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To cite this version:

Dimitrios Deslis, Andreas J. Stylianides, Mateja Jamnik. Primary school teachers' mathematical knowledge and views about Lakatos-style proving activity: a latent profile analysis. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04408299

HAL Id: hal-04408299

https://hal.science/hal-04408299v1

Submitted on 21 Jan 2024

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Primary school teachers' mathematical knowledge and views about Lakatos-style proving activity: a latent profile analysis

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Despite the importance of proving across all school years, little is known about primary teachers' knowledge and views about a special form of proving activity described as Lakatos-style and associated with authentic engagement with mathematics even in primary school. We aimed to identify profiles of primary teachers based on their mathematical knowledge (of content, students, and teaching practices) and views (related to value, cost, and expectancy) about Lakatos-style proving activity. 331 participants responded to a vignette-based survey with 66 Likert-scale items referring to comic-style classroom episodes. Latent profile analysis revealed six teacher profiles reflecting a wide range of views, various levels of knowledge, and combinations thereof. Future research and teacher education practice can consider the unique characteristics of each profile to provide appropriate support to teachers based on each profile's strengths and needs.

Keywords: Lakatos-style proving activity, conjecture refinement, primary school teachers' mathematical knowledge and views, comic-style vignettes, latent profile analysis

Introduction

Proof has long been established as a central concept in mathematics; more recently it has also been increasingly recognised as an important element of school mathematics (Stylianides & Stylianides, 2009). However, proving activity in school mathematics is scarce and usually limited to inauthentic tasks that prioritise formality over reasoning (Stylianides & Stylianides, 2022). In this paper we focus on the development and refinement of conjectures based on the examination of examples and counterexamples, which is a core aspect of what has been described as Lakatos-style proving activity following Lakatos' (1976) philosophical account of the role of proof in the evolution of mathematical knowledge.

Although Lakatos' account originally belonged to the field of the philosophy, some of its key ideas can be incorporated into school mathematics to involve students in authentic mathematical proving activities (Stylianides et al., 2022). In this way, the zig-zag path among conjectures, arguments, proofs, and refutations which Lakatos identified as an important disciplinary practice can also become part of a teaching and learning practice (Komatsu & Jones, 2022; Stylianides et al., 2022). Findings from studies with students suggest that this can be both feasible and beneficial (e.g., Komatsu & Jones, 2022), even at primary school level (e.g., Reid, 2002). These studies typically analysed the work of students taught by teacher-researchers or experts, and to the best of our knowledge there are no studies with a focus on teachers. Therefore, we still know little about the extent to which ordinary teachers, especially at primary school, are knowledgeable or willing to incorporate Lakatos-style proving activity into their teaching practice. Our study took a step towards addressing this research gap by tackling the following research question:

What are primary school teachers' mathematical knowledge and views about Lakatos-style proving activity, and how can they be described by means of teacher profiles?

Theoretical Frameworks

Based on Lakatos' (1976) theory and considering recent studies about its incorporation into the classroom (e.g., Komatsu & Jones, 2022), we developed *Framework LSI* (*L*akatos-style *I*nvestigation), which is an iterative method of conjecture refinement consisting of four phases (Deslis et al., 2021). After being presented with a proof task, the solvers develop a conjecture (Phase 1: Formulation). They start examining cases and during the exploration supportive examples might be discovered indicating that the conjecture might be true (Phase 2: Validation). However, counterexamples can also emerge and show that it is false (Phase 3: Refutation). Reflecting on the characteristics of the examples they examined, the solvers modify the initial conjecture either by restricting its domain, so that it becomes unaffected by the counterexamples, or by expanding it, so that the counterexamples become supportive examples (Phase 4: Modification). Then, a new investigation cycle begins to further investigate and refine the conjecture (see Figure 1). It is important to note that Lakatos-style investigation should be interpreted as an iterative and reflective process rather than a linear and one-way sequence of steps. LSI was the guiding framework of our study. To study teachers' knowledge and views about it, we developed two additional frameworks: *MaKTeLaP* (*Ma*thematical *K*nowledge for *T*eaching *La*katos-style *P*roving activity) and *VCE* (*V*alue-*C*ost-*E*xpectancy) (see Figure 2).

