq m, 1 \leq w \leq 5)$ denotes the membership value of the answer weight of the i th question Q_i belonging to the five fuzzy sets.

Step7 : List the fuzzy inference rules $\mathfrak{R}_E$ used for fuzzy inference in Table 4. Based on the fuzzy answer cost matrix $\mathbf{A}$, the fuzzy answer weight matrix $\mathbf{C}$ and fuzzy rules, perform the fuzzy inference to evaluate the adjustment value of each question.

Step8 : Based on the values of a_i and c_i , where $1 \leq i \leq m$, calculate the weight θ_a of the answer cost and the weight θ_c of the answer weight, respectively, shown as follows:

$$\theta_a = \frac{1}{2} \times \left(\frac{1}{2} + \frac{\sum_{i=1}^m \sum_{k=1}^m |a_i - a_k|}{\sum_{i=1}^m \sum_{k=1}^m |a_i - a_k| + \sum_{i=1}^m \sum_{k=1}^m |c_i - c_k|} \right), \quad (10)$$

$$\theta_c = \frac{1}{2} \times \left(\frac{1}{2} + \frac{\sum_{i=1}^m \sum_{k=1}^m |c_i - c_k|}{\sum_{i=1}^m \sum_{k=1}^m |a_i - a_k| + \sum_{i=1}^m \sum_{k=1}^m |c_i - c_k|} \right). \quad (11)$$

Step9 : Let us denote the fuzzy adjustment value matrix $\mathbf{E}$ as

$$\mathbf{E} = \begin{matrix} & \begin{matrix} \text{LW} & \text{ML} & \text{ME} & \text{MH} & \text{HI} \end{matrix} \\ \begin{matrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{matrix} & \begin{pmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e_{m1} & e_{m2} & e_{m3} & e_{m4} & e_{m5} \end{pmatrix} \end{matrix}$$

where $e_{iw} \in [0, 1]$ ($1 \leq i \leq m, 1 \leq w \leq 5$) denotes the membership value of the adjustment value of the i th question Q_i belonging to the five fuzzy sets. When the level of answer cost, w_a , and the level of answer weight, w_c , are given, the level of adjustment value, w , can be determined by the fuzzy inference rules $\mathfrak{R}_E$ given in Table 4. The value of e_{iw} is obtained by

$$e_{iw} = \max_{\{(w_a, w_c) | \mathfrak{R}_E(w_a, w_c) = w\}} \{\theta_a a_{i, w_a} + \theta_c c_{i, w_c}\}, \quad (12)$$

where $1 \leq i \leq m, 0 \leq w \leq 5$. Calculate the degree of adjustment value e_i of each question Q_i according to the COG defuzzification method as follows:

$$e_i = \frac{0.1 \times e_{i1} + 0.3 \times e_{i2} + 0.5 \times e_{i3} + 0.7 \times e_{i4} + 0.9 \times e_{i5}}{e_{i1} + e_{i2} + e_{i3} + e_{i4} + e_{i5}}, \quad (13)$$

where $1 \leq i \leq m$.

Step10 : Based on the adjustment value of each question, we obtain the adjusted mark of question Q_i is

$$\tilde{q}_i = q_i \times (1 + e_i). \quad (14)$$

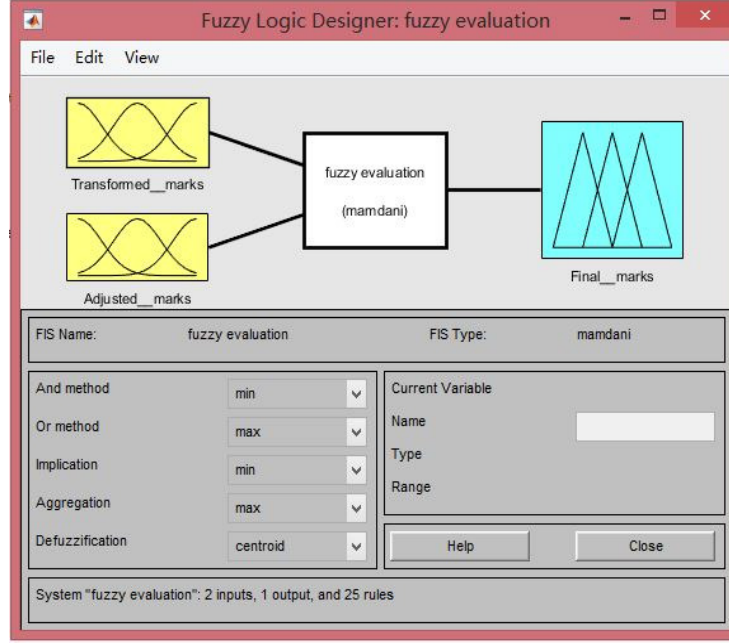


Figure 4: Mamdani fuzzy system created for final marks.

Then we obtain the adjusted total marks vector of dimension $l \times 1$

$$\mathbf{S}_A = \mathbf{M} \times \begin{pmatrix} \frac{1}{q_1} & 0 & \dots & 0 \\ 0 & \frac{1}{q_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{q_m} \end{pmatrix} \times \left(\frac{100\tilde{q}_1}{\sum_{i=1}^m \tilde{q}_i}, \frac{100\tilde{q}_2}{\sum_{i=1}^m \tilde{q}_i}, \dots, \frac{100\tilde{q}_m}{\sum_{i=1}^m \tilde{q}_i} \right)^T,$$

which is denoted as $\mathbf{S}_A = (\tilde{S}_1, \tilde{S}_2, \dots, \tilde{S}_l)$.

Module 5: Combining transformed marks and adjusted marks to final marks

In Modules 3 and 4, we obtained transformed marks and adjusted marks through considering the effect of teachers and answer sheets on fuzzy marks obtained in Module 2. In order to combine the obtained transformed marks and adjusted marks to final marks, we apply a Mamdani type fuzzy inference system for inference in this module. Using fuzzy tool box of MATLAB, two input-one output Mamdani fuzzy system are created as shown in Figure 4.

In this fuzzy system, each of the input and output variable is fuzzified with five linguistic variables (i.e., “low”, “more or less low”, “medium”, “more or

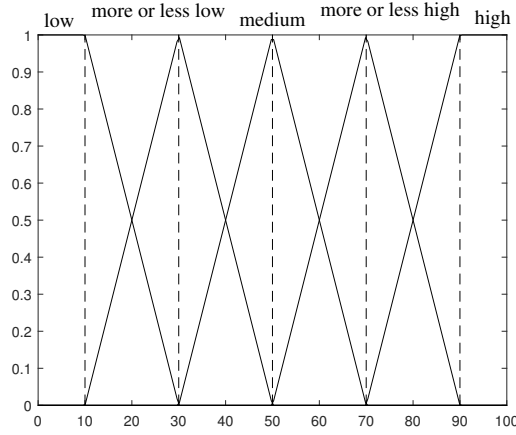


Figure 5: Membership functions of for input and output variables.

less high” and “high”) and given a universe of discourse of $[0, 100]$. A view of input membership functions for input variables, i.e., the transformed mark and the adjusted mark, are shown in Figure 5.

This study considers a total of 25 if-then fuzzy inference rules for all the controllers. A view of if-then fuzzy inference rules for final marks analysis are shown in Table 5. The fuzzy inference process for transformed mark of 72 and adjusted mark of 80 generating final mark of 77.7 is depicted in Figure 6. A surface viewer for final marks analysis is shown in Figure 7.

Table 5: If-then fuzzy inference rules for final marks

Adjusted marks	Transformed marks				
	LW	ML	ME	MH	HI
LW	LW	ML	ME	ME	MH
ML	ML	ML	ME	MH	MH
ME	ME	ME	ME	MH	HI
MH	ME	MH	MH	MH	HI
HI	MH	MH	HI	HI	HI

The formulation for the proposed fuzzy educational grading system containing five interrelated modules is shown as Figure 8.

