

Comments and Corrections

Correction to “Information Geometric Approach to Multisensor Estimation Fusion”

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I. INTRODUCTION

This correspondence corrects errors in the geodesic projection (GP) method in [1]. The changes include revising the first formula of Theorem 4 (namely, [1, (20)]) which shows the geodesic distance from a point to a submanifold with a fixed mean of multivariate Gaussian manifold. The proof in [1, App. A] has also been amended carefully. In addition, the formulae including [1, (25)], [1, (35)], and [1, (36)] in the GP method based on the preceding theorem are adjusted accordingly. The details of modifications are presented in Section II.

II. REVISED GEODESIC PROJECTION METHOD

As a remedy for the error in [1, (20)], [1, Theorem 4] is replaced by the following:

Theorem 1 (corrected Theorem 4 of [1]): The geodesic distance from a given point $P = N(\boldsymbol{\mu}_P, \boldsymbol{\Sigma}_P) \in \mathcal{N}^m$ to the submanifold $\mathcal{N}_{\boldsymbol{\mu}}^m$ with a constant mean $\boldsymbol{\mu} \in \mathbb{R}^m$ is given by

$$d(P, \mathcal{N}_{\boldsymbol{\mu}}^m) = \frac{1}{\sqrt{2}} \operatorname{arccosh} \left(1 + (\boldsymbol{\mu} - \boldsymbol{\mu}_P)^T \boldsymbol{\Sigma}_P^{-1} (\boldsymbol{\mu} - \boldsymbol{\mu}_P) \right), \quad (1)$$

and the projection of P onto $\mathcal{N}_{\boldsymbol{\mu}}^m$ is $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with

$$\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_P + \frac{1}{2} (\boldsymbol{\mu} - \boldsymbol{\mu}_P) (\boldsymbol{\mu} - \boldsymbol{\mu}_P)^T. \quad (2)$$

This error stems from a calculation mistake in [1, (74)], which should be corrected as

$$\begin{aligned} \operatorname{tr}(\mathbf{Cosh}(\mathbf{G}\rho)) &= \operatorname{tr} \left(\mathbf{I}_m + \sum_{i=1}^{\infty} \frac{\rho^{2i}}{(2i)!} \mathbf{G}^{2i} \right) \\ &= \operatorname{tr} \left(\mathbf{I}_m + \sum_{i=1}^{\infty} \frac{\rho^{2i}}{(2i)!} \underbrace{(2\mathbf{a}\mathbf{a}^T) \cdots (2\mathbf{a}\mathbf{a}^T)}_i \right) \\ &= \operatorname{tr} \left(\mathbf{I}_m + \sum_{i=1}^{\infty} \frac{(\sqrt{2}\rho)^{2i}}{(2i)!} \mathbf{a}\mathbf{a}^T \right) \end{aligned}$$

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$$\begin{aligned} &= m + \sum_{i=1}^{\infty} \frac{(\sqrt{2}\rho)^{2i}}{(2i)!} \\ &= \cosh(\sqrt{2}\rho) + m - 1, \end{aligned} \quad (3)$$

where $\mathbf{G}^2 = 2\mathbf{a}\mathbf{a}^T$ appeared after [1, (62)] and the normalizing condition $\mathbf{a}^T \mathbf{a} = 1$ was given by [1, (65)]. The article [1] mistakenly treated $\operatorname{tr}(\mathbf{I}_m)$ as a special term of the series $\sum_{i=0}^{\infty} \frac{(\sqrt{2}\rho)^{2i}}{(2i)!}$ for $i = 0$. Further, substituting (3) into [1, (72)] yields the corrected result (1) in Theorem 1.

According to the corrected theorem, some details of the GP method should also be adjusted, by simply replacing some m 's in [1] with 1. Specifically, the objective function in [1, (25)] is replaced by

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\boldsymbol{\mu} \in \mathbb{R}^m} \sum_{i=1}^N \operatorname{arccosh}^2 \left(1 + (\boldsymbol{\mu} - \hat{\mathbf{x}}_i)^T \mathbf{P}_i^{-1} (\boldsymbol{\mu} - \hat{\mathbf{x}}_i) \right), \quad (4)$$

and φ at the beginning of [1, Sec. IV-C] is redefined as $\varphi(x) = \operatorname{arccosh}^2(x + 1)$. Meanwhile, the first- and second-order derivatives of $\varphi(x)$ given in [1, (35)] and [1, (36)] should be revised as

$$\dot{\varphi}(x) = \frac{2 \operatorname{arccosh}(x + 1)}{\sqrt{(x + 1)^2 - 1}}, \quad (5)$$

and

$$\ddot{\varphi}(x) = \frac{2 \left(\sqrt{(x + 1)^2 - 1} - (x + 1) \operatorname{arccosh}(x + 1) \right)}{\left(\sqrt{(x + 1)^2 - 1} \right)^3}, \quad (6)$$

respectively.

Note that the above corrections do not severely affect the structure of the GP method, as can be seen by comparing the original objective function in [1, (25)] with the revised one in (4). For this reason, we have not observed any significant performance difference between the original and corrected GP methods in the numerical examples of [1, Sec. VII]. Specifically, the relative difference

$$\frac{|\operatorname{RMSE}_{\text{ori}} - \operatorname{RMSE}_{\text{cor}}|}{\operatorname{RMSE}_{\text{cor}}}$$

is within 0.73% in all cases, where $\operatorname{RMSE}_{\text{ori}}$ and $\operatorname{RMSE}_{\text{cor}}$ denote the (calculated or simulated) root mean squared errors given by the original and corrected GP methods, respectively. Therefore, for brevity of this correspondence, corrections to the figures in [1, Sec. VII] are not presented.

REFERENCE

- [1] M. Tang, Y. Rong, J. Zhou, and X. R. Li, “Information geometric approach to multisensor estimation fusion,” *IEEE Trans. Signal Process.*, vol. 67, no. 2, pp. 279–292, Jan. 2019.