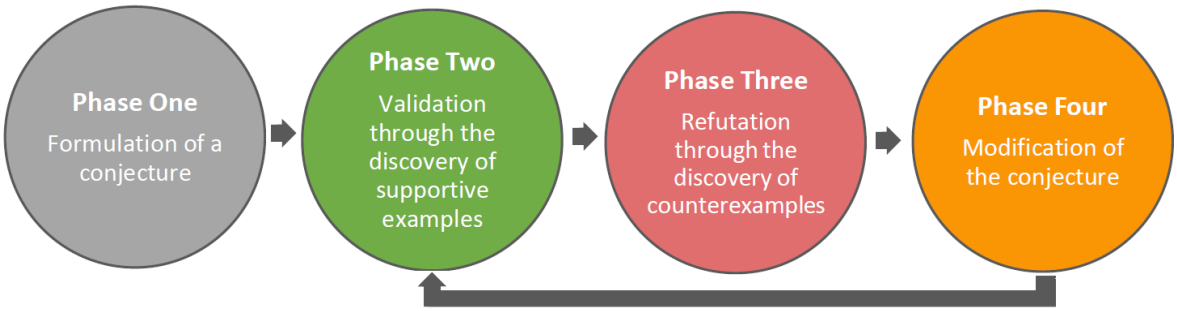

Figure 1: A visualisation of Lakatos-style investigation

With Framework MaKTeLaP we expanded previous theoretical constructs on teachers' generic professional knowledge (Shulman, 1986), and subject-specific professional knowledge, like that required to teach proof (e.g., Buchbinder et al., 2022) to identify three knowledge components of the mathematical knowledge relevant to the instruction of Lakatos-style proving activity (Deslis et al., 2022). (1) *CoLaP* is the *Co*ntent knowledge relevant to *La*katos-style *P*roving activity. It includes the knowledge of how examples can be used to validate or refute conjectures and is demonstrated by the ability to develop valid arguments or judge appropriately the arguments raised by others. (2) *StuLaP* is the knowledge of *Stu*dents' understandings about *La*katos-style *P*roving activity, especially the use of examples in validating and refuting conjectures. It is demonstrated by the ability to anticipate and appropriately analyse students' typical understandings and misconceptions. (3) *TeLaP* is the knowledge of teaching practices to facilitate students' involvement in *La*katos-style *P*roving activity. It is demonstrated by the ability to appropriately respond to students' contributions or arguments and suggest appropriate next steps. As

demonstrated earlier, there are two example types that play a crucial role in all LSI phases: supportive examples and counterexamples. To reflect that, we distinguish two dimensions (SE and CE) within each knowledge component, namely one for each example type (see Figure 2).

We based Framework *VCE* on previous constructs on views and motivation relevant to teaching and learning that have been used in the fields of educational psychology (e.g., Eccles & Wigfield, 2002) and mathematics education (e.g., O'Meara & Fitzmaurice, 2022; Schukajlow et al., 2017). VCE considers three view components, each consisting of three sub-components, that are associated with teachers' motivation to engage with Lakatos-style proving activity. (1) The *V*alue they believe this activity has, in terms of the perceived importance, usefulness and interest; (2) the *C*ost they believe that comes with it, in terms of the perceived cost in time and effort, and emotional cost; (3) their *E*xpectancies, specifically the perceived expected ease, competence and likelihood of success. We hypothesise that teachers consider the value, cost, and expectancy of a task, in this case engagement in Lakatos-style proving activity, both for themselves and their students. For example, they consider the effort it requires not only for them as teachers but also for their students. To reflect that, we have two dimensions (T and S) within each view component (see Figure 2).