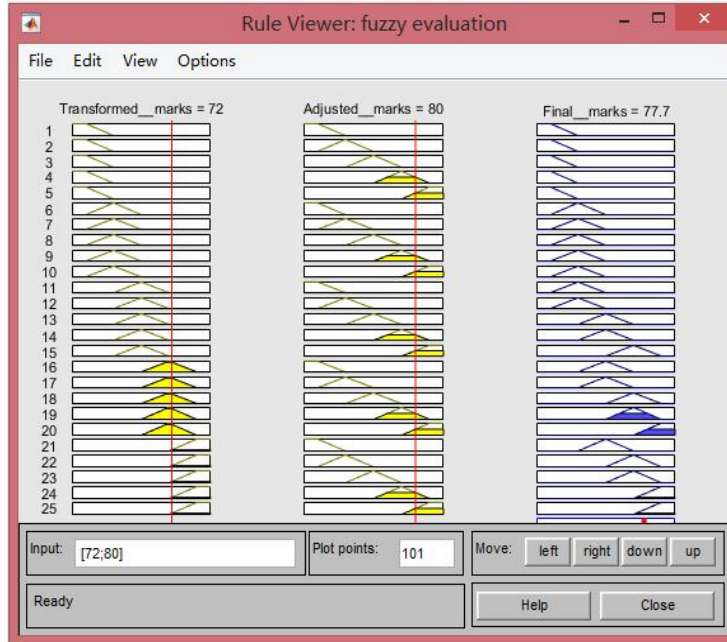


Figure 6: The fuzzy inference process.

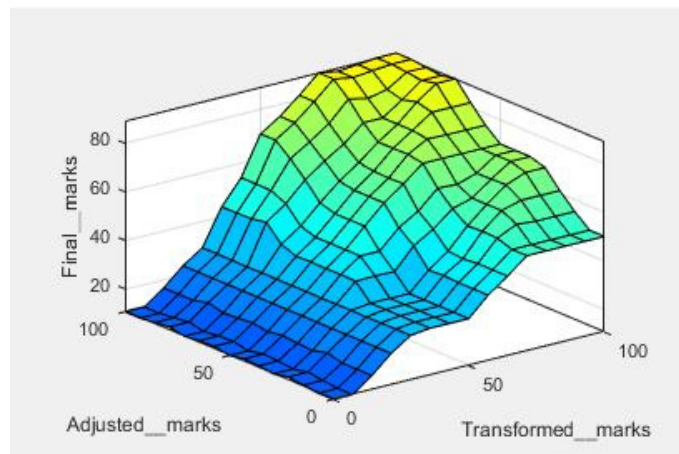


Figure 7: The surface viewer for final marks.

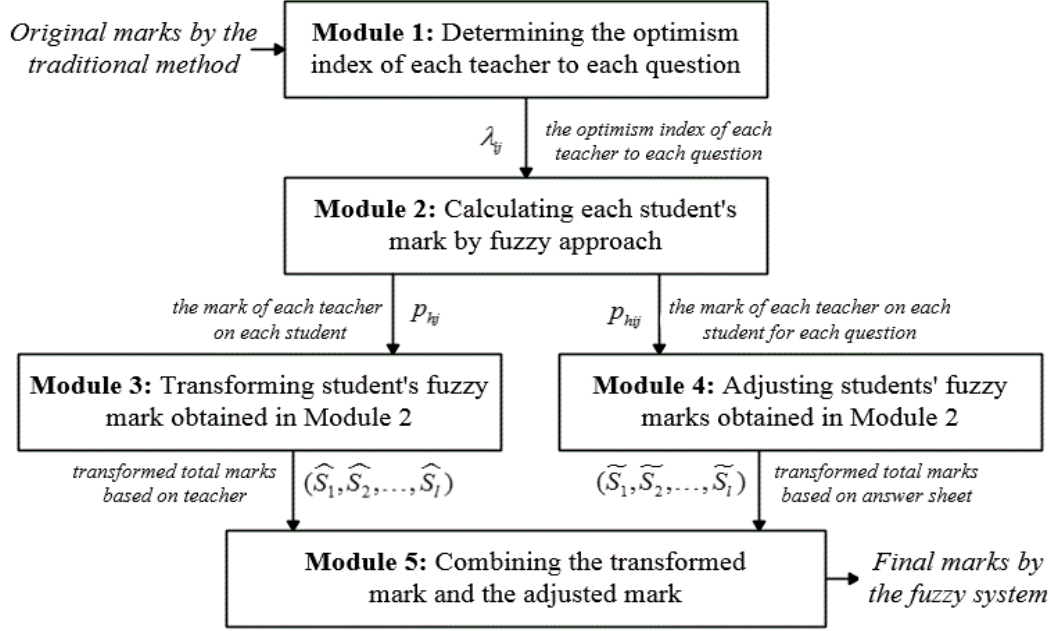


Figure 8: The formulation of the proposed fuzzy educational grading system.

3. An example to illustrate the process of the proposed fuzzy educational grading system

In this section, we use an example to illustrate the process of the proposed fuzzy educational grading system. Suppose that there are 8 teachers $T_1, T_2, \dots, T_8$ to evaluate 10 answer sheets $A_1, A_2, \dots, A_{10}$ with 5 questions Q_1, Q_2, Q_3, Q_4, Q_5 shown in Table A.1 in the appendix.

Module 1: *Determining the optimism index of each teacher to each question by the traditional method*

According to Table A.1, we get the mark matrix $\mathbf{G}$ listing the mean of the i th question $Q_i (1 \leq i \leq 5)$ evaluated by the j th teacher $T_j (1 \leq j \leq 8)$ as follows:

$$\mathbf{G} = \begin{matrix} & \begin{matrix} T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 \end{matrix} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \left(\begin{array}{ccccccccc} 6.8 & 7.0 & 7.3 & 7.6 & 7.9 & 8.4 & 8.6 & 8.9 \\ 10.1 & 10.6 & 11.1 & 11.4 & 12.0 & 12.2 & 12.6 & 12.9 \\ 13.0 & 14.1 & 14.6 & 15.1 & 15.7 & 16.1 & 16.7 & 17.3 \\ 16.6 & 17.7 & 18.3 & 19.1 & 19.7 & 20.6 & 21.1 & 21.5 \\ 19.5 & 20.9 & 21.4 & 21.9 & 22.5 & 23.1 & 23.6 & 24.3 \end{array} \right) \end{matrix}$$

Based on the mark matrix $\mathbf{G}$, calculate the optimism index of the teacher T_j to the question Q_i shown as follows:

$$\lambda = \begin{matrix} & T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{pmatrix} 0 & 0.12 & 0.26 & 0.38 & 0.55 & 0.76 & 0.88 & 1.0 \\ 0 & 0.19 & 0.37 & 0.46 & 0.67 & 0.75 & 0.89 & 1.0 \\ 0 & 0.26 & 0.36 & 0.48 & 0.62 & 0.71 & 0.86 & 1.0 \\ 0 & 0.22 & 0.34 & 0.51 & 0.64 & 0.81 & 0.92 & 1.0 \\ 0 & 0.29 & 0.41 & 0.51 & 0.64 & 0.75 & 0.86 & 1.0 \end{pmatrix} \end{matrix}$$

Module 2: Calculating each student's fuzzy mark based on the optimism index

Based on the optimism index of the teacher T_j to the question Q_i , calculate the expected fuzzy value $(E(x_k))_{ij}$ of each satisfaction level $x_k (1 \leq k \leq 9)$ decided by the teacher T_j to the question Q_i shown as Table A.2 in the appendix. Because of limitations of space, we only list the process of calculating the first student's fuzzy mark here. The process of calculating other students' fuzzy marks can be conducted in the same manner.

Based on the marks obtained by the traditional method shown in Table A.1, give the satisfaction degree of each question evaluated by each teacher in A_1 shown as Table A.3 in the appendix. Based on the optimism index of the teacher T_j to the question Q_i , calculate the expected fuzzy value $E(x_{1ij_k})$ of each interval value x_{1ij_k} of the satisfaction degree of question Q_i for A_1 , which is shown as Table A.4 in the appendix.

Based on the expected fuzzy values $(E(x_k))_{ij}$ and $E(x_{1ij_k})$, calculate the degree of satisfaction $D(x_{1ij})$ and the defuzzified crisp mark p_{1ij} of each question Q_i evaluated by each teacher T_j through Equations (1) and (2) respectively, which is shown as Table 6. The total mark of the first student's answer sheet can also be obtained in this table.

The marks of other students' answer sheets can be calculated in the same manner. Then the fuzzy mark matrix $\mathbf{P}$ for each teacher on each student is

Table 6: The degree of satisfaction $D(x_{1ij})$ and the defuzzified crisp mark p_{1ij}

Question No.	Teachers participating in the evaluation							
	T_1		T_2		T_3		T_4	
	$D(x_{1i1})$	p_{1i1}	$D(x_{1i2})$	p_{1i2}	$D(x_{1i3})$	p_{1i3}	$D(x_{1i4})$	p_{1i4}
Q_1	0.18	1.75	0.24	2.45	0.32	3.19	0.37	3.74
Q_2	0.20	2.93	0.25	3.82	0.31	4.63	0.38	5.74
Q_3	0.21	4.23	0.26	5.27	0.37	7.34	0.39	7.71
Q_4	0.19	4.76	0.26	6.47	0.32	8.01	0.39	9.63
Q_5	0.18	5.37	0.27	8.06	0.34	10.12	0.41	12.25
The total mark	19		26		33		39	

Question No.	Teachers participating in the evaluation							
	T_5		T_6		T_7		T_8	
	$D(x_{1i5})$	p_{1i5}	$D(x_{1i6})$	p_{1i6}	$D(x_{1i7})$	p_{1i7}	$D(x_{1i8})$	p_{1i8}
Q_1	0.45	4.47	0.53	5.28	0.54	5.39	0.55	5.50
Q_2	0.44	6.57	0.50	7.50	0.54	8.10	0.55	8.25
Q_3	0.43	8.64	0.51	10.13	0.54	10.74	0.55	11.00
Q_4	0.42	10.59	0.50	12.38	0.53	13.15	0.55	13.75
Q_5	0.43	13.03	0.50	15.00	0.52	15.63	0.55	16.50
The total mark	43		50		53		55	

Note: Assuming that no half mark is given in the total mark.

expressed as

$$\mathbf{P} = \begin{matrix} & \begin{matrix} T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{matrix} & \left(\begin{matrix} 19 & 26 & 33 & 39 & 43 & 50 & 53 & 55 \\ 30 & 45 & 50 & 52 & 57 & 63 & 67 & 72 \\ 57 & 64 & 66 & 68 & 72 & 76 & 79 & 84 \\ 55 & 61 & 68 & 74 & 77 & 81 & 85 & 88 \\ 68 & 73 & 76 & 79 & 86 & 89 & 91 & 95 \\ 69 & 75 & 78 & 83 & 86 & 88 & 92 & 94 \\ 73 & 79 & 82 & 85 & 88 & 91 & 95 & 97 \\ 77 & 82 & 85 & 87 & 91 & 93 & 94 & 96 \\ 78 & 84 & 86 & 88 & 92 & 94 & 96 & 98 \\ 82 & 89 & 90 & 92 & 94 & 97 & 99 & 100 \end{matrix} \right) \end{matrix}$$

Then the fuzzy total marks vector is obtained as follows:

$$\mathbf{S}_F = (40, 55, 71, 74, 82, 83, 86, 88, 90, 93).$$

Module 3: *Transforming students' fuzzy marks to normal-type marks*

Based on the fuzzy mark matrix $\mathbf{P}$, calculate the optimism index of each teacher to each answer sheet shown as follows:

$$\mu = \begin{matrix} & T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{matrix} & \left(\begin{array}{ccccccccc} 0 & 0.19 & 0.39 & 0.56 & 0.67 & 0.86 & 0.94 & 1.00 \\ 0 & 0.36 & 0.48 & 0.52 & 0.64 & 0.79 & 0.88 & 1.00 \\ 0 & 0.26 & 0.33 & 0.41 & 0.56 & 0.70 & 0.81 & 1.00 \\ 0 & 0.18 & 0.39 & 0.58 & 0.67 & 0.79 & 0.91 & 1.00 \\ 0 & 0.19 & 0.30 & 0.41 & 0.67 & 0.78 & 0.85 & 1.00 \\ 0 & 0.24 & 0.36 & 0.56 & 0.68 & 0.76 & 0.92 & 1.00 \\ 0 & 0.25 & 0.38 & 0.50 & 0.63 & 0.75 & 0.92 & 1.00 \\ 0 & 0.26 & 0.42 & 0.53 & 0.74 & 0.84 & 0.89 & 1.00 \\ 0 & 0.30 & 0.40 & 0.50 & 0.70 & 0.80 & 0.90 & 1.00 \\ 0 & 0.39 & 0.44 & 0.56 & 0.67 & 0.83 & 0.94 & 1.00 \end{array} \right) \end{matrix}$$

For each answer sheet A_h , where $1 \leq h \leq 10$, classify each teacher into the three types according to the value of μ_{hj} . The classification results are shown in Table 7.

For each answer sheet A_h , calculate respectively the mean $\bar{p}_{S_h}$ of strict-type teachers, the mean $\bar{p}_{L_h}$ of lenient-type teachers and the mean $\bar{p}_{N_h}$ of normal-type teachers according to Table 7 and the obtained results are shown in Table 8. Then the total average strict-type, lenient-type and normal-type marks are respectively $\bar{p}_S = 63.6$, $\bar{p}_L = 84.1$ and $\bar{p}_N = 73.6$. Then we can take the points $(0, 0)$, $(100, 100)$ and $(40, 50)$ to fit the curve representing the relational function between $\bar{p}_{S_h}$ and $\bar{p}_{N_h}$ and the points $(0, 0)$, $(100, 100)$ and $(60.5, 50)$ to fit the curve representing the relational function between $\bar{p}_{L_h}$ and $\bar{p}_{N_h}$.

By applying the Lagrange interpolation technique, we get the following equations

$$\begin{aligned} \bar{p}_{N_h} &= -\bar{p}_{S_h}^2/240 + 17\bar{p}_{S_h}/12, \\ \bar{p}_{N_h} &= 42\bar{p}_{L_h}^2/9559 + 5359\bar{p}_{L_h}/9559. \end{aligned}$$

Then the relationship between strict-type marks and normal-type marks and that between lenient-type marks and normal-type marks are respectively

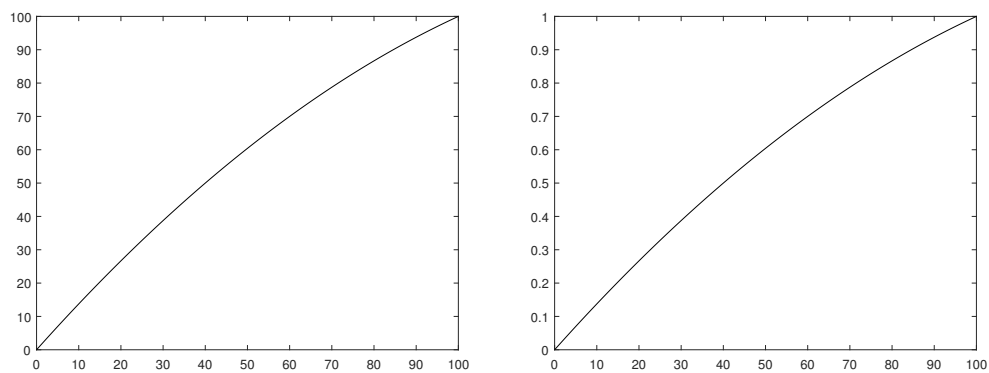


Figure 9: The relationship between strict-type marks and normal-type marks as well as the mark membership functions of the strict-type marks.

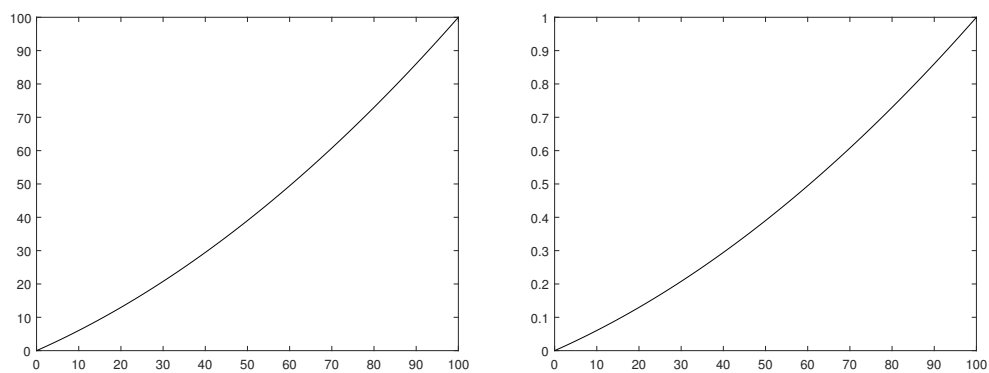


Figure 10: The relationship between lenient-type marks and normal-type marks as well as the mark membership functions of the lenient-type marks.