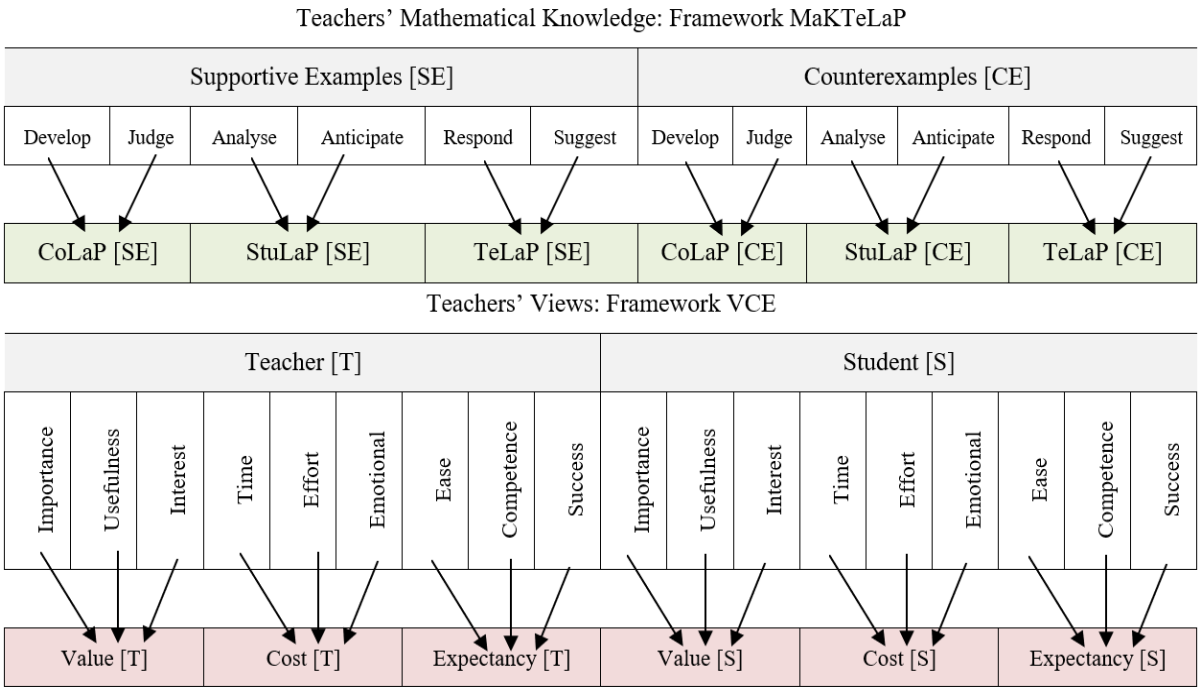

Figure 2: Frameworks MaKTeLaP & VCE- the components of teachers' knowledge and views

Methods

Data Collection

We collected our data through an online survey based on contextualised descriptions of classroom situations in the form of comic-style vignettes (see Figure 3). Our sample consisted of 149 primary teachers and 182 primary student teachers (N=331 participants). At the time of this study the former were teaching at Greek schools spread all over Greece, while the latter were studying at nine different Greek universities. Participants were presented with classroom episodes (adapted

from Zack, 1997 and Reid, 2002) depicting comic-style teacher and student characters discussing the "Count the Squares" problem (Zack, 1997; see Figures 3 and 4). They investigated the following conjecture: "The number of squares in an n-by-n grid is a multiple of five", going through all four LSI phases. All characters' appearance was generic to minimise the possibility of potential biases (e.g., colours resembling ethnicities, clothing etc.) and to enable teachers to identify themselves with the teacher character as well as their students with the student characters.

Figure 3: Two sample comic-style images included in the survey and the proof task that was investigated by the vignette characters