Table 7: The classification results of the three types of teachers in each answer sheet

Answer sheets	The classification results of the three types of teachers		
	Strict-type teacher	Normal-type teacher	Lenient-type teacher
A_1	T_1, T_2	T_3, T_4	T_5, T_6, T_7, T_8
A_2	T_1	T_2, T_3, T_4, T_5	T_6, T_7, T_8
A_3	T_1, T_2, T_3	T_4, T_5	T_6, T_7, T_8
A_4	T_1, T_2	T_3, T_4	T_5, T_6, T_7, T_8
A_5	T_1, T_2, T_3	T_4	T_5, T_6, T_7, T_8
A_6	T_1, T_2	T_3, T_4	T_5, T_6, T_7, T_8
A_7	T_1, T_2	T_3, T_4, T_5	T_6, T_7, T_8
A_8	T_1, T_2	T_3, T_4	T_5, T_6, T_7, T_8
A_9	T_1, T_2	T_3, T_4	T_5, T_6, T_7, T_8
A_{10}	T_1	T_2, T_3, T_4	T_5, T_6, T_7, T_8

Table 8: The means of the three types of teachers in each answer sheet

Answer sheets	The average marks of the three types of teachers		
	Strict-type $\bar{p}_{S_h}$	Normal-type $\bar{p}_{N_h}$	Lenient-type $\bar{p}_{L_h}$
A_1	22.5	36.0	50.3
A_2	30.0	51.0	67.3
A_3	62.3	70.0	79.7
A_4	58.0	71.0	82.8
A_5	72.3	79.0	90.3
A_6	72.0	80.5	90.0
A_7	76.0	85.0	94.3
A_8	79.5	86.0	93.5
A_9	81.0	87.0	95.0
A_{10}	82.0	90.3	97.5

shown in Figures 9 and 10. The mark membership functions of the strict-type

marks and lenient-type marks are also shown.

Based on the constructed mark membership functions, we perform fuzzy inference to transform strict-type and lenient-type marks to normal-type marks and the obtained transformed fuzzy mark matrix $\mathbf{P}'$ is shown as

$$\mathbf{P}' = \begin{matrix} & \begin{matrix} T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{matrix} & \begin{pmatrix} 25 & 34 & 33 & 39 & 32 & 39 & 42 & 44 \\ 39 & 45 & 50 & 52 & 57 & 53 & 57 & 63 \\ 67 & 74 & 75 & 68 & 72 & 68 & 72 & 78 \\ 65 & 71 & 68 & 74 & 69 & 74 & 79 & 83 \\ 77 & 81 & 84 & 79 & 81 & 85 & 87 & 93 \\ 78 & 83 & 78 & 83 & 81 & 83 & 89 & 92 \\ 81 & 86 & 82 & 85 & 88 & 87 & 93 & 96 \\ 84 & 88 & 85 & 87 & 87 & 90 & 92 & 94 \\ 85 & 90 & 86 & 88 & 89 & 92 & 94 & 97 \\ 88 & 89 & 90 & 92 & 92 & 96 & 99 & 100 \end{pmatrix} \end{matrix}$$

Then the transformed total marks vector is obtained as follows:

$$\mathbf{S}_T = (36, 52, 72, 73, 83, 83, 87, 88, 90, 93).$$

Module 4: *An adjustment of students' fuzzy marks*

Take the weighted average of the marks given by all teachers in Step 3 of Module 2 as students' marks in each question, we obtain the mark matrix $\mathbf{M}$ for each question on each answer sheet as follows:

$$\mathbf{M} = \begin{matrix} & \begin{matrix} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{matrix} & \begin{pmatrix} 4.0 & 5.9 & 8.1 & 9.8 & 12.0 \\ 5.9 & 8.3 & 11.8 & 13.5 & 14.9 \\ 6.7 & 10.2 & 13.3 & 17.7 & 22.7 \\ 7.3 & 10.6 & 16.0 & 18.7 & 20.9 \\ 8.5 & 12.7 & 15.7 & 21.5 & 23.7 \\ 8.4 & 13.0 & 16.8 & 21.4 & 23.5 \\ 8.6 & 13.8 & 17.7 & 21.0 & 25.0 \\ 9.0 & 13.3 & 17.6 & 22.5 & 25.6 \\ 8.9 & 13.2 & 17.7 & 22.9 & 26.8 \\ 9.5 & 14.1 & 18.3 & 23.4 & 27.6 \end{pmatrix} \end{matrix}$$

Based on the mark matrix $\mathbf{M}$, calculate the difficulty of each question Q_i by Equation (3) and then we have $d_1 = 0.29, d_2 = 0.31, d_3 = 0.30, d_4 = 0.30$

and $d_5 = 0.32$. Fuzzify each d_i based on the five fuzzy sets shown in Figure 3 and then obtain the fuzzy difficulty matrix $\mathbf{D}$ as follows:

$$\mathbf{D} = \begin{matrix} & \text{LW} & \text{ML} & \text{ME} & \text{MHHI} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{pmatrix} 0.05 & 0.95 & 0 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0.90 & 0.10 & 0 & 0 \end{pmatrix} \end{matrix}$$

Based on the mark matrix $\mathbf{M}$, calculate the discrimination index of question Q_i by Equation (4) and then we have $f_1 = 0.43, f_2 = 0.44, f_3 = 0.40, f_4 = 0.46$ and $f_5 = 0.46$. Fuzzify each f_i based on the five fuzzy sets shown in Figure 3 and then obtain the fuzzy discrimination index matrix $\mathbf{F}$ as follows:

$$\mathbf{F} = \begin{matrix} & \text{LW} & \text{ML} & \text{ME} & \text{MHHI} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{pmatrix} 0 & 0.35 & 0.65 & 0 & 0 \\ 0 & 0.30 & 0.70 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0.20 & 0.80 & 0 & 0 \\ 0 & 0.20 & 0.80 & 0 & 0 \end{pmatrix} \end{matrix}$$

Based on the values of d_i and f_i , calculate the weight θ_d of the difficulty and the weight θ_f of the discrimination index respectively by Equations (5) and (6) and obtain $\theta_d = 0.42$ and $\theta_f = 0.58$.

Based on the fuzzy difficulty matrix $\mathbf{D}$ and fuzzy discrimination index matrix $\mathbf{F}$, the fuzzy inference rules shown in Table 4 and Equation (7), we perform fuzzy inference to evaluate the answer cost of each question to get the fuzzy answer cost matrix $\mathbf{A}$ as

$$\mathbf{A} = \begin{matrix} & \text{LW} & \text{ML} & \text{ME} & \text{MH} & \text{HI} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{pmatrix} 0.40 & 0.78 & 0.40 & 0.40 & 0 \\ 0.40 & 0.81 & 0.43 & 0.41 & 0 \\ 0.42 & 0.71 & 0.42 & 0.42 & 0 \\ 0.42 & 0.89 & 0.46 & 0.46 & 0 \\ 0.38 & 0.84 & 0.51 & 0.46 & 0 \end{pmatrix} \end{matrix}$$

Based on the fuzzy answer cost matrix $\mathbf{A}$ and Equation (8), we calculate the degree of answer cost a_i of each question Q_i and obtain $a_1 = 0.38, a_2 = 0.38, a_3 = 0.39, a_4 = 0.39$ and $a_5 = 0.40$.

Based on the mark matrix $\mathbf{M}$, calculate the answer weight of question Q_i by Equation (9) and then we have $c_1 = 0.15, c_2 = 0.18, c_3 = 0.19, c_4 = 0.23$ and $c_5 = 0.25$. Fuzzify each c_i based on the five fuzzy sets shown in Figure 3 and then obtain the fuzzy answer weight matrix $\mathbf{C}$ as follows:

$$\mathbf{C} = \begin{matrix} & \text{LW} & \text{ML} & \text{MEM} & \text{HHI} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{pmatrix} 0.75 & 0.25 & 0 & 0 & 0 \\ 0.60 & 0.40 & 0 & 0 & 0 \\ 0.55 & 0.45 & 0 & 0 & 0 \\ 0.35 & 0.65 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Based on the values of a_i and c_i , calculate the weight θ_a of the answer cost and the weight θ_c of the answer weight respectively by Equations (10) and (11) and obtain $\theta_a = 0.33$ and $\theta_c = 0.67$.

Based on the fuzzy answer cost matrix $\mathbf{A}$ and fuzzy answer weight matrix $\mathbf{C}$, the fuzzy inference rules shown in Table 4 and Equation (12), we perform fuzzy inference to evaluate the adjustment value of each question to get the fuzzy adjustment value matrix $\mathbf{E}$ as

$$\mathbf{E} = \begin{matrix} & \text{LW} & \text{ML} & \text{ME} & \text{MH} & \text{HI} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{pmatrix} 0.76 & 0.63 & 0.50 & 0.26 & 0.13 \\ 0.67 & 0.54 & 0.40 & 0.27 & 0.14 \\ 0.60 & 0.54 & 0.44 & 0.30 & 0.14 \\ 0.57 & 0.73 & 0.59 & 0.44 & 0.15 \\ 0.63 & 0.78 & 0.65 & 0.50 & 0.15 \end{pmatrix} \end{matrix}$$

Based on the fuzzy answer cost matrix $\mathbf{E}$ and Equation (13), we calculate the degree of adjustment value e_i of each question Q_i and obtain $e_1 = 0.36, e_2 = 0.37, e_3 = 0.38, e_4 = 0.41$ and $e_5 = 0.41$.

According to Equation (14), we obtain the adjusted marks of each question Q_i are respectively $\tilde{q}_1 = 13.6, \tilde{q}_2 = 20.5, \tilde{q}_3 = 27.7, \tilde{q}_4 = 35.2$ and $\tilde{q}_5 = 42.3$. Then we obtain the adjusted total marks vector as follows:

$$\mathbf{S}_A = (39, 53, 69, 72, 80, 81, 84, 86, 87, 91).$$

Module 5: Combining transformed marks and adjusted marks to final marks

In Modules 3 and 4, we obtained transformed marks and adjusted marks through considering the effect of teachers and answer sheets on fuzzy marks

obtained in Module 2. In this module, we apply a Mamdani type fuzzy inference system to combine these two types of marks to final marks through the fuzzy tool box of MATLAB. In the applied fuzzy system, each of the input and output variable is fuzzified with five linguistic variables shown in Figure 5. Table 9 shows all the final marks combined by transformed marks and adjusted marks in the Mamdani type fuzzy inference system.

Table 9: The marks by fuzzy evaluation method and traditional method

Answer sheets	The marks by fuzzy evaluation method				Traditional marks
	Former fuzzy marks	Transformed marks	Adjusted marks	Our final marks	
A_1	40	36	39	39	40
A_2	55	52	53	54	54
A_3	71	72	69	71	70
A_4	74	73	72	72	73
A_5	82	83	80	79	82
A_6	83	83	81	80	83
A_7	86	87	84	84	86
A_8	88	88	86	86	88
A_9	90	90	87	89	90
A_{10}	93	93	91	90	95
The mean	76	76	74	74	76
Standard deviation	16.2	17.5	15.8	15.6	16.5

4. A discussion of the marks by the proposed fuzzy educational grading system and traditional marks

According to Table 9 and Figure 11, we observe the comparison between the marks by fuzzy evaluation methods and the marks by the traditional method.

In Module 2, for fuzzy marks around the mean 76, their values are equal to the marks by the traditional method. It means that the fuzzy marks can reflect the subjective judgment of teachers to a certain extent. For fuzzy

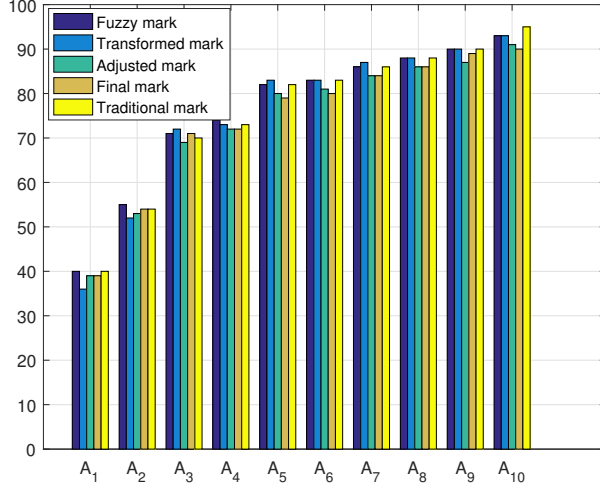


Figure 11: A comparison between the marks by fuzzy evaluation methods and the marks by the traditional method.

marks below the mean 76, their values are larger than the marks by the traditional method; For fuzzy marks too above the mean 76, their values are smaller than the marks by the traditional method. Both the fuzzy marks and the marks by the traditional method have the same mean. However, the standard deviation of the fuzzy marks is smaller than that of the marks by the traditional method, which indicates that the fuzzy marks are clustered more closely. Meanwhile, the fuzzy marks are obtained based on the optimism index of each teacher to each question, which could reduce the impact of subjectivity in the process of grading.

Considering that the fuzzy marks obtained in Module 2 are calculated based on the optimism index, it will inevitably make some marks strict or lenient. Thus we need to transform the strict-type or lenient-type marks to normal-type marks in Module 3. For answer sheets A_1, A_2 and A_4 , their transformed marks are smaller than their fuzzy marks. This is because there are more lenient-type marks than strict-type marks in these answer sheets. However, although there are also more lenient-type marks than strict-type marks in answer sheets A_3, A_5 and A_7 , their transformed marks are larger than their fuzzy marks. This is because their lenient-type marks distribute intensively but their strict-type marks distribute sparsely. Thus the transformed strict-type marks have greater impact on the mean. For answer sheets

A_6, A_8, A_9 and A_{10} , their transformed marks are equal to their fuzzy marks. This means that the change of their lenient-type marks and strict-type marks is balanced. In this sense, transforming students' fuzzy marks to normal-type marks through Module 3 is reliable. It is shown that the means of transformed marks and fuzzy marks are the same but the standard deviation of the transformed marks is larger than that of the fuzzy marks. In this module, although the values of fuzzy marks are scattered after being transformed, the transformed marks have considered teachers' influences on students' marks, which could solve the subjective judging problem of teachers.

The marks given in the above-mentioned modules are obtained totally from teachers' perspective without considering the effect of the quality of answer sheets on students' marks. Thus we need to consider the influence of answer sheets on students' marks in Module 4. Four factors from answer sheets are applied to adjust the fuzzy marks, i.e., the difficulty, the discrimination index, the answer cost, and the answer weight. In this module, the adjusted marks of all answer sheets are smaller than their fuzzy marks. This is because questions Q_1, Q_2 and Q_3 have high grading rate among all questions, but the marks of them are reduced after being adjusted. It is shown that the standard deviation of the adjusted marks is smaller than that of the fuzzy marks, which indicates that the fuzzy marks may be clustered more closely around the means after being adjusted.

In order to combine the transformed marks and the adjusted marks obtained in Modules 3 and 4, we apply a Mamdani type fuzzy inference system in Modules 5 to combine these two types of marks to final marks through the fuzzy tool box of MATLAB instead of solving the weighted mean of them. The superiority of fuzzy inference approach over traditional average methodology is that it may provide an additional advantage of allocating different weigh to each attribute according to needs and requirements of the organization through fuzzy inference rules. The standard deviation of the final marks is smaller than those of the transformed marks and adjusted marks, which indicates that the combination of transformed marks and adjusted marks through fuzzy inference system may make the distribution of marks more centralized. It is found that there is a significant difference between the final marks and traditional marks. All the marks by the traditional method above the mean 76 have decreased. For the marks by the traditional method below the mean, the marks of some answer sheets have decreased, those of some have increased while there is no change in some of them. What makes this change is that the fuzzy marks have decreased in the process of adjustment

in Module 4. The standard deviation of the final marks is smaller than that of the marks by the traditional method, which indicates that the proposed fuzzy educational grading system has an advantage to make the distribution of marks more centralized over the traditional method.

Now we compare the fuzzy evaluation method in students' learning achievement evaluation with the traditional method to explain why we use the fuzzy evaluation method in our work in Table 10. Then we compare the existing fuzzy evaluation methods in students' evaluation with our proposed fuzzy educational grading system to show the advantages of our work in Tables 11 and 12.

Table 10: A comparison between the traditional method and the fuzzy evaluation method

The traditional method	The fuzzy evaluation method
1. For the answer sheet contains narrative responses, it is directly evaluated by the subjective judgment of the teacher.	1. Fuzzy evaluation method includes the potential of modeling linguistic variables and hence can provide a mechanism for capturing the uncertainty introduced by subjectivity.
2. It is not easy to remove the existing subjectivity from the evaluation, especially when the answer sheet is highly subjective by nature.	2. It can reduce the impact of subjectivity in the process of grading.
3. The experiences of teachers are not considered in the process of grading.	3. The fuzzy logic and technologies are developed in view of human intelligence and behavior, and thus have the ability to consider teachers' views and experiences.
4. The multiple teachers have to gather for the evaluation of the skills of the presentations of students.	4. The differences of views and personal mistakes of teachers can be eliminated in the process of grading.

5. Conclusions

The evaluation of students' learning achievement is significant in the process of education. Before the proposal of fuzzy theory, the answer sheets were evaluated by the traditional method, which was simple and direct. However, it ignored some important information: (1) Each student's mark was directly evaluated by the subjective judgment of teachers without considering that some questions were neither correct nor incorrect, which could only be evaluated by the mark of linguistic data. (2) The mark of an answer sheet evaluated by some teacher was calculated by directly accumulating the marks of all questions without considering the optimism index of the teacher to each question in the process of grading. (3) The total mark of each answer sheet was obtained from taking the weighted mean of all marks given by all teachers without considering the types of teachers. (4) The number of students getting the same mark might be high, where the knowledge levels, skills sets of two students were really different. How to consider these factors and remove these differences was not considered in the existing literature. Several fuzzy educational grading system have been proposed to consider one or more of the four issues mentioned above. However, there is no existing fuzzy educational grading system that considers all of the four issues.