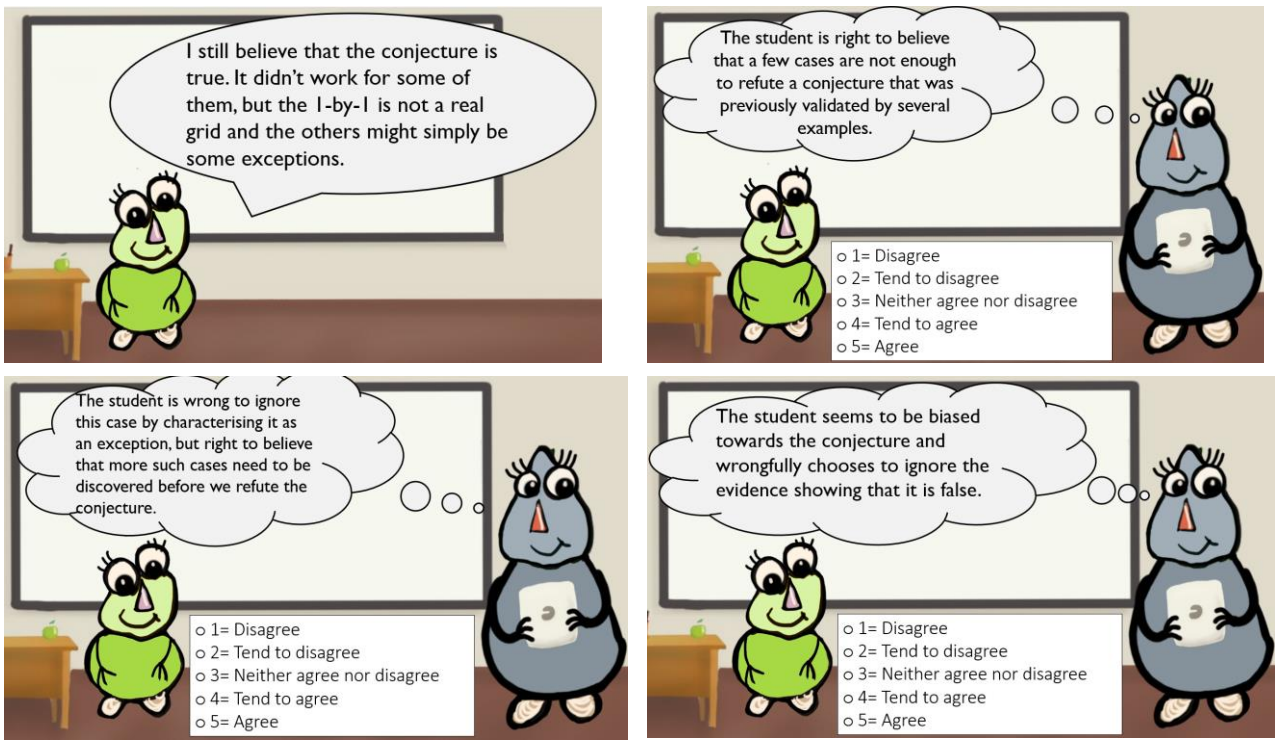

Figure 4: Sample episode and survey items (StuLaP [CE]-Analyse); each item reflects a different level of sophistication for teachers' ability to analyse students' understandings

Participants responded to 66 Likert-scale items (1: disagree- 5: agree). The first 48 items were based on 16 brief comic-style classroom episodes. After each episode teachers were presented with three items presenting reasoning strategies (CoLaP component), student arguments (StuLaP component) or teaching practices (TeLaP component) of different levels of sophistication (see

Figure 4). To identify these levels and design the relevant items we relied on the findings of an interview study on teachers' mathematical knowledge about Lakatos-style proving activity (Deslis et al., 2021), which was based on the same comic-style classroom episodes. We used the responses to these 48 items to calculate scores (0-1; continuous) for their knowledge of content, students, and teaching practices based on Framework MaKTeLaP (see Figure 2). Participants were then asked to reflect on these classroom situations and respond to 18 items (one for each sub-component of VCE, see Figure 2) examining their views about the incorporation of Lakatos-style proving activity into the classroom. We calculated scores (0-1; continuous) for their perceived value, cost, and expectancy views about the students and themselves, as described in Framework VCE.