This study proposed a new fuzzy educational grading system considering the four above-mentioned issues in five interrelated modules. In the first module, the optimism index of each teacher to each question was determined. In the second module, a fuzzy evaluation method was proposed to calculate each student's mark based on the optimism index given in Module 1. In the third module, the fuzzy marks obtained in Module 2 was transformed into normal-type marks according to the classification of teachers. In the fourth module, the fuzzy marks obtained in Module 2 was adjusted through applying four factors from answer sheets in the adjustment. In the last module, the transformed marks and adjusted marks respectively obtained in Modules 3 and 4 were combined to final marks through a Mamdani type fuzzy inference system in the fuzzy tool box of MATLAB.

The main advantage of the proposed fuzzy educational grading system is that the evaluation resources are used efficiently and effectively in the system. It has the transparency, objectivity, logical inference, and easy computer implementation, which may provide a useful way to automatically evaluate students' achievement in a more objective and more intelligent manner. The main limitation of this system is that it requires domain experts to decide

the fuzzy inference rules used for fuzzy inference in Modules 4 and 5. How to guarantee the objectivity in the process of deciding the fuzzy inference rules is a challenging task for domain experts.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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Table 11: A comparison between the existing fuzzy evaluation methods and our proposed fuzzy educational grading system

The existing fuzzy evaluation methods	Our fuzzy educational grading system
1. The existing fuzzy evaluation methods were classified into two groups: methods applying fuzzy inference and methods applying fuzzy arithmetic.	1. Both fuzzy inference and fuzzy arithmetic are applied in the proposed five interrelated modules.
2. Although the optimism index of each teacher to each question has been considered in some studies [5, 13], it was determined subjectively by teachers.	2. In Module 2, nine satisfaction levels are given to evaluate each question based on the optimism index of each teacher to each question which is determined by the traditional method in Module 1. In this sense, the proposed approach unifies the traditional method and the fuzzy evaluation method. The obtained optimism index can be stored in the database for the next use.
3. The teachers participated in an evaluation were classified into three types based on the given mark matrix. The marks were transformed based on the classification of teachers [2]. In this method, only teachers' influences on marks were considered.	3. In Module 3, teachers participated in the evaluation are classified into three types based on the optimism index given in Module 1. The fuzzy marks obtained in Module 2 are transformed through the Lagrange interpolation technique and then transformed marks are obtained.
4. To consider the effect of the quality of answer sheets on marks, four factors from teachers and answer sheets were applied to adjust the original marks, i.e., the accuracy rate, the time rate, the complexity, and the importance in some studies [1, 7, 11]. However, there was no formalized way to determine the complexity, the importance, and the time rate of each question.	4. For avoiding the problems of determining the complexity, the importance, and the time rate, we apply four new factors to replace the old ones in Module 4. They are the difficulty, the discrimination index, the answer cost, and the answer weight, which can be directly determined by the information given in Module 1.

Table 12: A comparison between the existing fuzzy evaluation methods and our proposed fuzzy educational grading system

The existing fuzzy evaluation methods	Our fuzzy educational grading system
<p>5. According to Figure 1, we know that there were three nodes in some existing mark adjustment systems. In the first node, the difficulty was obtained through the fuzzy inference between the accuracy rate and the time rate. In the second node, the answer cost was obtained through the fuzzy inference between the difficulty and the complexity, which was subjectively determined by the teacher. In the third node, the adjustment value was obtained through the fuzzy inference between the answer cost and the importance, which was subjectively determined by the teacher.</p> <p>6. The effect of teachers and the effect of answer sheets to final marks were considered separately in different studies [1, 2, 5, 7, 11, 13].</p>	<p>5. According to Figure 2, we know that there are two nodes in our proposed mark adjustment system. In the first node, the difficulty is directly calculated through its calculation formula Equation (3). We use the discrimination index, which is directly calculated through its calculation formula Equation (4) to evaluate the complexity of the answer sheet. Then the answer cost is obtained through the fuzzy inference between the difficulty and the discrimination index. Thus the problem that should be solved through two nodes in the original system can be solved in only one node in the proposed system. In the second node, the adjustment value is obtained through the fuzzy inference between the answer cost and the answer weight, which is directly calculated through its calculation formula Equation (9). Since all factors are based on known information, the proposed mark adjustment system is objective.</p> <p>6. The transformed marks and the adjusted marks are obtained through considering the effect of teachers and answer sheets to marks in Modules 3 and 4. In Module 5, these two types of marks are combined to final marks through a Mamdani type fuzzy inference system.</p>

Appendix A. 1

Table A.1: Marks of 10 answer sheets evaluated by 8 teachers by the traditional method

		T_1		T_2		T_3		T_4		T_5		T_6		T_7		T_8		S_h
A_1	Q_1	2.5	Q_1	3.0	Q_1	3.5	Q_1	4.0	Q_1	4.5	Q_1	5.0	Q_1	5.0	Q_1	5.0		
	Q_2	4.0	Q_2	4.5	Q_2	5.0	Q_2	6.0	Q_2	6.5	Q_2	7.0	Q_2	7.5	Q_2	7.5		
	Q_3	5.5	Q_3	6.0	Q_3	7.5	Q_3	8.0	Q_3	8.5	Q_3	9.5	Q_3	10.0	Q_3	10.0	40	
	Q_4	6.5	Q_4	7.5	Q_4	8.5	Q_4	9.5	Q_4	10.5	Q_4	11.5	Q_4	12.0	Q_4	12.5		
	Q_5	7.5	Q_5	9.0	Q_5	10.5	Q_5	12.5	Q_5	13.0	Q_5	14.0	Q_5	14.5	Q_5	15.0		
A_2	g_{11}	26	g_{12}	30	g_{13}	35	g_{14}	40	g_{15}	43	g_{16}	47	g_{17}	49	g_{18}	50		
	Q_1	4.5	Q_1	5.0	Q_1	5.5	Q_1	5.5	Q_1	6.0	Q_1	6.5	Q_1	7.0	Q_1	7.0		
	Q_2	5.5	Q_2	7.0	Q_2	7.5	Q_2	8.0	Q_2	8.5	Q_2	9.0	Q_2	10.0	Q_2	10.0		
	Q_3	7.0	Q_3	11.0	Q_3	11.5	Q_3	12.0	Q_3	12.5	Q_3	13.0	Q_3	13.5	Q_3	14.0	54	
	Q_4	9.0	Q_4	11.5	Q_4	12.5	Q_4	12.5	Q_4	13.0	Q_4	15.0	Q_4	16.0	Q_4	18.0		
A_3	Q_5	10.0	Q_5	13.5	Q_5	14.0	Q_5	14.0	Q_5	15.0	Q_5	16.5	Q_5	16.5	Q_5	18.0		
	g_{11}	36	g_{12}	48	g_{13}	51	g_{14}	52	g_{15}	55	g_{16}	60	g_{17}	63	g_{18}	67		
	Q_1	6.0	Q_1	6.0	Q_1	6.0	Q_1	6.5	Q_1	6.5	Q_1	7.0	Q_1	7.0	Q_1	8.5		
	Q_2	9.5	Q_2	9.5	Q_2	9.5	Q_2	9.5	Q_2	10.0	Q_2	10.5	Q_2	11.0	Q_2	11.5		
	Q_3	11.0	Q_3	12.5	Q_3	13.0	Q_3	13.0	Q_3	13.5	Q_3	13.5	Q_3	14.0	Q_3	15.0	70	
A_4	Q_4	16.0	Q_4	16.5	Q_4	16.5	Q_4	16.5	Q_4	17.5	Q_4	18.5	Q_4	19.0	Q_4	19.5		
	Q_5	19.5	Q_5	21.5	Q_5	22.0	Q_5	22.5	Q_5	22.5	Q_5	23.5	Q_5	24.0	Q_5	24.5		
	g_{11}	62	g_{12}	66	g_{13}	67	g_{14}	68	g_{15}	70	g_{16}	73	g_{17}	75	g_{18}	79		
	Q_1	6.0	Q_1	6.0	Q_1	7.0	Q_1	7.0	Q_1	7.5	Q_1	8.0	Q_1	8.0	Q_1	8.5		
	Q_2	9.0	Q_2	9.0	Q_2	10.0	Q_2	10.5	Q_2	10.5	Q_2	11.0	Q_2	11.5	Q_2	12.5		
A_5	Q_3	12.5	Q_3	13.0	Q_3	14.0	Q_3	15.5	Q_3	17.0	Q_3	17.5	Q_3	19.5	Q_3	19.5	73	
	Q_4	13.5	Q_4	16.0	Q_4	17.0	Q_4	19.5	Q_4	20.0	Q_4	20.0	Q_4	21.0	Q_4	21.0		
	Q_5	19.0	Q_5	19.0	Q_5	20.0	Q_5	20.5	Q_5	2.0	Q_5	21.5	Q_5	22.0	Q_5	22.5		
	g_{11}	60	g_{12}	63	g_{13}	68	g_{14}	73	g_{15}	76	g_{16}	78	g_{17}	82	g_{18}	84		
	Q_1	7.5	Q_1	7.5	Q_1	8.0	Q_1	8.0	Q_1	8.5	Q_1	9.5	Q_1	10.0	Q_1	10.0		
A_5	Q_2	11.0	Q_2	11.5	Q_2	12.0	Q_2	12.0	Q_2	13.5	Q_2	13.5	Q_2	14.0	Q_2	14.5		
	Q_3	14.0	Q_3	14.5	Q_3	14.5	Q_3	15.0	Q_3	15.5	Q_3	16.0	Q_3	16.5	Q_3	18.0	82	
	Q_4	18.5	Q_4	20.0	Q_4	20.5	Q_4	21.0	Q_4	22.5	Q_4	23.0	Q_4	23.5	Q_4	24.0		
	Q_5	22.0	Q_5	21.5	Q_5	22.0	Q_5	23.0	Q_5	24.0	Q_5	25.0	Q_5	25.0	Q_5	25.5		
	g_{11}	73	g_{12}	75	g_{13}	77	g_{14}	79	g_{15}	84	g_{16}	87	g_{17}	89	g_{18}	92		

	T_1		T_2		T_3		T_4		T_5		T_6		T_7		T_8		S_h
A_6	Q_1	7.5	Q_1	7.5	Q_1	8.0	Q_1	8.5	Q_1	8.5	Q_1	9.0	Q_1	9.0	Q_1	9.5	
	Q_2	11.5	Q_2	12.5	Q_2	12.5	Q_2	13.0	Q_2	13.5	Q_2	13.5	Q_2	14.0	Q_2	14.5	
	Q_3	15.0	Q_3	15.5	Q_3	16.0	Q_3	16.5	Q_3	17.0	Q_3	17.0	Q_3	18.5	Q_3	19.0	83
	Q_4	19.0	Q_4	19.5	Q_4	19.5	Q_4	21.0	Q_4	21.5	Q_4	23.0	Q_4	23.5	Q_4	24.0	
	Q_5	21.0	Q_5	22.0	Q_5	23.0	Q_5	23.0	Q_5	23.5	Q_5	23.5	Q_5	25.0	Q_5	25.0	
	g_{11}	74	g_{12}	77	g_{13}	79	g_{14}	82	g_{15}	84	g_{16}	86	g_{17}	90	g_{18}	92	
A_7	Q_1	8.0	Q_1	8.0	Q_1	8.0	Q_1	8.5	Q_1	8.5	Q_1	9.0	Q_1	10.0	Q_1	10.0	
	Q_2	12.5	Q_2	13.0	Q_2	14.0	Q_2	14.0	Q_2	14.5	Q_2	15.0	Q_2	15.0	Q_2	15.0	
	Q_3	16.0	Q_3	17.0	Q_3	17.5	Q_3	17.5	Q_3	17.5	Q_3	17.5	Q_3	18.5	Q_3	19.0	86
	Q_4	19.0	Q_4	19.0	Q_4	19.5	Q_4	20.5	Q_4	21.0	Q_4	22.0	Q_4	22.5	Q_4	23.0	
	Q_5	21.5	Q_5	24.0	Q_5	24.0	Q_5	24.5	Q_5	25.5	Q_5	25.5	Q_5	26.0	Q_5	27.0	
	g_{11}	77	g_{12}	81	g_{13}	83	g_{14}	85	g_{15}	87	g_{16}	89	g_{17}	92	g_{18}	94	
A_8	Q_1	8.0	Q_1	8.5	Q_1	8.5	Q_1	9.0	Q_1	10.0	Q_1	10.0	Q_1	10.0	Q_1	10.0	
	Q_2	12.5	Q_2	12.5	Q_2	13.0	Q_2	13.0	Q_2	13.5	Q_2	13.5	Q_2	14.0	Q_2	14.5	
	Q_3	16.5	Q_3	17.0	Q_3	17.0	Q_3	17.0	Q_3	18.0	Q_3	18.0	Q_3	18.0	Q_3	18.5	88
	Q_4	20.5	Q_4	21.5	Q_4	22.5	Q_4	23.0	Q_4	23.0	Q_4	23.5	Q_4	24.0	Q_4	23.0	
	Q_5	23.5	Q_5	24.5	Q_5	25.0	Q_5	25.0	Q_5	25.5	Q_5	26.0	Q_5	26.0	Q_5	27.0	
	g_{11}	81	g_{12}	84	g_{13}	86	g_{14}	87	g_{15}	90	g_{16}	91	g_{17}	92	g_{18}	93	
A_9	Q_1	8.5	Q_1	8.5	Q_1	8.5	Q_1	8.5	Q_1	9.0	Q_1	9.5	Q_1	10.0	Q_1	10.0	
	Q_2	12.0	Q_2	12.5	Q_2	13.0	Q_2	13.0	Q_2	14.0	Q_2	14.0	Q_2	14.0	Q_2	14.0	
	Q_3	16.5	Q_3	17.0	Q_3	17.0	Q_3	17.0	Q_3	18.0	Q_3	18.5	Q_3	18.5	Q_3	20.0	90
	Q_4	21.0	Q_4	22.0	Q_4	22.5	Q_4	23.0	Q_4	24.0	Q_4	24.0	Q_4	24.5	Q_4	25.0	
	Q_5	25.0	Q_5	26.0	Q_5	26.0	Q_5	26.5	Q_5	27.0	Q_5	27.0	Q_5	27.0	Q_5	28.0	
	g_{11}	83	g_{12}	86	g_{13}	87	g_{14}	88	g_{15}	92	g_{16}	93	g_{17}	94	g_{18}	97	
A_{10}	Q_1	9.0	Q_1	10.0	Q_1	10.0	Q_1	10.0	Q_1	10.0	Q_1	10.0	Q_1	10.0	Q_1	10.0	
	Q_2	13.0	Q_2	14.0	Q_2	14.5	Q_2	14.5	Q_2	15.0	Q_2	15.0	Q_2	15.0	Q_2	15.0	
	Q_3	16.0	Q_3	17.5	Q_3	17.5	Q_3	19.0	Q_3	19.0	Q_3	20.0	Q_3	20.0	Q_3	20.0	95
	Q_4	22.5	Q_4	23.0	Q_4	23.5	Q_4	24.0	Q_4	24.0	Q_4	25.0	Q_4	25.0	Q_4	25.0	
	Q_5	25.5	Q_5	27.5	Q_5	27.5	Q_5	27.5	Q_5	28.0	Q_5	28.0	Q_5	30.0	Q_5	30.0	
	g_{11}	86	g_{12}	92	g_{13}	93	g_{14}	95	g_{15}	96	g_{16}	98	g_{17}	100	g_{18}	100	

Table A.2: The expected fuzzy value of each satisfaction level

Question No.	Satisfaction levels evaluated by T_1								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0.910	0.810	0.710	0.610	0.510	0.410	0.250	0.100	0
Q_2	0.910	0.810	0.710	0.610	0.510	0.410	0.250	0.100	0
Q_3	0.910	0.810	0.710	0.610	0.510	0.410	0.250	0.100	0
Q_4	0.910	0.810	0.710	0.610	0.510	0.410	0.250	0.100	0
Q_5	0.910	0.810	0.710	0.610	0.510	0.410	0.250	0.100	0
Question No.	Satisfaction levels evaluated by T_2								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0.921	0.821	0.721	0.621	0.521	0.421	0.268	0.117	0.011
Q_2	0.927	0.827	0.727	0.627	0.527	0.427	0.279	0.127	0.017
Q_3	0.933	0.833	0.733	0.633	0.533	0.433	0.288	0.136	0.023
Q_4	0.930	0.830	0.730	0.630	0.530	0.430	0.283	0.131	0.020
Q_5	0.936	0.836	0.736	0.636	0.536	0.436	0.294	0.141	0.026
Question No.	Satisfaction levels evaluated by T_3								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0.934	0.834	0.734	0.634	0.534	0.434	0.289	0.137	0.024
Q_2	0.943	0.843	0.743	0.643	0.543	0.443	0.305	0.152	0.033
Q_3	0.942	0.842	0.742	0.642	0.542	0.442	0.304	0.150	0.032
Q_4	0.941	0.841	0.741	0.641	0.541	0.441	0.302	0.148	0.031
Q_5	0.947	0.847	0.747	0.647	0.547	0.447	0.311	0.157	0.037
Question No.	Satisfaction levels evaluated by T_4								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0.944	0.844	0.744	0.644	0.544	0.444	0.307	0.153	0.034
Q_2	0.951	0.851	0.751	0.651	0.551	0.451	0.318	0.164	0.041
Q_3	0.953	0.853	0.753	0.653	0.553	0.453	0.322	0.167	0.043
Q_4	0.955	0.855	0.755	0.655	0.555	0.455	0.326	0.171	0.045
Q_5	0.956	0.856	0.756	0.656	0.556	0.456	0.327	0.171	0.046