Data Analysis

Latent profile analysis is a technique of statistical analysis that groups participants with similar response patterns into profiles, ensuring that the dissimilarity of participants between different profiles is maximal. Unlike most traditional similar techniques, latent profile analysis is model-based and can offer rigorous criteria for determining the number of the classes; it also allows partial membership to a class, thus giving richer information than techniques which classify participants as either members or non-members of a class (Pastor et al., 2007). We used the six knowledge and the six view scores (see the coloured rows in Figure 2) as indicator variables. Following the common practice, we tried all possible solutions from 1- to 7-profile models and guided by appropriate statistical criteria (AIC, BIC, SSBIC, CAIC) we identified the six-profile model as the optimal solution, which was also validated as highly accurate by the average latent class posterior probabilities and the entropy statistic (E=0.934).

Findings

In this section we present the six different profiles and their most important distinctive characteristics. As shown in Figure 5, there are profiles with notably higher and lower knowledge scores. We can also observe a wide variation in the view scores of the different profiles.

Profile 1- Moderately Knowledgeable Enthusiasts (n=63) have moderate knowledge scores (.51-.57) but are characterised by an enthusiasm about the incorporation of Lakatos-style proving activity into the classroom which is demonstrated by the high perceived value (T=.88; S=.89), the low perceived cost (T=.32; S=.43), as well as the high expectancies (T=.82; S=.74).

Profile 2- Slightly Knowledgeable Hyper-enthusiasts (n=22) have below average knowledge scores (.34-.42), but they believe that the cost of teaching and learning Lakatos-style proving activity is relatively low (T=.38; S=.52). They have high expectancies (T=.83; S=.77), and what is even more notable is their exceptionally high value scores (T=.97; S=.99).

Profile 3- Moderately Knowledgeable and Neutrally Positioned (n=102) constitute the most sizeable group of participants (nearly 1/3 of the sample). These teachers have moderate knowledge scores (.54-.59). They see a balance between the value of incorporating Lakatos-style proving activity into the classroom (T=.63; S=.65) and the cost that comes with it (T=.60; S=.66). Finally, as demonstrated by their modest expectancies (T=.52; S=.48), they are neither optimistic nor pessimistic about the prospect of incorporating Lakatos-style proving activity into the classroom.

Profile 4- Knowledgeable Enthusiasts (n=67) are the second most sizeable group. This profile is characterised by relatively high knowledge scores in four of the six knowledge scores, three of which are the SE-related components (CoLaP[SE]=.66; StuLaP[SE]=.63; TeLaP[SE]=.67; CoLaP[CE]=.60). In line with their good knowledge scores, they consider the cost for themselves as manageable (.54), but they believe that the cost is considerable for students (.64). Similarly, they have high expectancies for themselves (.72), but their expectancies for the students are more modest (.55). Finally, they believe that engaging with Lakatos-style proving activity would be valuable both for themselves and the students (T=.86; S=.87).

Profile 5- Moderately Knowledgeable Sceptics (n=17) constitute the least sizeable group of participants. Other than the moderate knowledge scores (.51-.55), what characterises this group is the belief that the cost of incorporating of Lakatos-style proving activity into the classroom is too high (T=.55; S=.60) for its value (T=.28; S=.28). Additionally, they have high expectancies for themselves (.61) but not for the students (.38).

Profile 6- Moderately Knowledgeable and Cautious Supporters (n=60) have moderate knowledge scores (.46-.57) and moderate expectancy scores (T=.55; S=.50), but they believe that Lakatos-style proving activity is of high value (T=.89; S=.90). Unlike the other profiles with similarly high value scores, this profile also believes that the incorporation of this activity into the classroom comes with a high cost (T=.79; S=.78).