Question No.	Satisfaction levels evaluated by T_5								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0.959	0.859	0.759	0.659	0.559	0.459	0.332	0.177	0.049
Q_2	0.970	0.870	0.770	0.670	0.570	0.470	0.350	0.193	0.060
Q_3	0.965	0.865	0.765	0.665	0.565	0.465	0.342	0.186	0.055
Q_4	0.967	0.867	0.767	0.667	0.567	0.467	0.345	0.189	0.057
Q_5	0.967	0.867	0.767	0.667	0.567	0.467	0.345	0.189	0.057
Question No.	Satisfaction levels evaluated by T_6								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0.979	0.879	0.779	0.679	0.579	0.479	0.364	0.207	0.069
Q_2	0.978	0.878	0.778	0.678	0.578	0.478	0.363	0.206	0.068
Q_3	0.974	0.874	0.774	0.674	0.574	0.474	0.356	0.199	0.064
Q_4	0.983	0.883	0.783	0.683	0.583	0.483	0.371	0.213	0.073
Q_5	0.978	0.878	0.778	0.678	0.578	0.478	0.363	0.205	0.068
Question No.	Satisfaction levels evaluated by T_7								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0.989	0.889	0.789	0.689	0.589	0.489	0.382	0.223	0.079
Q_2	0.991	0.891	0.791	0.691	0.591	0.491	0.384	0.225	0.081
Q_3	0.987	0.887	0.787	0.687	0.587	0.487	0.379	0.220	0.077
Q_4	0.993	0.893	0.793	0.693	0.593	0.493	0.388	0.229	0.083
Q_5	0.988	0.888	0.788	0.688	0.588	0.488	0.380	0.221	0.078
Question No.	Satisfaction levels evaluated by T_8								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.240	0.090
Q_2	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.240	0.090
Q_3	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.240	0.090
Q_4	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.240	0.090
Q_5	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.240	0.090

Table A.3: The satisfaction degree of each question evaluated by each teacher in A_1

Question No.	Satisfaction levels evaluated by T_1								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[1, 1]	[0.9, 1]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.1, 0.2]	[1, 1]	[0.8, 0.9]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[1, 1]	[0.7, 0.8]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.1, 0.2]	[1, 1]	[0.9, 1]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[1, 1]	[0.9, 1]	[0, 0]
Question No.	Satisfaction levels evaluated by T_2								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.3, 0.4]	[1, 1]	[0.6, 0.7]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.3, 0.4]	[1, 1]	[0.6, 0.7]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.3, 0.4]	[1, 1]	[0.6, 0.7]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.3, 0.4]	[1, 1]	[0.6, 0.7]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.3, 0.4]	[1, 1]	[0.6, 0.7]	[0, 0]
Question No.	Satisfaction levels evaluated by T_3								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.6, 0.7]	[1, 1]	[0.2, 0.3]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.5, 0.6]	[1, 1]	[0.4, 0.5]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.8, 0.9]	[1, 1]	[0, 0]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.6, 0.7]	[1, 1]	[0.3, 0.4]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.6, 0.7]	[1, 1]	[0.2, 0.3]	[0, 0]
Question No.	Satisfaction levels evaluated by T_4								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.8, 0.9]	[1, 1]	[0, 0]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.1, 0.2]	[1, 1]	[0.8, 0.9]	[0, 0]	[0, 0]

Question No.	Satisfaction levels evaluated by T_5								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.4, 0.5]	[1, 1]	[0.5, 0.6]	[0, 0]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[1, 1]	[0.7, 0.8]	[0, 0]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[1, 1]	[0.7, 0.8]	[0, 0]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.1, 0.2]	[1, 1]	[0.8, 0.9]	[0, 0]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[1, 1]	[0.7, 0.8]	[0, 0]	[0, 0]
Question No.	Satisfaction levels evaluated by T_6								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.7, 0.8]	[1, 1]	[0.2, 0.3]	[0, 0]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.8, 0.9]	[1, 1]	[0.1, 0.2]	[0, 0]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.6, 0.7]	[1, 1]	[0.3, 0.4]	[0, 0]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.7, 0.8]	[1, 1]	[0.2, 0.3]	[0, 0]	[0, 0]
Question No.	Satisfaction levels evaluated by T_7								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.8, 0.9]	[1, 1]	[0.1, 0.2]	[0, 0]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.8, 0.9]	[1, 1]	[0.1, 0.2]	[0, 0]	[0, 0]
Question No.	Satisfaction levels evaluated by T_8								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]
Q_5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.9, 1]	[1, 1]	[0, 0]	[0, 0]	[0, 0]

Table A.4: The expected fuzzy value of each interval value obtained in Table A.3

Question No.	Satisfaction levels evaluated by T_1								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	0	0	1.00	1.00	0
Q_2	0	0	0	0	0	0.10	1.00	0.80	0
Q_3	0	0	0	0	0	0.20	1.00	0.70	0
Q_4	0	0	0	0	0	0.10	1.00	0.90	0
Q_5	0	0	0	0	0	0	1.00	0.90	0
Question No.	Satisfaction levels evaluated by T_2								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	0	0.31	1.00	0.61	0
Q_2	0	0	0	0	0	0.32	1.00	0.62	0
Q_3	0	0	0	0	0	0.33	1.00	0.63	0
Q_4	0	0	0	0	0	0.32	1.00	0.62	0
Q_5	0	0	0	0	0	0.33	1.00	0.63	0
Question No.	Satisfaction levels evaluated by T_3								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	0	0.63	1.00	0.23	0
Q_2	0	0	0	0	0	0.54	1.00	0.44	0
Q_3	0	0	0	0	0	0.84	1.00	0	0
Q_4	0	0	0	0	0	0.63	1.00	0.33	0
Q_5	0	0	0	0	0	0.64	1.00	0.24	0
Question No.	Satisfaction levels evaluated by T_4								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	0	0.94	1.00	0	0
Q_2	0	0	0	0	0	0.95	1.00	0	0
Q_3	0	0	0	0	0	0.95	1.00	0	0
Q_4	0	0	0	0	0	0.85	1.00	0	0
Q_5	0	0	0	0	0.15	1.00	0.85	0	0

Question No.	Satisfaction levels evaluated by T_5								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	0.45	1.00	0.55	0	0
Q_2	0	0	0	0	0.27	1.00	0.77	0	0
Q_3	0	0	0	0	0.26	1.00	0.76	0	0
Q_4	0	0	0	0	0.16	1.00	0.86	0	0
Q_5	0	0	0	0	0.26	1.00	0.76	0	0
Question No.	Satisfaction levels evaluated by T_6								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	0.98	1.00	0	0	0
Q_2	0	0	0	0	0.78	1.00	0.28	0	0
Q_3	0	0	0	0	0.87	1.00	0.17	0	0
Q_4	0	0	0	0	0.68	1.00	0.38	0	0
Q_5	0	0	0	0	0.78	1.00	0.28	0	0
Question No.	Satisfaction levels evaluated by T_7								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	0.99	1.00	0	0	0
Q_2	0	0	0	0	0.99	1.00	0	0	0
Q_3	0	0	0	0	0.99	1.00	0	0	0
Q_4	0	0	0	0	0.89	1.00	0.19	0	0
Q_5	0	0	0	0	0.89	1.00	0.19	0	0
Question No.	Satisfaction levels evaluated by T_8								
	EG	VG	GD	MG	FR	MB	BD	VB	EB
Q_1	0	0	0	0	1.00	1.00	0	0	0
Q_2	0	0	0	0	1.00	1.00	0	0	0
Q_3	0	0	0	0	1.00	1.00	0	0	0
Q_4	0	0	0	0	1.00	1.00	0	0	0
Q_5	0	0	0	0	1.00	1.00	0	0	0