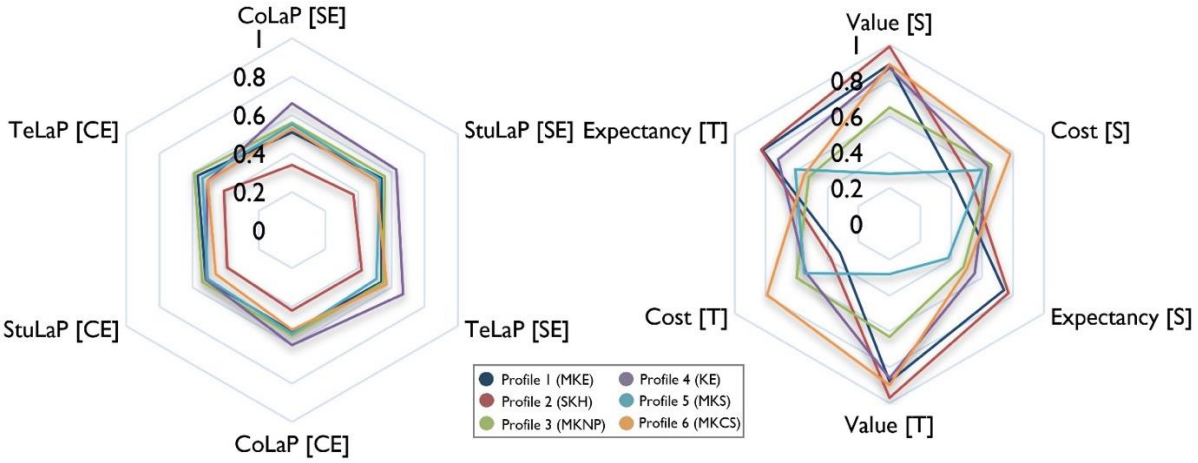

Figure 5: An overview of the average knowledge and view scores of the six teacher profiles (N=331)

Discussion

With this study we extend our previous work on teachers' mathematical knowledge about Lakatos-style proving activity (Deslis et al., 2021; 2022) by considering also teachers' views about it. We analysed 331 primary teachers' responses to a comic-style vignette-based survey and discovered six teacher profiles, each characterised by a unique combination of knowledge and view scores.

The findings can have implications for teacher education practice, enabling the design of instructional interventions tailored to teachers' needs. Despite the potential of interventions to promote the place of Lakatos-style proving activity in the classroom (Stylianides et al., 2022), there are few intervention studies with students (e.g., Komatsu & Jones, 2022) and a lack of

interventions targeted at teachers. Schukajlow et al. (2017) point out that intervention studies in mathematics education rarely pay attention to the important aspect of teachers' views, especially those that are related to their motivation (e.g., O'Meara & Fitzmaurice, 2022). The striking variation in views across the different profiles which was found in our study highlights how crucial the consideration of this aspect can be for the design of effective interventions. Past interventions with students highlighted that helping them see an intellectual need to learn about proof can be crucial in later development of their understandings (Stylianides & Stylianides, 2009). In a similar way, interventions with teachers may start by helping them see why learning about Lakatos-style proving activity is worthwhile, and then provide them with support to develop their relevant mathematical knowledge. This can be particularly relevant to teacher profiles that have reservations or are not as enthusiastic as the other profiles about the incorporation of Lakatos-style proving activity into the classroom (e.g., Profiles 3-MKNP, 5-MKS, 6-MKCS). On the other hand, interventions for teachers who already have a high level of motivation can directly target the development of knowledge. Considering the variation in the knowledge scores, these can again be tailored to the unique needs of each profile. For example, Profiles 1-MKE and 2-SKHE would both need to improve their understandings about all knowledge components, but the latter would probably need a more intensive treatment. Meanwhile, an intervention for teachers of Profile 4-KE could aim explicitly at the knowledge components that have the most room for improvement.

Directions for future research may include surveys with teachers from other countries to investigate whether the findings we report would be replicated. Furthermore, it would be useful to explore if similar profiles would emerge from studies with teachers at different levels of education (e.g., secondary school). Finally, this study considers teachers' relationship with Lakatos-style proving activity from a broad perspective by considering both their knowledge and their views, thus bringing together two important research strands in educational research that are usually examined separately (e.g., Buchbinder et al., 2022 and O'Meara & Fitzmaurice, 2022, respectively). Researchers may find it informative to employ a similar approach to co-examine teachers' knowledge and views about other subjects of interest. In this way, detailed accounts of teachers' relationship with these subjects can be generated which can reliably suggest appropriate ways of offering support to teachers and helping them prepare for teaching.

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